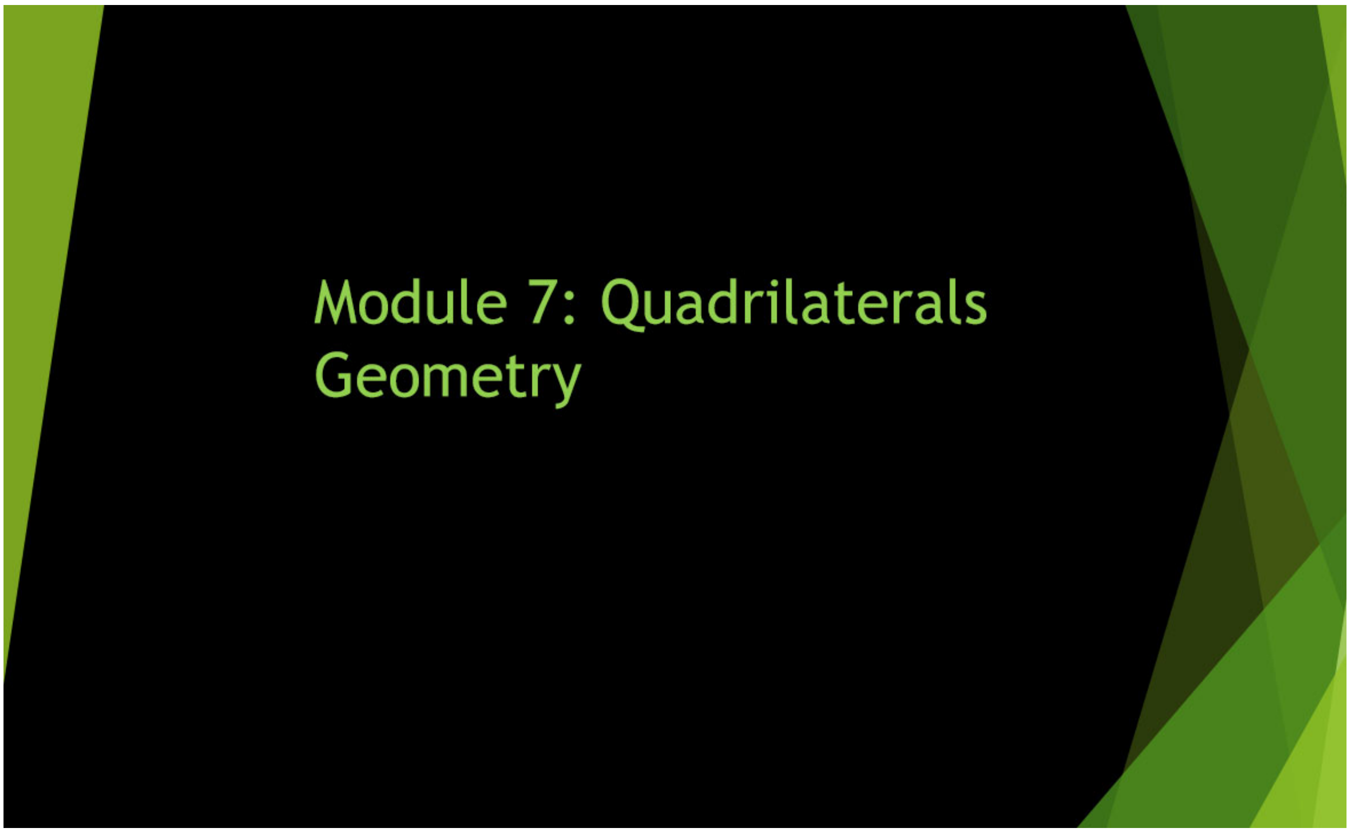
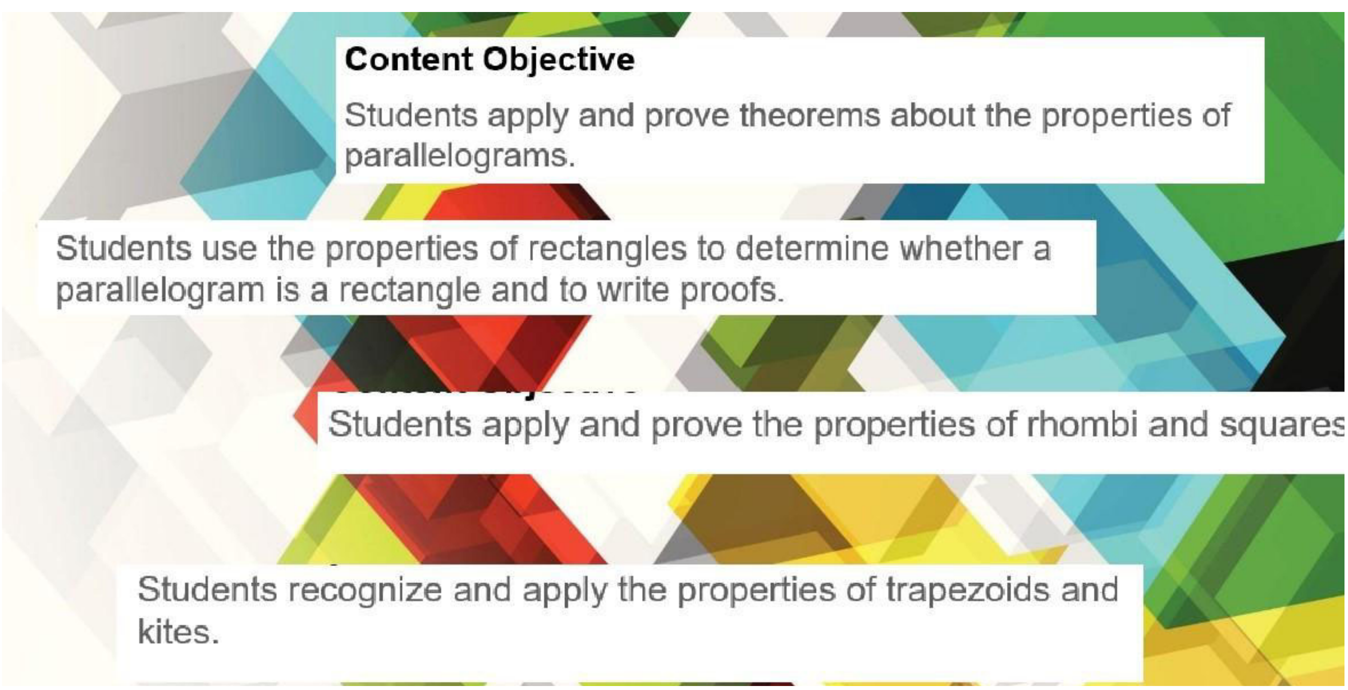
Module 7: Quadrilaterals

