Module 7 Quadrilaterals

Tuesday, April 08, 2025 12:09 AM

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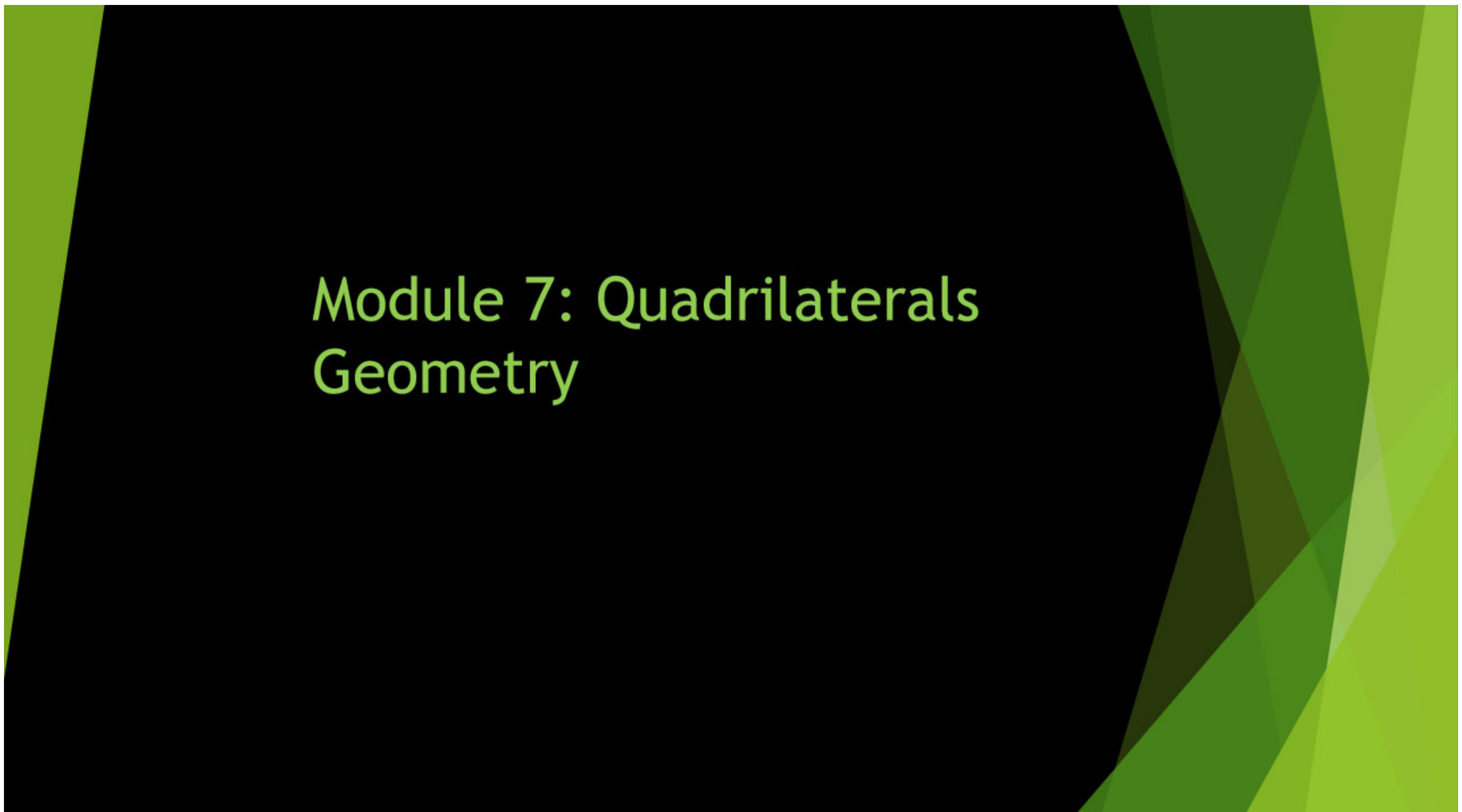
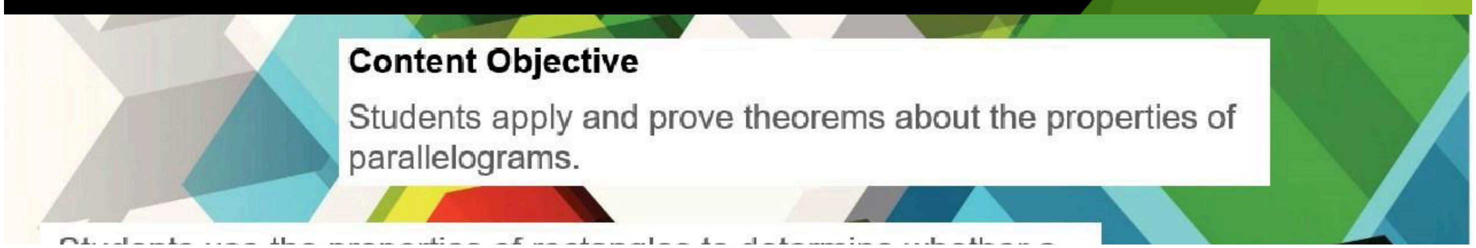
Quadrilater...





