Lesson 1.5 Exponents and Properties of Real Numbers

Sunday, September 15, 2024 9:59 PM

*ALL EVEN PROBLEMS MUST BE TURNED IN FOR A GRADE!

- Write expressions in exponential form and evaluate exponential expressions.
- Evaluate expressions using the order of operations.
- Identify and use the properties of real numbers.

Exponents

Repeated multiplication can be described in exponential form.

Repeated Multiplication	*Exponential Form*
$\underbrace{7 \cdot 7 \cdot 7 \cdot 7}_{\text{4 factors of 7}}$	7^4

In the exponential form, **7** is the **base** and it specifies the repeated factor. The number **4** is the **exponent** and it indicates how many times the base occurs as a factor. When you write the exponential form, you can say that you are raising **7** to the fourth **power**.

Example 1

Evaluating Exponential Expressions

a. $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ — Rewrite expression as a product.
$= 32$ — Simplify.

b. $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$ — Rewrite expression as a product.
$= \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3}$ — Multiply fractions.
$= \frac{16}{81}$ — Simplify.

Order of Operations

The accepted priorities for the **order of operations** are summarized below.

1. Perform operations inside symbols of grouping: **P**arentheses, brackets, or absolute value symbols, starting with the innermost symbols.
2. Evaluate all **E**xponential expressions.
3. Perform all **M**ultiplications and **D**ivisions from left to right.
4. Perform all **A**dditions and **S**ubtractions from left to right.

P
E
M → L → R
D
A → L → R
S

3. 3^2 $3 \cdot 3 = 9$

Answer

4. 4^3

5. 2^6 $= 64$

Answer

6. 6^3

7. $\left(\frac{1}{4}\right)^3$ $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$ $\frac{1}{64}$

Answer

8. $\left(\frac{4}{5}\right)^4$

9. $(-5)^3$ $= -125$

Answer

10. $(-4)^2$

11. -4^2 $-(4^2) = -(16) = -16$

Answer

12. $-(-7)^3$

13. $(-1.2)^3$ -1.728

Answer

14. $(-1.5)^4$

20. $7 + 8 - 11$

21. $5 - (8 - 15)$

Answer

22. $13 - (12 - 3)$

23. $15 + 3 \cdot 4$

Answer

24. $9 - 5 \cdot 2$

25. $25 - 32 \div 4$

Answer

26. $12 + 36 \div 9$

27. $(45 \div 10) \cdot 2$

Answer

28. $(38 \div 5) \cdot 4$

29. $(16 - 5) \div (3 - 5)$

Answer

30. $(19 - 4) \div (7 - 2)$

31. $(10 - 16) \cdot (20 - 26)$

Answer

32. $(10 - 12)(15 - 19)$

33. $17 - |2 - (6 + 5)|$

Answer

34. $125 - |10 - (25 - 3)|$

35. $[360 - (8 + 12)] \div 5$

Answer

36. $[127 - (13 + 4)] \div 10$

37. $5 + (2^2 \cdot 3)$

Answer

38. $181 - (13 \cdot 3^2)$

39. $(-6)^2 - (48 \div 4^2)$

Answer

40. $(-4)^3 + (27 \div 3^2)$

41. $\left(\frac{3}{4} \cdot \frac{5}{9}\right) + 1 - \frac{1}{3}$

Answer

42. $\frac{2}{3}\left(\frac{3}{4}\right) + 2 - \frac{3}{2}$

43. $18\left(\frac{1}{2} + \frac{2}{3}\right)$

Answer

44. $4\left(-\frac{2}{3} + \frac{4}{3}\right)$

45. $\frac{7}{25}\left(\frac{7}{16} - \frac{1}{8}\right)$

Answer

46. $\frac{4}{3}\left(\frac{3}{4} + \frac{1}{8}\right)$

47. $\frac{7}{3}\left(\frac{2}{3}\right) \div \frac{28}{15}$

Answer

48. $\frac{3}{8}\left(\frac{1}{5}\right) \div \frac{25}{32}$

49. $\frac{3 + [15 \div (-3)]}{16}$

Answer

50. $\frac{5 + [(-12) \div 4]}{24}$

51. $\frac{1 - 3^2}{-2}$

Answer

52. $\frac{3^2 + 2^3}{34}$

53. $\frac{7^2 - 4^2}{0}$

Answer

54. $\frac{0}{3^2 - 1^2}$

55. $\frac{0}{6^2 + 1}$

Answer

56. $\frac{4^4 + 1}{0}$

57. $\frac{5^2 + 12^2}{13}$

Answer

58. $\frac{8^2 - 2^3}{4}$

Properties of Real Numbers

Let a, b, and c be real numbers.