Tuesday, March 26, 2024 11:14 PM

Click Link Below to Open the Interactive Pear Deck PowerPoint
https://app.peardeck.com/student/tmmgxgvcz

Quadrilat...





MA.912.GR.1.4
Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms.

MA.912.GR. 3.2
Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

MA.912.GR.3.3
Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

MA.912.GR.1.5
Prove relationships and theorems about trapezoids. Solve mathematical and real-world problems involving postulates, relationships and theorems of trapezoids.

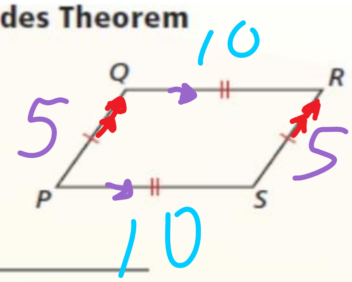
Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If *PQRS* is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 368

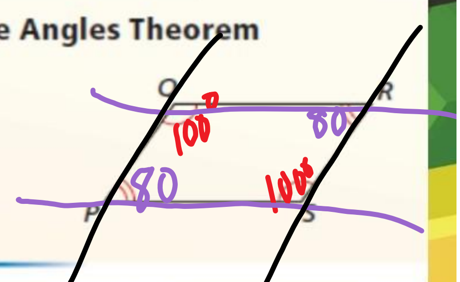


Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If *PQRS* is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