Students use the properties of rectangles to determine whether a parallelogram is a rectangle and to write proofs.

Students apply and prove the properties of rhombi and squares.

Students recognize and apply the properties of trapezoids and kites.

MA.912.GR.1.4
Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms.

MA.912.GR. 3.2
Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

MA.912.GR.3.3
Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

MA.912.GR.1.5
Prove relationships and theorems about trapezoids. Solve mathematical and real-world problems involving postulates, relationships and theorems of trapezoids.

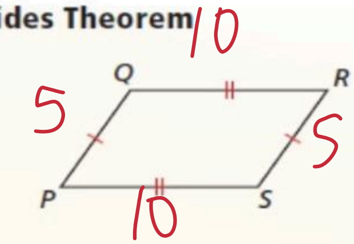
Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 368

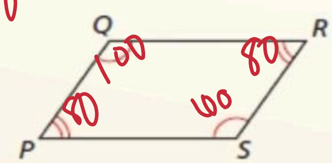


Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent. Quad 360°

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 373



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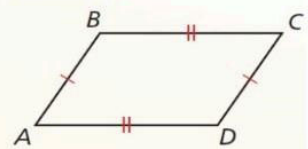
Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

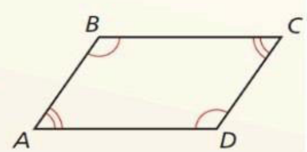


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383



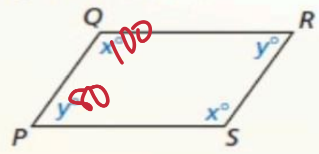
Theorems

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 373

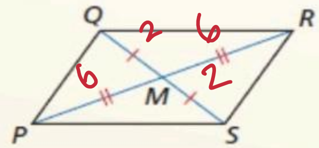


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 370



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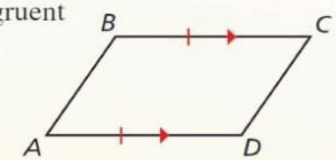
Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

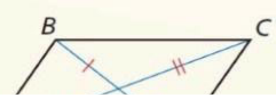
If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof Ex. 40, p. 383



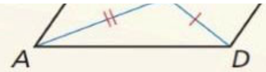
Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



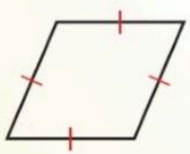
If BD and AC bisect each other, then $ABCD$ is a parallelogram.

Proof Ex. 41, p. 383

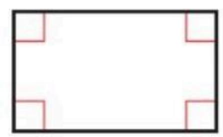


Core Concept

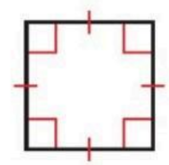
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

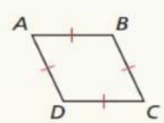
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

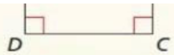
Proof Ex. 81, p. 396



Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if
$\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

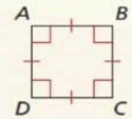


Proof Ex. 82, p. 396

Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if
$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Proof Ex. 83, p. 396

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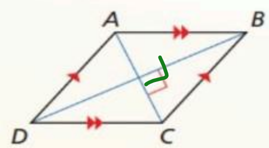
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular. 90°

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

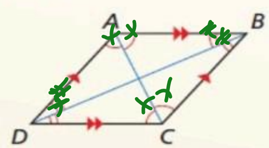


Proof p. 390; Ex. 72, p. 395

Theorem 7.12 Rhombus Opposite Angles Thoerem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.



