Lesson 5.1 Exponents and Scientific Notation

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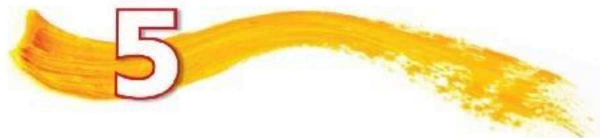
MCA Lesson 5.1



Exponents and Polynomials

5.1 Integer Exponents and Scientific Notation

What You Will Learn

- Use the rules of exponents to simplify expressions.
- Rewrite exponential expressions involving negative and zero exponents.
- Write very large and very small numbers in scientific notation.

Rules of Exponents

Rules of Exponents

Let m and n be positive integers, and let a and b represent real numbers, variables, or algebraic expressions.

Rule	*Example*
1. Product: $a^m \cdot a^n = a^{m+n}$	$x^5(x^4) = x^{5+4} = x^9$
2. Product-to-Power: $(ab)^m = a^m \cdot b^m$	$(2x)^3 = 2^3(x^3) = 8x^3$
3. Power-to-Power: $(a^m)^n = a^{mn}$	$(x^2)^3 = x^{2 \cdot 3} = x^6$

4. Quotient:	$\frac{a^m}{a^n} = a^{m-n},\ m > n,\ a \neq 0$	$\frac{x^5}{x^3} = x^{5-3} = x^2,\ x \neq 0$
5. Quotient-to-Power:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0$	$\left(\frac{x}{4}\right)^2 = \frac{x^2}{4^2} = \frac{x^2}{16}$

Example 1 – Using Rules of Exponents

a. $(x^2y^4)(3x) = 3x^3y^4$

b. $-2(y^2)^3 = -2y^6$

c. $(3x^2)(-5x)^3 = (3x^2)(-125x^3) = -375x^5$

d. $\frac{14a^5b^3}{7a^2b^2} = \boxed{2a^3b}$

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Example 1 – Using Rules of Exponents cont'd

Example 1 – Using Rules of Exponents cont'd

f. $\left(\frac{x^2}{2y}\right)^3 = \boxed{\frac{x^6}{8y^3}}$

2^3

g. $\cancel{\frac{x^n y^{3n}}{x^2 y^4}} = \boxed{}$

Integer Exponents 1

Definitions of Zero Exponents and Negative Exponents

Let *a* and *b* real numbers such that $a \neq 0$ and $b \neq 0$, and let *m* be an integer.

1. $a^0 = 1$

2. $a^{-m} = \frac{1}{a^m}$ $\quad 6^{-2} \quad \frac{1}{6^2} = \frac{1}{36}$

3. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$ $\quad \left(\frac{2}{3}\right)^{-2} \quad \frac{2^{-2}}{3^{-2}} \quad 3^2$

Notice that by definition, $a^0 = 1$ for all real *nonzero* values of *a*.

Zero cannot have a zero exponent, because the expression 0^0 is undefined.

$\frac{1}{2^2}$

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Example 2 – Using Rules of Exponents

a. $3^0 =$ 1 — Definition of zero exponents

b. $3^{-2} = \boxed{\frac{1}{3^2} =} \frac{1}{9}$ — Definition of negative exponents

c. $\left(\frac{3}{4}\right)^{-1} = \boxed{\frac{4^1}{3^1} =} \frac{4}{3}$ — Definition of negative exponents

$\frac{3^{-1}}{4^{-1}}$

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Integer Exponents 2

Summary of Rules of Exponents

Let *m* and *n* be integers, and let *a* and *b* represent real numbers, variables, or algebraic expressions. (All denominators and bases are nonzero.)

Product and Quotient Rules	*Example*
1. $a^m \cdot a^n = a^{m+n}$	$x^4(x^3) = x^{4+3} = x^7$
2. $\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^3}{x} = x^{3-1} = x^2$
Power Rules	
3. $(ab)^m = a^m \cdot b^m$	$(3x)^2 = 3^2(x^2) = 9x^2$
4. $(a^m)^n = a^{mn}$	$(x^3)^3 = x^{3 \cdot 3} = x^9$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{x}{3}\right)^2 = \frac{x^2}{3^2} = \frac{x^2}{9}$
Zero and Negative Exponent Rules	
6. $a^0 = 1$	$(x^2 + 1)^0 = 1$
7. $a^{-m} = \frac{1}{a^m}$	$x^{-2} = \frac{1}{x^2}$
8. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$	$\left(\frac{x}{3}\right)^{-2} = \left(\frac{3}{x}\right)^2 = \frac{3^2}{x^2} = \frac{9}{x^2}$

Example 3 – Using Rules of Exponents

Rewrite each expression using only positive exponents.
(Assume that $x \neq 0$.)

a. $2x^{-1} = \boxed{\frac{2}{x^1}}$ Use negative exponent rule and simplify.

b. $(2x)^{-1} = \boxed{\frac{1}{2x}}$ Use negative exponent rule and simplify.

c. $\frac{3}{x^{-2}} = \boxed{}$ $3x^2$ Use negative exponent rule.