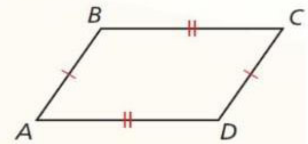
Proof Ex. 37, p. 373



Theorems

Theorem 7.7 **Parallelogram Opposite Sides Converse**

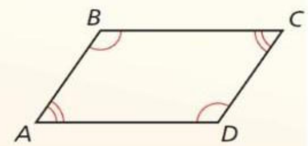
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

Theorem 7.8 **Parallelogram Opposite Angles Converse**

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383

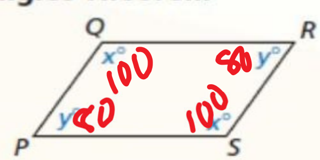
Theorems

Theorem 7.5 **Parallelogram Consecutive Angles Theorem**

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 373 $100 + 80 = 180$

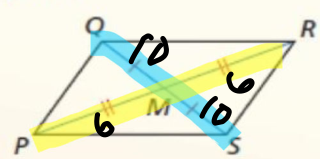


Theorem 7.6 **Parallelogram Diagonals Theorem**

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

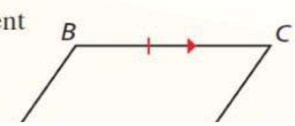
Proof p. 370



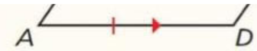
Theorems

Theorem 7.9 **Opposite Sides Parallel and Congruent Theorem**

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

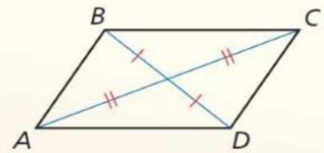


Proof Ex. 40, p. 383

Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

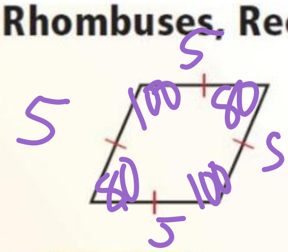
If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.



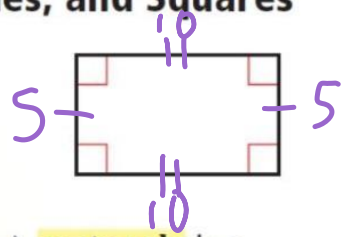
Proof Ex. 41, p. 383

Core Concept

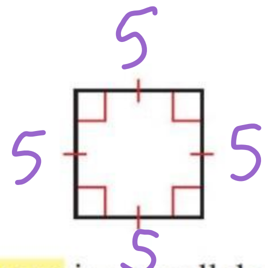
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



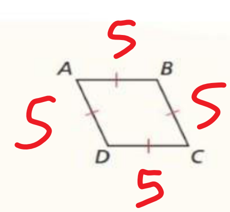
A **square** is a parallelogram with four congruent sides and four right angles.

Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

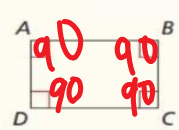


Proof Ex. 81, p. 396

Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



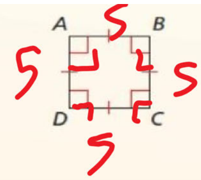
Proof Ex. 82, p. 396

Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



Students, draw anywhere on this slide!

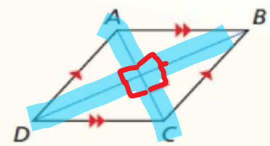
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular. 90°

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

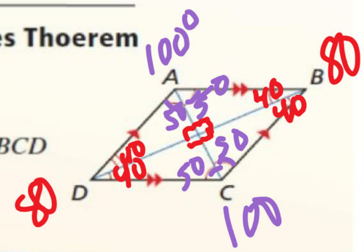


Theorem 7.12 Rhombus Opposite Angles Thoerem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395



Students, draw anywhere on this slide!

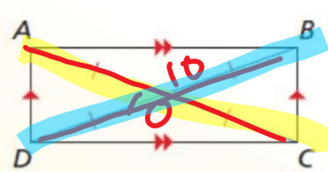
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent. 10 10

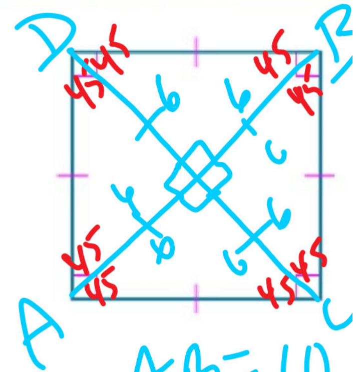
$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



Rhombus
Rectangle

A **square** is a parallelogram with all four sides and all four angles congruent. All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).



