Lesson 4.3 Functions

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MCA 4.3 Functions

Graphs and Functions

4.3 Relations, Functions, and Graphs

 1

What You Will Learn

- Identify the domain and range of a relation.
- Determine whether relations are functions.
- Use function notation and evaluate functions.
- Identify the domain and range of a function.

- Identify the domain and range of a function.
- Determine intervals on which functions are increasing or decreasing.
- Identify even and odd functions.
- Identify and graph step and other piecewise-defined functions.

Relations

Definition of Relation

A **relation** is any set of ordered pairs. The set of first components in the ordered pairs is the **domain** of the relation. The set of second components is the **range** of the relation.

In mathematics, relations are commonly described by ordered pairs of numbers. The set of x-coordinates is the domain, and the set of y-coordinates is the range.

In the relation {(3,5), (1,2), (4,4), (0,3)}, the domain D and range R are the sets D = {3, 1, 4, 0} and R = {5, 2, 4, 3}.

Example 1 – Analyzing a Relation

Find the domain and range of the relation {(0, 1), (1, 3), (2, 5), (3, 5), (0, 3)}. Then sketch a graphical representation of the relation.

Solution:

Then sketch a graphical representation of the relation.

Solution:

The domain is the set of all first components of the relation, and the range is the set of all second components. So, the domain D and the range R are the sets

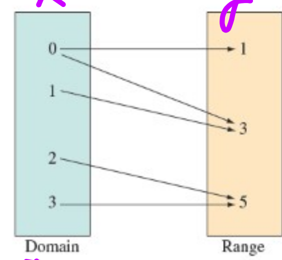
$$D = \{0, 1, 2, 3\}$$

D R
x y

and

$$R = \{1, 3, 5\}.$$

x y



x y
(0, 1)
(0, 3)
(1, 3)
(2, 5)
(3, 5)

not a function

A graphical representation of the relation is shown.

You should note that it is not necessary to list repeated components of the domain and range of a relation.

Functions 1

Definition of Function

A **function** is a relation in which no two ordered pairs have the same first component and different second components.

This definition means that a given first component cannot be paired with two different second components. For instance, the pairs (1,3) and (1,−1) could not be ordered pairs of a function.

Functions 2

Functions 2

The ordered pairs of a relation can be thought of in the form (input, output). For a function, a given input cannot yield two different outputs. For instance, if the input is a person's name and the output is that person's month of birth, then your name as the input can yield only your month of birth as the output.

Example 2 – Testing Whether a Relation Is a Function

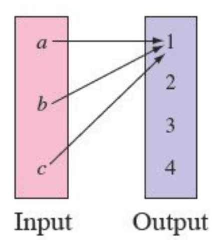
Decide whether each relation represents a function.

a. Input: *a*, *b*, *c*   yes

Output: 2, 3, 4

{(*a*, 2), (*b*, 3), (*c*, 4)}

b.



{(a,1)
(b,1)
(c,1)}   yes

c.

Input, *x*	Output, *y*	(*x*, *y*)
3	1	(3,1)
4	3	(4,3)
5	4	(5,4)
3	2	(3,2)

No

Example 2 – Testing Whether a Relation Is a Function cont'd

Solution:

a. This set of ordered pairs *does* represent a function. No first component has two different second components.

b. This diagram *does* represent a function. No first component has two different second components.

c. This table *does not* represent a function. The first component 3 is paired with two different second components, 1 and 2.

Functions 3

In algebra, it is common to represent functions by equations in two variables rather than by ordered pairs. The equation $y = x^2$ represents the variable y as a function of x.

The variable x is the **independent variable** (the input) and y is the **dependent variable** (the output). In this content, the domain of the function is the set of all *allowable* values of x, and the range is the *resulting* set all the values taken on by the dependent variable y.

Vertical Line Test

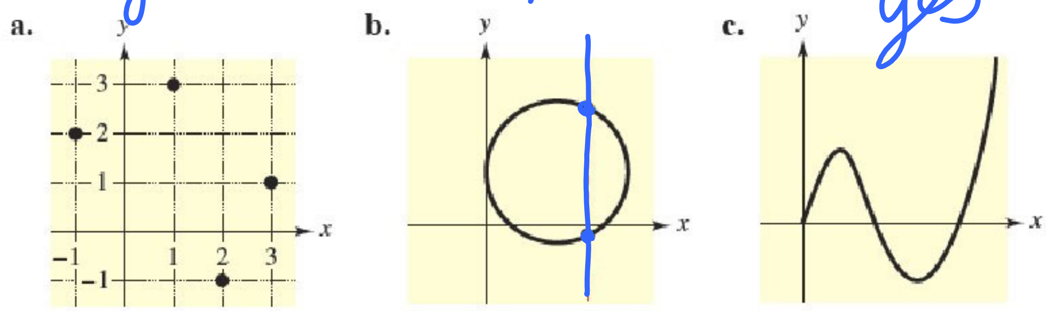
A set of points on a rectangular coordinate system is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

than one point.