Property	Example
1. *Commutative Property of Addition:* Two real numbers can be added in either order. $a + b = b + a$	$3 + 5 = 5 + 3$
2. *Commutative Property of Multiplication:* Two real numbers can be multiplied in either order. $ab = ba$	$4 \cdot (-7) = -7 \cdot 4$

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Two real numbers can be multiplied in either order.

$ab = ba$ $\qquad 4 \cdot (-7) = -7 \cdot 4$

1·2 2·1

a

3. *Associative Property of Addition:*
When three real numbers are added, it makes no difference which two are added first.

$(a + b) + c = a + (b + c)$ $\qquad (2 + 6) + 5 = 2 + (6 + 5)$

4. *Associative Property of Multiplication:*
When three real numbers are multiplied, it makes no difference which two are multiplied first.

$(ab)c = a(bc)$ $\qquad (3 \cdot 5) \cdot 2 = 3 \cdot (5 \cdot 2)$

5. *Distributive Property:*
Multiplication distributes over addition.

$a(b + c) = ab + ac$ $\qquad 3(8 + 5) = 3 \cdot 8 + 3 \cdot 5$
$(a + b)c = ac + bc$ $\qquad (3 + 8)5 = 3 \cdot 5 + 8 \cdot 5$

6. *Additive Identity Property:*
The sum of zero and a real number equals the number itself.

$a + 0 = 0 + a = a$ $\qquad 3 + 0 = 0 + 3 = 3$

1+0=1

7. *Multiplicative Identity Property:*
The product of 1 and a real number equals the number itself.

$a \cdot 1 = 1 \cdot a = a$ $\qquad 4 \cdot 1 = 1 \cdot 4 = 4$

3·1=3

8. *Additive Inverse Property:*
The sum of a real number and its opposite is zero.

$a + (-a) = 0$ $\qquad 3 + (-3) = 0$

5-5=0

9. *Multiplicative Inverse Property:*
The product of a nonzero real number and its reciprocal is 1.

$a \cdot \frac{1}{a} = 1, a \neq 0$ $\qquad 8 \cdot \frac{1}{8} = 1$

$\frac{3}{2}$ $\frac{2}{3}$

59. $(10 + 3) + 2 = 10 + (3 + 2)$ associative

Answer

60. $(23 + 6) + 4 = 23 + (6 + 4)$

61. $6(-3) = -3(6)$ commutative

Answer

62. $16 + 10 = 10 + 16$

63. $5 + 10 = 10 + 5$ commutative

Answer

64. $-2(8) = 8(-2)$

65. $6(3 + 13) = 6 \cdot 3 + 6 \cdot 13$ distributive

Answer

66. $(14 + 2)3 = 14 \cdot 3 + 2 \cdot 3$

67. $7\left(\frac{1}{7}\right) = 1$ inverse (reciprocal)

Answer

68. $1 \cdot 9 = 9$

69. $0 + 15 = 15$ identity

Answer

70. $18 + (-18) = 0$

Using Order of Operations In Exercises 81–84, evaluate the expression. If it is not possible, state the reason. Write fractional answers in simplest form.

81. $\frac{4 \cdot 3 - 6 \cdot 6}{4 - 4}$ $\frac{12-36}{0} = \frac{-24}{0}$ undefined

Answer

82. $\frac{5 \cdot 3 + 5 \cdot 6}{7 - 2}$

83. $7 - \frac{4 + 6}{2^2 + 1} + 5$ $7 - \frac{10}{5} + 5 = 7 - 2 + 5 = 5 + 5 = 10$

4+1

Answer

84. $11 - \frac{3^3 - 30}{8 + 1} + 1$

Using a Calculator In Exercises 85–88, use a calculator to evaluate the expression. Round your

Using a Calculator In Exercises 85–88, use a calculator to evaluate the expression. Round your answer to two decimal places.

85. $200\left(1+\frac{0.1}{8}\right)^{36}$

200 (1 + 0.0125)^36 = 312.79

1.0125

Answer

86. $1000 \div \left(1+\frac{0.09}{4}\right)^{8}$

87. $\frac{1.28+3(4.36)}{2.25}$

$\frac{1.28+13.08}{2.25} = \frac{14.36}{2.25} = 6.38$

Answer

88. $\frac{4.19-7(2.27)}{14.8}$

89. **Finding Inverses** Find

a. the additive inverse and

Answer −20

b. the multiplicative inverse of 20.

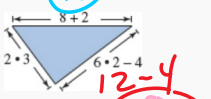
Answer $\frac{1}{20}$

90. **Finding Inverses** Find

a. the additive inverse and

b. the multiplicative inverse of −8.

91. **Geometry** Write and evaluate an expression for the perimeter of the triangle.



10

24

6

12 − 4

8

Add

92. **Geometry** Find the area of the region.