AB=10
CD=10

Theorems: Conditions for Rhombi and Squares

Theorem 7.17

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a Square & rhombus

Theorem 7.18

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a Square & rhombus

Theorem 7.19

If two consecutive sides of a parallelogram are congruent, then the parallelogram is a Square / rhombus

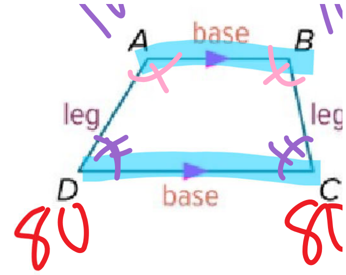
Theorem 7.20

If a quadrilateral is both a rectangle and a rhombus, then it is a Square & parallelogram

A **trapezoid** is a quadrilateral with at least one

A **trapezoid** is a quadrilateral with at least one pair of parallel sides. In a trapezoid that is not a parallelogram, the parallel sides are called the **bases** and the nonparallel sides are called **legs**.

A **base angle** is formed by a base and a leg. In trapezoid *ABCD*, $\angle A$ and $\angle B$ are one pair of base angles, and $\angle C$ and $\angle D$ are the other pair. If the legs are congruent, then a trapezoid is an **isosceles trapezoid**.

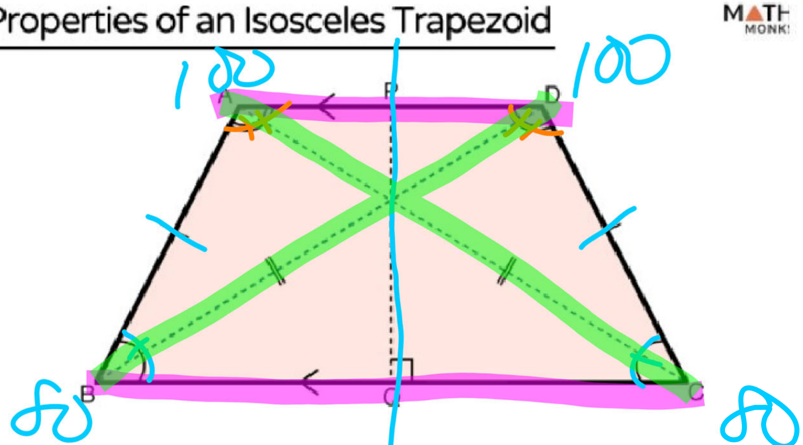


Students, draw anywhere on this slide!

Pear Deck Interactive Slide
Do not remove this bar

Properties of an Isosceles Trapezoid

MATH MONKS



① Has one pair of parallel and unequal opposite sides (bases)
② Has one pair of congruent non-parallel sides (legs)
③ Lower base angles & upper base angles are congruent
④ Diagonals are congruent
⑤ Any lower base angle is supplementary to any upper base angle
⑥ Has one line of symmetry connecting the bases at their midpoints

Students, draw anywhere on this slide!

Pear Deck Interactive Slide
Do not remove this bar

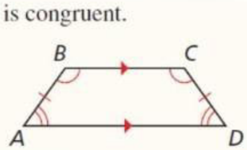
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

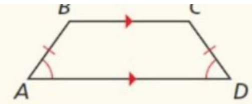


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof Ex. 40, p. 405

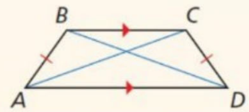


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



Theorem

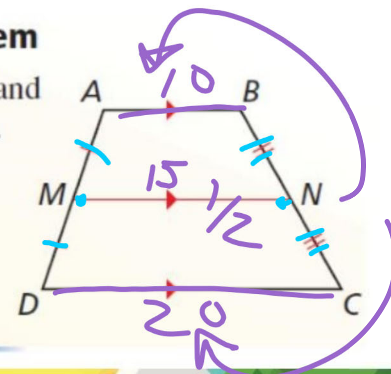
Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 406