Proof Exs. 73 and 74, p. 395

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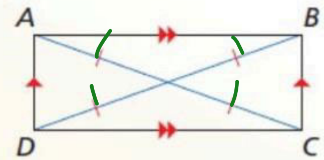
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

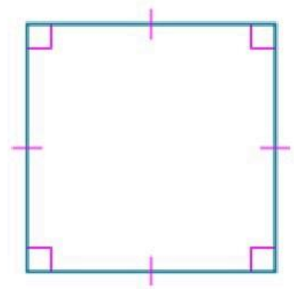
A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



A **square** is a parallelogram with all four sides and all four angles congruent. All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).



Theorems: Conditions for Rhombi and Squares

Theorem 7.17

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a Square, rhombus

Theorem 7.18

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus, Square

Theorem 7.19

If two consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus, Square

Theorem 7.20

If a quadrilateral is both a rectangle and a rhombus, then it is a SQuaRE

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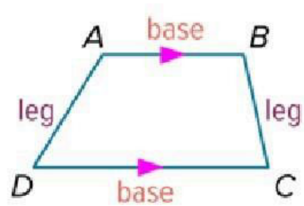
A **trapezoid** is a quadrilateral with at least one pair of parallel sides. In a trapezoid that is not a parallelogram, the parallel sides are called the **bases** and the nonparallel sides are called **legs**.



A **base angle** is formed by a base and a leg. In trapezoid $ABCD$, $\angle A$ and $\angle B$ are one pair of base angles, and $\angle C$ and $\angle D$ are the other pair. If the legs are congruent, then a trapezoid is an **isosceles trapezoid**.

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Properties of an Isosceles Trapezoid

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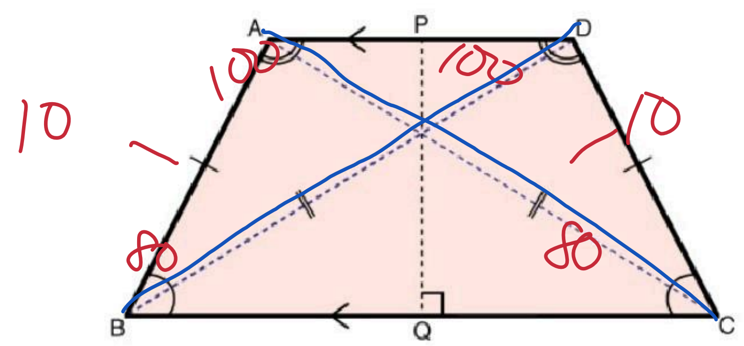

① Has one pair of parallel and unequal opposite sides (bases)

② Has one pair of congruent non-parallel sides (legs)

③ Lower base angles & upper base angles are congruent

④ Diagonals are congruent $DB \cong AC$

P2 ⑤ Any lower base angle is supplementary to any upper base angle

⑥ Has one line of symmetry connecting the bases at their midpoints

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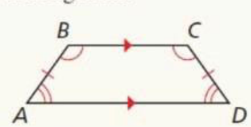
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

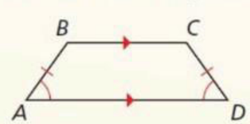


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

Proof Ex. 40, p. 405



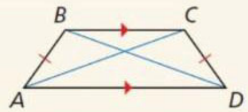
Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



Theorem

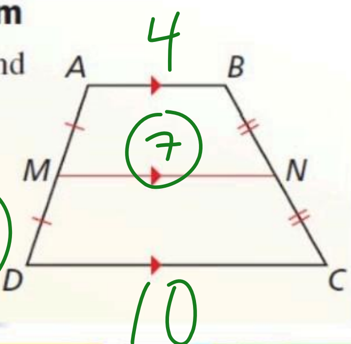
Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

$\frac{1}{2}(4+10) = 7$

Proof Ex. 49, p. 406



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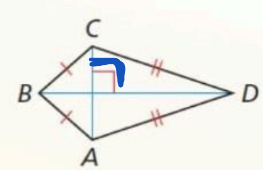
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular. 90°

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

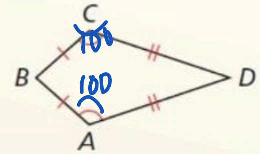


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406



Kites

A **kite** is a convex quadrilateral with exactly two distinct pairs of adjacent congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

Theorems: Kites

Theorem 7.25

If a quadrilateral is a kite, then its diagonals are perpendicular.