Example 3 – Using the Vertical Line Test

Use the Vertical Line Test to determine whether *y* is a function of *x*.

yes NO yes



Example 3 – Using the Vertical Line Test cont’d

Solution:

a. From the graph, you can see that no vertical line intersects more than one point on the graph. So, the relation *does* represent *y* as a function of *x*.

b. From the graph, you can see that a vertical line intersects more than one point on the graph. So, the relation *does not* represent y as a function of *x*.

c. From the graph, you can see that no vertical line relation *does not* represent y as a function of *x*.

c. From the graph, you can see that no vertical line intersects more than one point on the graph. So, the relation *does* represent *y* as a function of *x*.

Function Notation

Function Notation

In the notation *f*(*x*):

f is the **name** of the function.

x is the **domain** (or input) value.

f(*x*) is a **range** (or output) value *y* for a given *x*.

The symbol *f*(*x*) is read as *the value of f at x* or simplify *f of x*.

The process of finding the value of f(x) for a given value of *x* is called **evaluating a function**.

This is accomplished by substituting a given *x*-value (input) into the equation to obtain the value of *f*(*x*) (output).

Example 4 – Evaluating a Function

Let $f(x) = x^2 + 1$. Find each value of the function.

a. $f(-2)$ b. $f(0)$

Solution:

a. $f(x) = x^2 + 1$ Write original function.

$f(-2) = (-2)^2 + 1$ Substitute − 2 for *x*.

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$f(-2) = (-2)^2 + 1$ Substitute − 2 for *x*.

$4 + 1$

$y = 5$ Simplify.

b. $f(x) = x^2 + 1$ Write original function.

$f(0) = 0^2 + 1$ Substitute 0 for *x*.

$0 + 1$

$y = 1$ Simplify.

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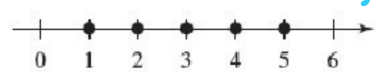
The Domain and Range of a Function 2

The domain of a function can be discrete or continuous. A **discrete domain** consists of only certain numbers in an interval, and a **continuous domain** consists of all numbers in an interval.

integers

continues
all #
"connects"
decimals

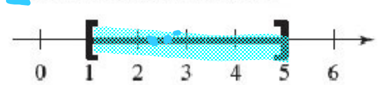
Discrete Domain



Integers from 1 to 5

Continuous Domain

connects



All real numbers from 1 to 5

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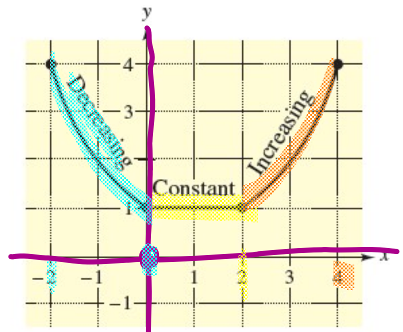
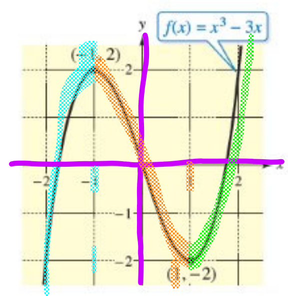
Increasing and Decreasing Functions 1

The more you know about the graph of a function, the more you know about the function itself. Consider the graph shown in the figure. As you move from *left to* more you know about the function itself. Consider the graph shown in the figure. As you move from *left to right*, this graph falls from $x = -2$ to $x = 0$, is constant from $x = 0$ to $x = 2$, and rises from $x = 2$ to $x = 4$.



Example 8 – Describing Function Behavior

Determine the open intervals on which the function is increasing or decreasing.



Solution:

The function is increasing on the interval $(-\infty, -1)$, decreasing on the interval $(-1, 1)$, and increasing on the interval $(1, \infty)$.

Even and Odd Functions

In the terminology of functions, a function is said to be **even** when its graph is symmetric with respect to the *y*-axis and **odd** when its graph is symmetric with respect to the origin.

Tests for Even and Odd Functions

A function $y = f(x)$ is **even** when, for each x in the domain of f, $f(-x) = f(x)$.