$\frac{1}{2}(10+20)$



$\frac{1}{2}(30) = 15$

Theorems

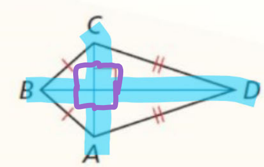
Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

90°

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

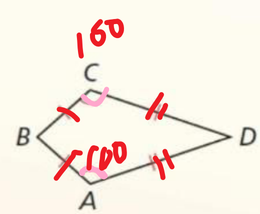


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406



Kites

A **kite** is a convex quadrilateral with exactly two distinct pairs of adjacent congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

Theorems: Kites

Theorem 7.25

If a quadrilateral is a kite, then its diagonals are perpendicular.

Theorem 7.26

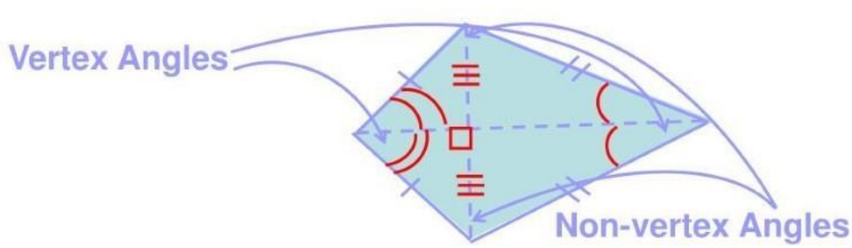
If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Students, draw anywhere on this slide!
Pear Deck Interactive Slide
Do not remove this bar

Properties of Kites and Trapezoids

Kite:

- *2 distinct pairs of consecutive congruent sides.*
- One diagonal is the $\perp$ bisector of the other.
- Non-vertex angles are congruent.
- One diagonal bisects both vertex angles.

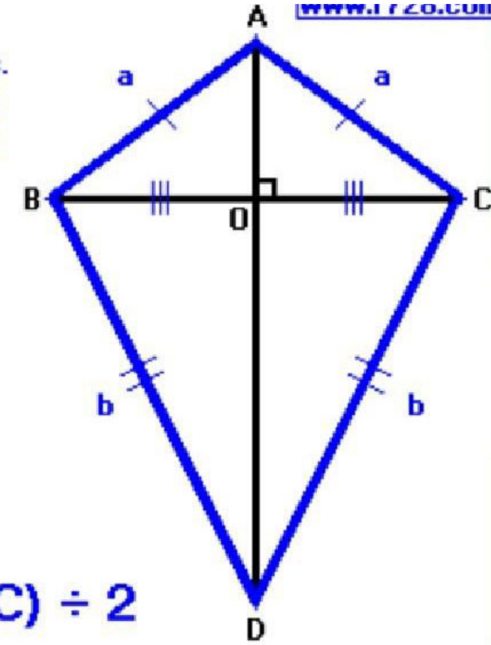


Students, draw anywhere on this slide!
Pear Deck Interactive Slide
Do not remove this bar

∠A and ∠D are vertex angles.
∠B and ∠C are the non-vertex angles.
Lines AD and BC are diagonals and always meet at right angles
Line AD, the axis of symmetry, bisects diagonal BC, bisects ∠A and ∠D and bisects the kite into 2 congruent triangles:
△ABD and △ACD

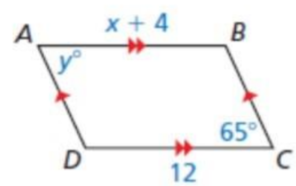
Side AB = side AC
Side BD = side CD
Line OB = Line OC
Diagonal BC bisects the kite into 2 isoceles triangles

Kite Area = (AD × BC) ÷ 2

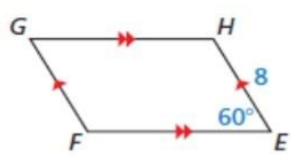


*Find all angles and side measures for all problems on this page!

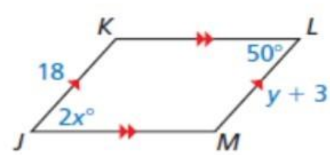
Find the values of x and y.



1. Find FG and $m\angle G$.



2. Find the values of x and y.



Example 2
Use Properties of Rectangles and Algebra

Check