Theorem 7.26

If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

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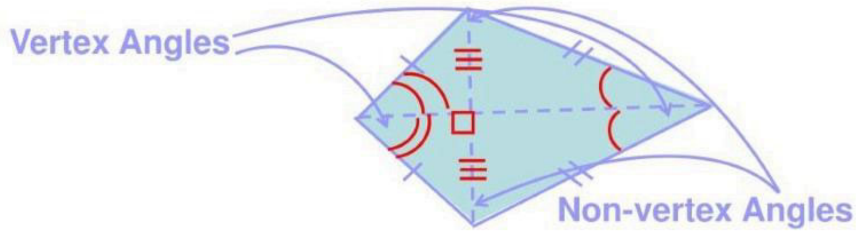
Properties of Kites and Trapezoids

Kite:

- *2 distinct pairs of consecutive congruent sides.*

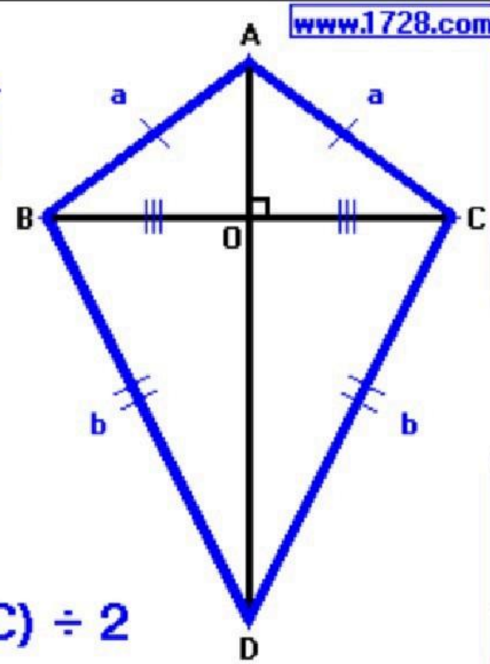
- One diagonal is the ⊥ bisector of the other.
- Non-vertex angles are congruent.
- One diagonal bisects both vertex angles.



∠A and ∠D are vertex angles.
∠B and ∠C are the non-vertex angles.
Lines AD and BC are diagonals and always meet at right angles
Line AD, the axis of symmetry, bisects diagonal BC, bisects ∠A and ∠D and bisects the kite into 2 congruent triangles:
△ABD and △ACD

Side AB = side AC
Side BD = side CD
Line OB = Line OC
Diagonal BC bisects the kite into 2 isoceles triangles

Kite Area = (AD × BC) ÷ 2



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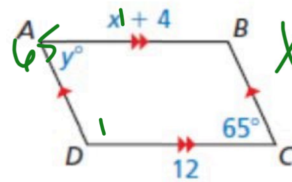
*Find all angles and side measures for all problems on this page!

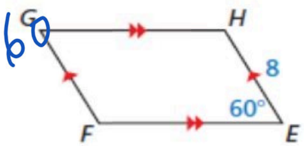
Find the values of x and y.



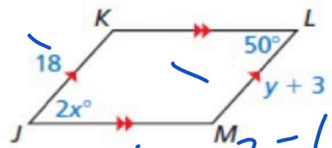
$x+4=12$
$-4 \quad -4$
$x=8$

1. Find FG and $m\angle G$.

8 60



2. Find the values of x and y.



$2x=50$
$2 \quad 2$
$x=25$

$y+3=18$
$-3 \quad -3$
$y=15$

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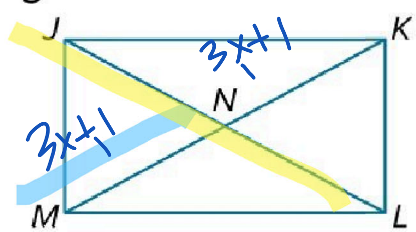
Example 2

Use Properties of Rectangles and Algebra

Check

Quadrilateral *JKLM* is a rectangle.