Invert divisor and multiply.

Example 3 – Using Rules of Exponents cont'd

d. $\frac{1}{(3x)^{-2}} = (3x)^2$

Use negative exponent rule.

$= 9x^2$

Use product-to-power rule and simplify.

Invert divisor and multiply

$=$

Simplify.

Example 4 – Using Rules of Exponents 1

Rewrite each expression using only positive exponents.
(Assume that $x \neq 0$ and $y \neq 0$.)

a. $(-5x^{-3})^2 = 25x^{-6}$

-5^2

$\frac{25}{x^6}$

Product-to-power rule

Power-to-power rule

Negative exponent rule

b. $-\left(\frac{7x}{y^2}\right)^{-2} = \boxed{\frac{7^{-2}x^{-2}}{y^{-4}}}$ $\frac{y^4}{7^2x^2}$

$-\frac{y^4}{49x^2}$

Negative exponent rule

Quotient-to-power rule

Power-to-power and product-to product rules

Example 4 – Using Rules of Exponents 2

c. $\frac{12x^2y^{-4}}{6x^{-1}y^2} = \boxed{2x^3y^{-6}}$ $\frac{2x^3}{y^6}$

Quotient rule

$2+1=3$

$-4-2=-6$

Negative exponent rule

d. $\left(\frac{8x^{-1}y^4}{4x^3y^2}\right)^{-3} = \boxed{}$

$8^{-3}x^3y^{-12}$

$\frac{x^3 4^3 x^9 y^6}{}$

Simplify

$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

$\frac{64x^{12}y^6}{}$

Negative exponent rule

$4^{-3}x^{-9}y^{-6}$

$\frac{x^{12}}{8y^6}$ $8y^{12}$ $\frac{1x^{12}}{8y^6}$

Quotient-to-power rule

$512y^{12}$

Power-to-power and product-to-product rules

$6-12$

y^{-6}

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Example 4 – Using Rules of Exponents 3

$\frac{\cancel{x}}{\cancel{x} \cdot x}$

e. $\frac{3x\cancel{y^0}}{x^2(\cancel{5y})^0} =$ $\boxed{}$ Zero exponent rule

$\frac{3x^1}{x^2}$ $1-2=-1$ Simplify. $3x^{-1}$ $\frac{3}{x^1}$ $\frac{3}{x}$

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Scientific Notation 1

Exponents provide an efficient way of writing and computing with very large and very small numbers.

For instance, a drop of water contains more than 33 billion molecules—that is, 33 followed by 18 zeros.

It is convenient to write such numbers in **scientific notation.**

This notation has the form

$$c \times 10^{n}$$

Scientific Notation 2

where $1 \leq c < 10$ and n is an integer. So, the number of molecules in a drop of water can be written in scientific notation as follows.

$$\underbrace{33{,}000{,}000{,}000{,}000{,}000{,}000}_{\text{19 places}} = 3.3 \times 10^{19}$$

Example 5 – Writing in Scientific Notation

a. 0.0000684 = 6.84×10^{-5} Small number ⇨ negative exponent

Five places

b. 937,200,000.0 = 9.37×10^{8} Large number ⇨ positive exponent

Eight places

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Example 5 – Writing in Scientific Notation cont'd

a. $2.486 \times 10^2 =$ 248.6

Positive exponent ⇨ large number

Right

b. $1.81 \times 10^{-6} =$ 0.00000181

Negative exponent ⇨ small number

.000001.81

Left

Example 7 – Writing in Decimal Notation

$$\frac{(2{,}400{,}000{,}000)(0.0000045)}{(0.00003)(1{,}500)} = \frac{(2.4\times10^{9})(4.5\times10^{-6})}{(3\times10^{-5})(1.5\times10^{3})}$$

$$= \frac{10.8\times10^{3}}{4.5\times10^{-2}}$$

$$=$$

3 − −2

240000

240000

$\rightarrow (2.4 \times 10$