Lesson 7.1 Rational Expressions & Functions

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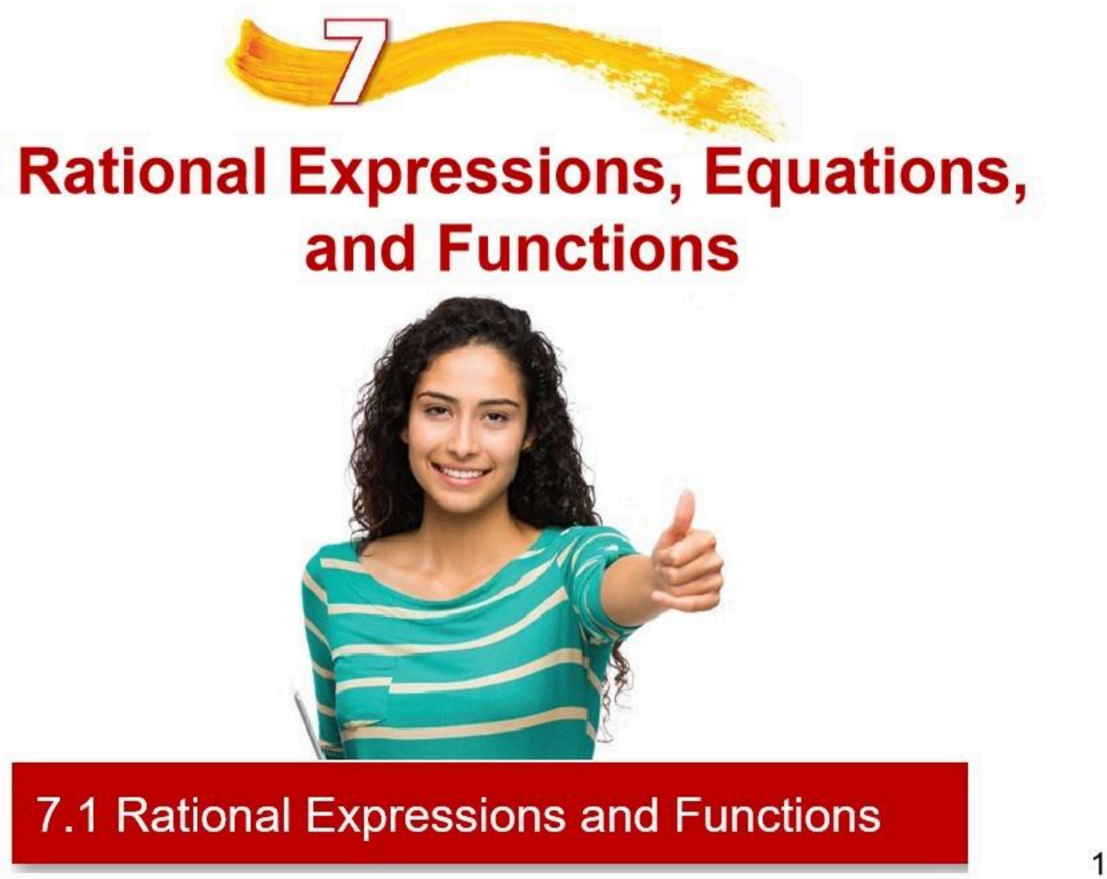
MCA Lesson 7.1



What You Will Learn

- Find the domain of a rational function.
- Simplify rational expressions.
- Use rational expressions to model and solve real-life problems.

Find the domain of each rational function.

$x+7=0$ $\quad x-7=0$

$x=-7$ $\quad x=7$

a. $f(x) = \dfrac{4}{x-2}$ $\quad = \dfrac{4}{0}$ $\qquad$ **b.** $g(x) = \dfrac{8x}{x^2-49}$ $\quad (x+7)(x-7)$

$x \neq 2$ $\qquad$ $x \neq -7$ $\quad$ $x \neq 7$

Solution: $x-2=0$ $\quad x=2$

a. The denominator can not be zero. The denominator is 0 when x-2 = 0. Solve for x.

The domain is all real values of x such that $x \neq \alpha$

b. The denominator is 0 when $x^2 - 49 = 0$. Solving this equation by factoring.

The domain is all real values of x such that $x \neq -7$ and $x \neq 7$

Example 2 – An Application Involving a Restricted Domain 1

You have started a small business that manufactures lamps. The initial investment for the business is \$120,000. The cost of manufacturing each lamp is \$15. So, your total cost of producing x lamps is

$$C = 15x + 120{,}000. \qquad \text{Cost function}$$

Your average cost per lamp depends on the number of lamps produced. For instance, the average cost per lamp $\overline{C}$ of producing 100 lamps is

$$1500 + 120{,}000 = \frac{121{,}500}{100} = 1{,}215$$

$$\overline{C} = \frac{15(100) + 120{,}000}{100} \qquad \text{Substitute 100 for } x.$$

Example 2 – An Application Involving a Restricted Domain 2

Example 2 – An Application Involving a Restricted Domain 2

$$= \frac{121{,}500}{100}$$ Simplify.

= $ 1,215 Average cost per lamp for 100 lamps

The average cost per lamp decreases as the number of lamps increases. For instance, the average cost per lamp $\overline{C}$ of producing 1000 lamps is

$$\overline{C} = \frac{15(1000) + 120{,}000}{1000}$$ Substitute 1000 for x.

Simplify.