Part A

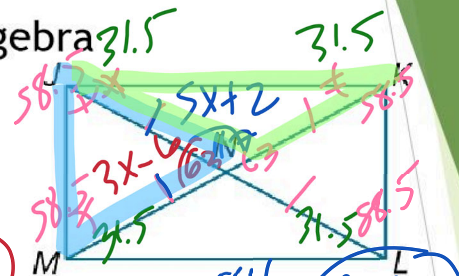
If $MN = 3x + 1$ and $JL = 2x + 9$, find MK. Round to the nearest tenth if necessary.

$6x+2=2x+9$
$-2x \quad -2x$
$4x+2=9$
$-2 \quad -2$
$4x=7$

$4x=7$
$4 \quad 4$
$x=1.75$

Example 2

Use Properties of Rectangles and Algebra



Check

Quadrilateral *JKLM* is a rectangle.

$5x+2+3x-6=180$

$8x-4=180$
$+4 \quad +4$

$\frac{8x}{8}=\frac{184}{8}$

$x=23$

Part B

If $m\angle JNK = (5x + 2)^\circ$ and $m\angle JNM = (3x - 6)^\circ$,

$5(23)+2=117$ $3(23)-6=63$

find $m\angle JNK$ and $m\angle JNM$. **Part C:** Find all angle degrees!

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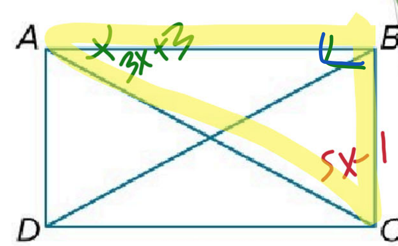
Example 2

180
-90
90



Quadrilateral *ABCD* is a rectangle.

If $m\angle BAC = (3x + 3)^\circ$ and

$m\angle ACB = (5x - 1)^\circ$, find the value of x.

$3x+3+5x-1=90$

$8x+2=90$
$-2 \quad -2$

$\frac{8x}{8}=\frac{88}{8}$

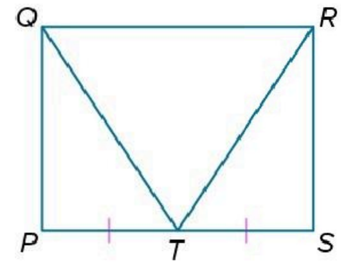
$x=11$

$8x = 88$

Example 3

Prove Rectangular Relationships

Given: $PQRS$ is a rectangle; $\overline{PT} \cong \overline{ST}$.
Prove: $\overline{QT} \cong \overline{RT}$

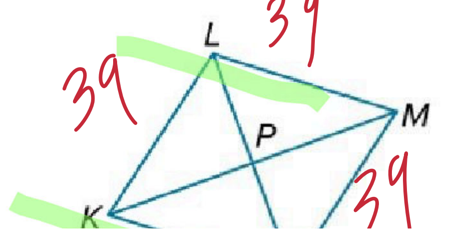


Statements	Reasons
1. $PQRS$ is a rectangle; $\overline{PT} \cong \overline{ST}$	1. Given
2. $PQRS$ is a parallelogram	2. Definition of rectangle
3.	3. Opp. sides of a ▱ are ≅.
4.	4. Definition of rectangle
5. $\angle S \cong \angle P$	5. All right angles are congruent.
6.	6. SAS
7.	7. CPCTC

$2(24)-9$ $\quad$ $24+15$

If $LM = 2x - 9$ and $KN = x + 15$ in rhombus $KLMN$, find the value of x.

Find all side lengths!



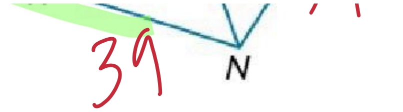
$2x - 9 = x + 15$

$-x \qquad -x$

$x - 9 = 15$

$+9 \quad +9$

$x = 24$



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For what values of x and y is quadrilateral $PQRS$ a parallelogram?

Find the lengths of all the sides.

