Lesson 2.4 Algebraic Equations

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Lesson 2.4 Equations

Fundamentals of Algebra

2.4 Introduction to Equations

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What You Will Learn

- Check whether a given value is a solution of an equation.
- Use properties of equality to form equivalent equations.
- Use a verbal model to write an algebraic equation

equations.

- Use a verbal model to write an algebraic equation.

Checking Solutions of Equations 1

An **equation** is a statement that two algebraic expressions are equal. For instance, the following are equations:

$x = 3$, $5x - 2 = 8$, $\frac{x}{4} = 7$, and $x^2 - 9 = 0$

$+2 \quad +2$

$5x = 10$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

$4 \quad 4$

$x = 28$

$+9 \quad +9$

$x^2 = 9$

$x = \sqrt{9}$

$x = 3$

To **solve** an equation involving the variable *x* means to find all values of *x* for which the equation is true.

Such values are called **solutions**

Checking Solutions of Equations 2

For instance, $x = 2$ is a solution of the equation

$$5x - 2 = 8$$

$$5x - 2 = 8$$

because

$$5(2) - 2 = 8$$

is a true statement.

The solutions of an equation are said to **satisfy** the equation.

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Example 1 – Checking Solutions of an Equation

Determine whether (a) $x = -2$ and (b) $x = 2$ are solutions of $x^2 - 5 = 4x + 7$.

Solution:

a. $x^2 - 5 = 4x + 7$ Write original equation.

$-2^2 - 5 \stackrel{?}{=} 4(-2) + 7$ Substitute −2 for x.

$4 - 5 \stackrel{?}{=} -8 + 7$ Simplify.

$-1 = -1$ See if solution checks. ✓

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Example 1 – Checking Solutions of an Equation cont'd

Example 1 – Checking Solutions of an Equation cont'd

a. $x^2 - 5 = 4x + 7$ Write original equation.

$2^2 - 5 \stackrel{?}{=} 4(2) + 7$ Substitute 2 for x.

$4 - 5 \stackrel{?}{=} 8 + 7$ Simplify.

$-1 \neq 15$ See if solution checks. NO

$x^2 - 4x - 12 = 0$

$(x+2)(x-6)$

$x^2 - 6x + 2x - 12$

$x^2 - 4x - 12$

$(x+2) = 0$

$x = -2$

$x - 6 = 0$

$x = 6$

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Example 2 – Comparing Equations and Expressions

Make a table that compares algebraic expressions and algebraic equations.

Solution:

Algebraic Expression	Algebraic Equation
• Example: $4(x - 1)$	• Example: $4(x - 1) = 12$
• Contains no equal sign	• Contains an equal sign and is true for only certain values of the variable
• Can be evaluated for any real number for which the expression is defined (example *substitute the variable x for 3 and evaluate) $4(3-1)$ $4(2) = 8$	• Solution is found by forming equivalent equations using the properties of equality: $4(x - 1) = 12$ $4x - 4 = 12$ $+4 \quad +4$
• Can sometimes be simplified to an equivalent form: $4(x - 1)$ simplifies to	$\frac{4x}{4} = \frac{16}{4}$ $x = 4$

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Forming Equivalent Equations 1

Forming Equivalent Equations: Properties of Equality

An equation can be transformed into an **equivalent equation** using one or more of the following procedures.

	Original Equation	*Equivalent Equation(s)*
1. *Simplify either side:* Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation.	$3x - x = 8$	$2x = 8$
2. *Apply the Addition Property of Equality:* Add (or subtract) the same quantity to (from) *each* side of the equation.	$x - 2 = 5$	$x - 2 + 2 = 5 + 2$ $x = 7$
3. *Apply the Multiplication Property of Equality:* Multiply (or divide) each side of the equation by the same *nonzero* quantity.	$3x = 9$	$\frac{3x}{3} = \frac{9}{3}$ $x = 3$
4. *Interchange the two sides of the equation.*	$7 = x$	$x = 7$

Example 4 – Identifying Properties of Equality 1

Identify the property of equality used to solve each equation.

a. $x - 5 = 0$

$+5 \quad +5$

$x = 5$

b. $5\left(\frac{x}{5} = -2\right)5$

$x = -10$

c. $4x = 9$

$\frac{4x}{4} = \frac{9}{4}$

$x = 2.25$

$x = 2.25$

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Example 5 – Using a Verbal Model to Write an Equation

Write an algebraic equation for the following problem.

The total income that an employee received in a year was $40,950. How much was the employee paid each week? Assume that each weekly paycheck contained the same amount and that the year consisted of 52 weeks.

$\frac{52x}{52} = \frac{40950}{52}$

$x = 787.5$

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Example 5 – Using a Verbal Model to Write an Equation cont'd

Solution:

Verbal Model: Income for year = Number of weeks in a year · Weekly pay

Labels:	Income for year = 40,950	(dollars)
	Weekly pay = *x*	(dollars per week)
	Number of weeks = 52	(weeks)

Equation: $\frac{40,950}{52} = \frac{52x}{52}$

Equation: $40{,}950 = 52x$

$$\frac{40{,}950}{52} = \frac{52x}{52}$$

Solve:

$x = 787.50$

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Example 6 – Using a Verbal Model to Write an Equation

Write an algebraic equation for the following problem.

> Returning to college after spring break, you travel 3 hours and stop for lunch. You know that it ~~takes 45 minutes~~ to complete the last 36 miles of the 180-mile trip. What was the average speed during the first 3 hours of the trip?

r

$d = rt$

$$\frac{144}{3} = \frac{r\,3}{3}$$

$r = 48$ mph

$$\begin{array}{r} 180 \\ -\ 36 \\ \hline 144 \end{array}$$ miles

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Example 6 – Using a Verbal Model to Write an Equation cont'd

Solution:

Verbal Model: Distance = Rate · Time

Labels: Distance = 180 − 36 = 144 (miles)

Rate = r (miles per hour)

Time = 3 (hours)

Rate = r (miles per hour)

Time = 3 (weeks)

Equation: $144 = 3r$

$\frac{144}{3} = \frac{3r}{3}$

Solve:

$r = 48$ mph

Example 7 – Using a Verbal Model to Write an Equation

Write an algebraic equation for the following problem.

Tickets for a concert cost $175 for each floor seat and $95 for each stadium seat. There were 2500 seats on the main floor, and these were sold out. The total revenue from ticket sales was $865,000. How many stadium seats were sold?

?

$175(2500) + 95x = 865,000$

$437,500 + 95x = 865,000$

Example 7 – Using a Verbal Model to Write an Equation cont'd

Solution:

Verbal Model: Total revenue = Revenue from floor seats + Revenue from stadium seats

Verbal Model: Total revenue = Revenue from floor seats + Revenue from stadium seats

Labels:

Total Revenue = 865,000	(dollars)
Price per floor seat = 175	(dollars per seat)
Number of floor seats = 2500	(seats)
Price per stadium seat = 95	(dollars per seat)
Number of stadium seats = x	(seats)

Equation: $865{,}000 = 175(2500) + 95x$

Solve:

$$865{,}000 = \cancel{437{,}500} + 95x$$

$$-437{,}500 \quad -\cancel{437{,}500}$$

$$\frac{427{,}500}{95} = \frac{95x}{95}$$

$$\boxed{x = 4{,}500}$$