$y = x^2 + 2$

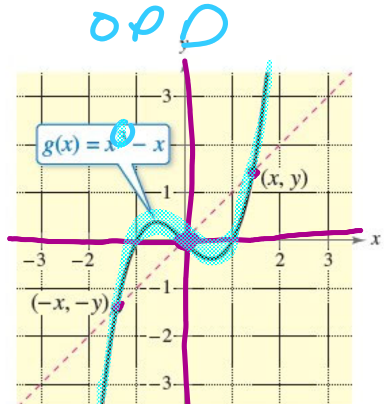
A function $y = f(x)$ is **odd** when, for each x in the domain of f, $f(-x) = -f(x)$.

$y = x^3 + 1$

Example 9 – Even and Odd Functions cont'd

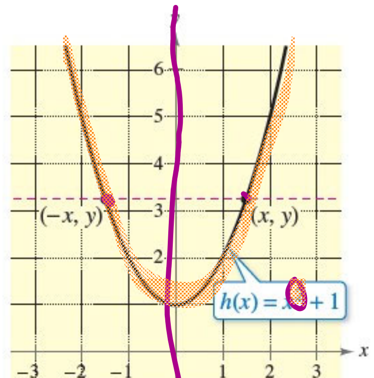
The figures below show the graphs and symmetry of these two functions.

odd



Symmetric to origin: Odd Function

even

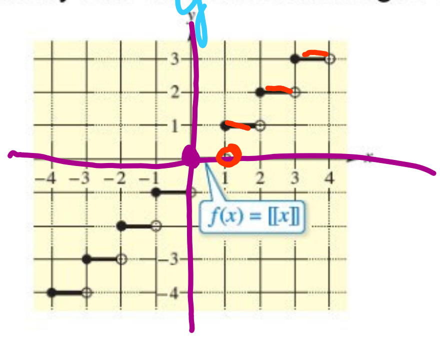


Symmetric to *y*-axis: Even Function

Step and Piecewise-Defined Functions 2

The graph of the greatest integer function $f(x) = [\![x]\!]$ has the characteristics below, as shown in the figure.

- x The domain of the function is the set of all real numbers.
- y The range of the function is the set of all integers.
- The graph has a y-intercept at (0, 0) and x-intercepts in the interval (0, 1).
- The graph is constant between each pair of consecutive integer values of x.
- The graph jumps vertically one unit at each integer value of x.

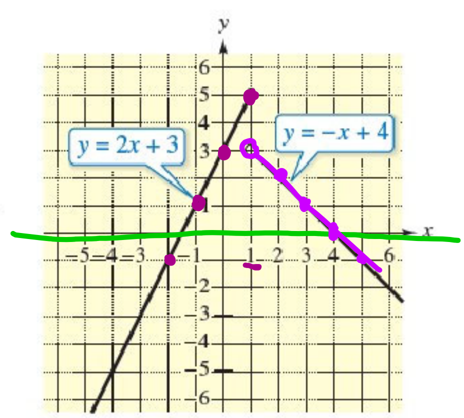


Example 11 – Graphing a Piecewise-Defined Function

Sketch the graph of $f(x) = \begin{cases} 2x + 3, & x \le 1 \\ -x + 4, & x > 1 \end{cases}$.

Solution:

This piecewise-defined function consists of two linear functions. At x = 1 and to the left of x = 1, the graph is the line y = 2x + 3, and to the right of x = 1, the graph is the line y = −x + 4, as shown in the figure. Notice that the point (1, 5) is a solid dot and the point (1, 3) is an open dot. This is because f(1) = 2(1) + 3 = 5.



Absolute Value Functions 4

- One common type of piecewise-defined function is the **absolute value function.**

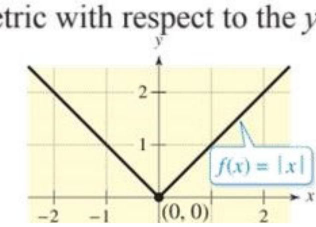
The absolute value function can be defined by two linear pieces as

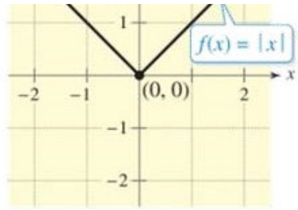
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Absolute Value Functions 5

The graph of the absolute value function $f(x) = |x|$ has the characteristics below, as shown in the figure.