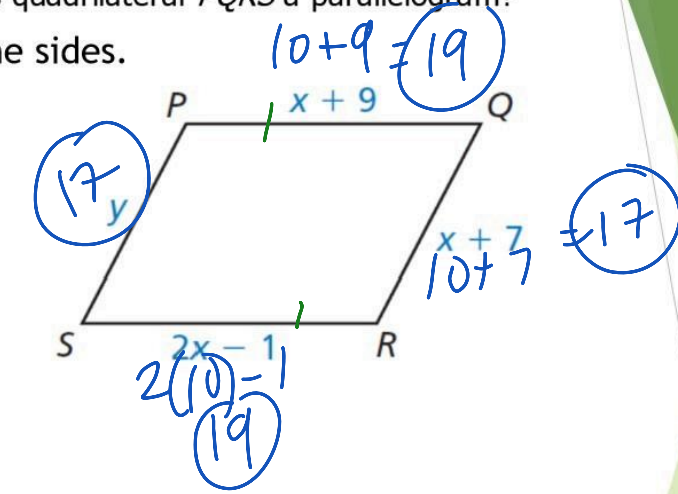
$x + 9 = 2x - 1$

$-x \qquad -x$

$9 = x - 1$

$+1 \qquad +1$

$10 = x$



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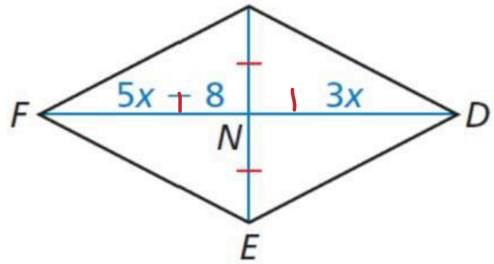
For what value of x is quadrilateral $CDEF$ a parallelogram?

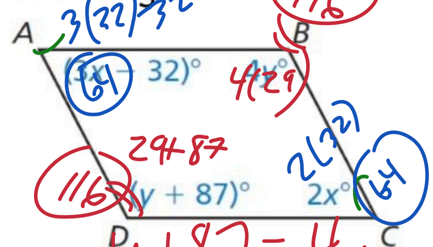
C

$15 \quad \dots = 3x$

$$5x - 8 = 3x$$
$$-5x \qquad -5x$$
$$-8 = -2x$$
$$\frac{-8}{-2} = \frac{-2x}{-2}$$
$$4 = x$$



For what values of x and y is quadrilateral *ABCD* a parallelogram? Determine the measures of each angle.

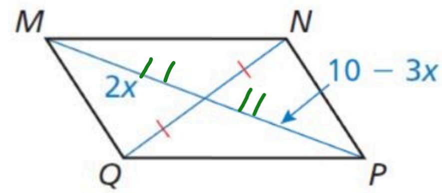


$$3x - 32 = 2x$$
$$-3x \qquad -3x$$
$$-32 = -1x$$
$$\frac{-32}{-1} = \frac{-1x}{-1}$$
$$32 = x$$

$$y + 87 = 4y$$
$$-y \qquad -y$$
$$87 = 3y$$
$$\frac{87}{3} = \frac{3y}{3}$$
$$29 = y$$

For what value of x is quadrilateral *MNPQ* a parallelogram?

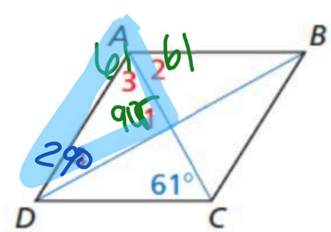
$$2x = 10 - 3x$$
$$+3x \qquad +3x$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$x = 2$$



Find the measures of the numbered angles in rhombus *ABCD*.

$$90 + 61 = 151$$

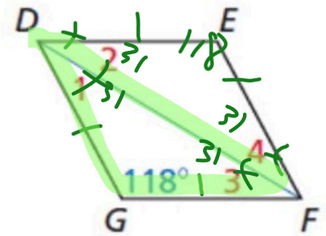
$$\begin{array}{r} 180 \\ -151 \\ \hline 29^\circ \end{array}$$



Find the measures of the numbered angles in rhombus *DEFG*.

$$\begin{array}{r} 180 \\ -118 \\ \hline 62 \end{array}$$

$$\frac{62}{2} = 31$$



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$5(14)-31$ $2(14)+11$

In rectangle *QRST*, *QS* = 5*x* – 31 and *RT* = 2*x* + 11. Find the lengths of the diagonals of *QRST*.

$$5x-31=2x+11$$
$$-2x \qquad -2x$$
$$3x-31=11$$
$$+31 \qquad +31$$
$$\frac{3x}{3}=\frac{42}{3}$$
$$x=14$$