= $135 Average cost per lamp for 1000 lamps

Example 2 – An Application Involving a Restricted Domain 3

In general, the average cost of producing *x* lamps is

$$\overline{C} = \frac{15x + 120{,}000}{x}.$$ Average cost per lamp for *x* lamps

Describe the domain of this rational function.

Solution:

If you were considering this function from only a mathematical point of view, you would say that the domain is all real values of point of view, you would say that the domain is all real values of x such that $x \neq 0$. However, because this function is a mathematical model representing a real-life situation, you must decide which values of x make sense in real life. For this model, the variable x represents the number of lamps that you produce.

Example 2 – An Application Involving a Restricted Domain 4

Assuming that you can only produce a whole number of lamps, the domain is the set of positive integers from 1 to some maximum number L, where L depends on practical constraints such as time and resources. That is, the domain is $\{1, 2, 3, \ldots, L\}$.

Example 3 – Simplifying a Rational Expression

Simplify the rational expression $\frac{2x^3 - 6x}{6x^2}$.

$$\frac{\cancel{2x}(x^2-3)}{\cancel{6x} \cdot x} \qquad \frac{x^2-3}{3x}$$

Solution:

First note that the domain of the rational expression is all real values of x such that $x \neq 0$.

$$\frac{3x}{3} = \frac{0}{3} \qquad x = 0 \qquad x \neq 0$$

Then, completely factor both the numerator and denominator.

Factor numerator and denominator.

Example 3 – Simplifying a Rational Expression cont'd

$$= \frac{\cancel{2x}(x^2 - 3)}{\cancel{2x}(3x)}$$ Divide out common factor $2x$.

$$= \frac{x^2 - 3}{3x}$$ Simplified form

In simplified form, the domain of the rational expression is

In simplified form, the domain of the rational expression is the same as that of the original expression—all real values of x such that $x \neq 0$.

Example 4 – Simplifying a Rational Expression

Simplify the rational expression $\frac{x^2 + 2x - 15}{3x - 9}$.

$$\frac{(x+5)\cancel{(x-3)}}{3\cancel{(x-3)}}$$

Solution:

The domain of the rational expression is all real values of x such that $x \neq$.

$$\frac{x+5}{3}$$

Factor numerator and denominator.

Divide out common factor

Simplified form

Example 5 – Simplifying a Rational Expression

Simplify the rational expression $\dfrac{x^3 - 16x}{x^2 - 2x - 8}$.

$$\frac{x(x^2-16)}{}$$

$$\frac{x(x+4)\cancel{(x-4)}}{\cancel{(x-4)}(x+2)}$$

Solution:

The domain of the rational expression is all real values of x such that $x \neq$ and $x \neq$

$$\frac{x(x+4)}{(x+2)}$$

Partially factor.

Factor completely.

$x + 2 = 0$

$x = -2$

Divide out common factor

Simplified form $x \neq -2$

Example 7 – Rational Expressions Involving Two Variables

a. $\dfrac{3xy + y^2}{2y} = \dfrac{\cancel{y}(3x + y)}{2\cancel{y}}$ Factor numerator.

Divide out common factor

$$\frac{3x+y}{2}$$

Simplified form

b. $\dfrac{4x^2y - y^3}{2x^2y - xy^2} = \dfrac{(4x^2 - y^2)y}{(2x - y)xy}$ Partially factor.

$$\frac{y(4x^2 - y^2)}{}$$

Factor completely.

$$xy(2x - y)$$

$$\frac{y(2x+y)(2x-y)}{xy(2x-y)}$$

Divide out common factors

Simplified form

$$\frac{(2x+y)}{x}$$

Example 7 – Rational Expressions Involving Two Variables cont'd

The domain of the original rational expression is all real values of *x* and *y* such that $x \neq 0$, $y \neq 0$, and $y \neq 2x$.

c. $\dfrac{2x^2 + 2xy - 4y^2}{5x^3 - 5xy^2}$

$$2(x^2 + xy - 2y^2)$$

$$\begin{array}{r} 2xy \\ -1xy \\ \hline 1xy \end{array}$$

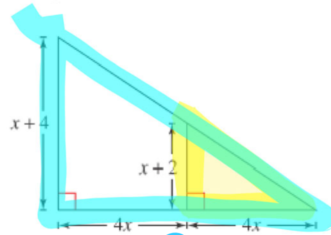
$$\frac{2(x+2y)(x-1y)}{5x(x^2-y^2)}$$

$$5x(x+y)(x-y)$$

$$\frac{2(x+2y)}{5x(x+y)}$$

Example 8 – Geometry: Finding a Ratio 1

Find the ratio of the area of the shaded portion of the triangle to the total area of the triangle.



8x

Solution:

The area of the shaded portion of the triangle is given by

$$\text{Area} = \frac{1}{2}(4x)(x + 2)$$

$2x(x+2)$

$= 2x^2+4$

Example 8 – Geometry: Finding a Ratio 2

The total area of the triangle is given by

$$\text{Area} = \frac{1}{2}(4x + 4x)(x + 4)$$

$(2x+2x)(x+4)$

$(4x)(x+4)$

$4x^2+16x$

Example 8 – Geometry: Finding a Ratio 3

So, the ratio of the area of the shaded portion of the triangle to the total area of the triangle is

$$\frac{2x^2 + 4x}{4x^2 + 16x} = \frac{2x(x+2)}{4x(x+4)}$$

$\frac{2}{4}$

$$\frac{1(x+2)}{2(x+4)}$$