- The domain of the function is the set of all real numbers.
- The range of the function is the set of all real numbers greater than or equal to 0.
- The graph has a y-intercept at the point (0, 0).
- The function is decreasing on the interval $(-\infty, 0)$.
- The function is increasing on the interval $(0, \infty)$.
- The function is symmetric with respect to the y-axis. So, the function is even.





Example 12 Evaluating an Absolute Value Function

Evaluate the function $f(x) = 2|x + 3| - 1$

when $x = -4, 0, and\ \frac{1}{2}$.

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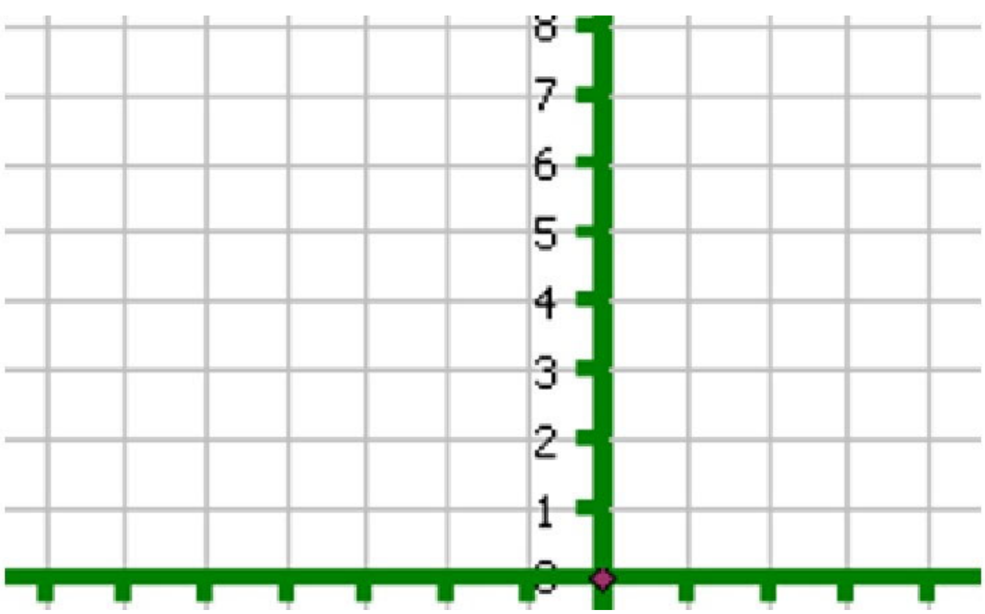

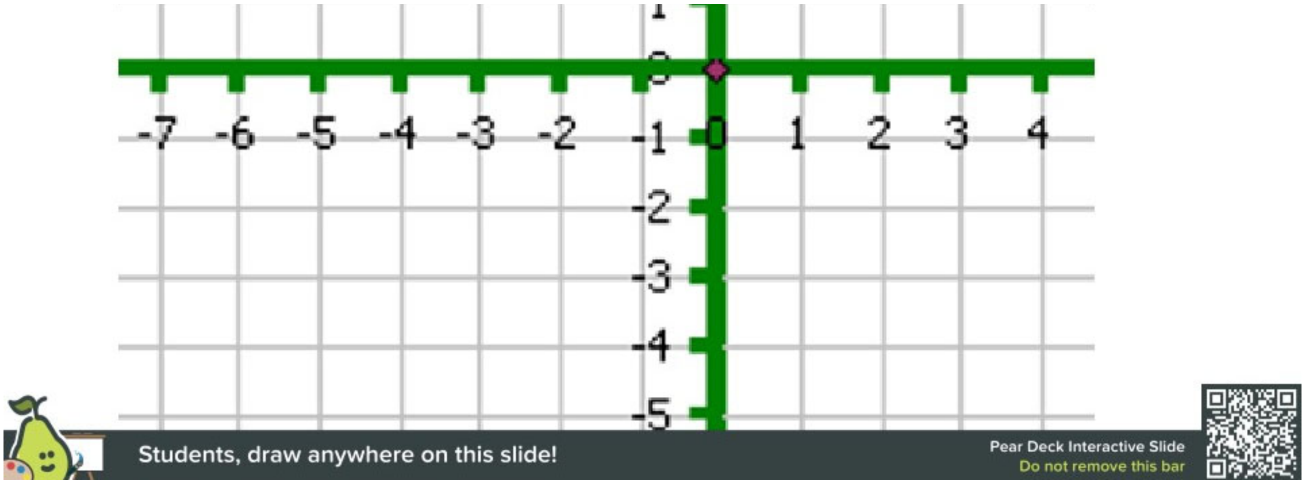
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