Module 7: Quadrilaterals

Tuesday, March 26, 2024 11:14 PM

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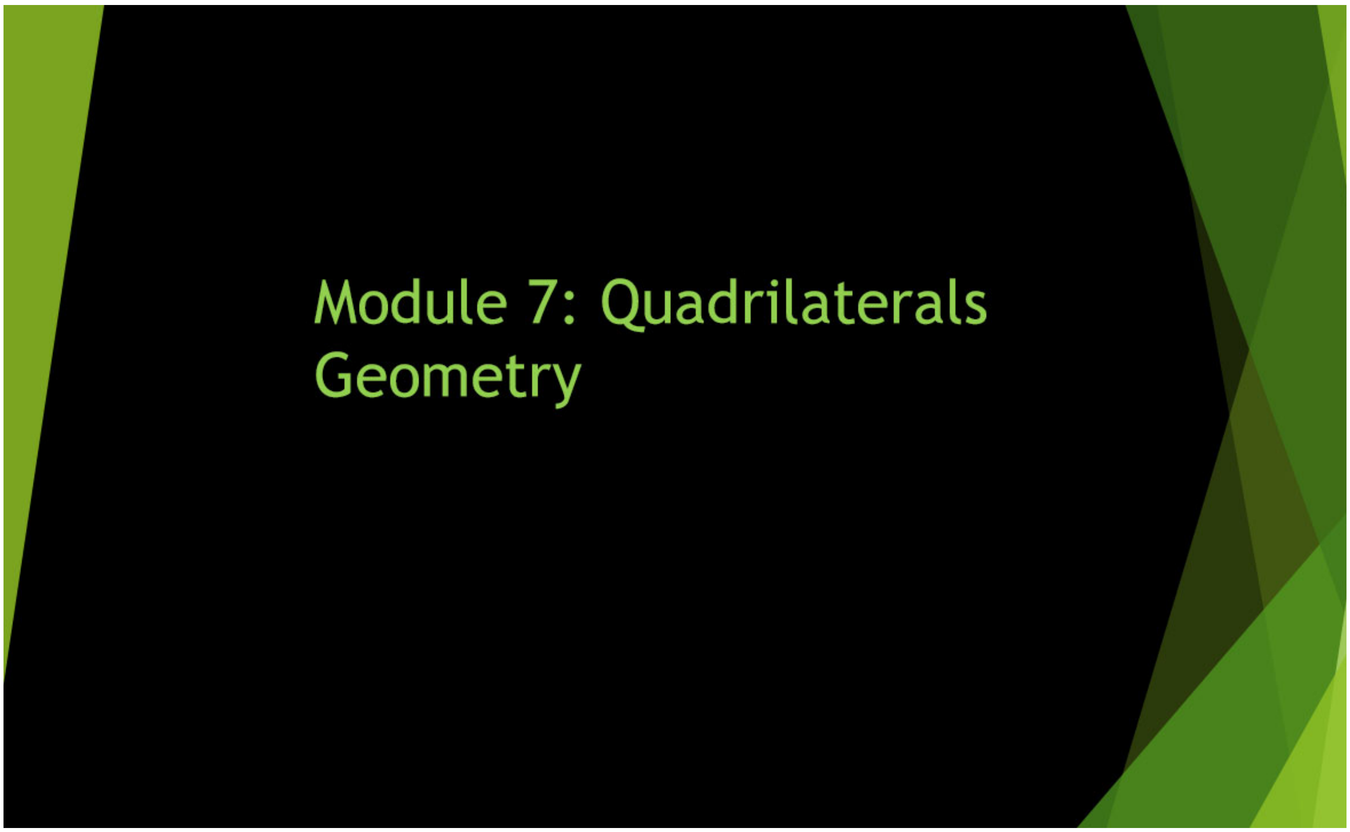
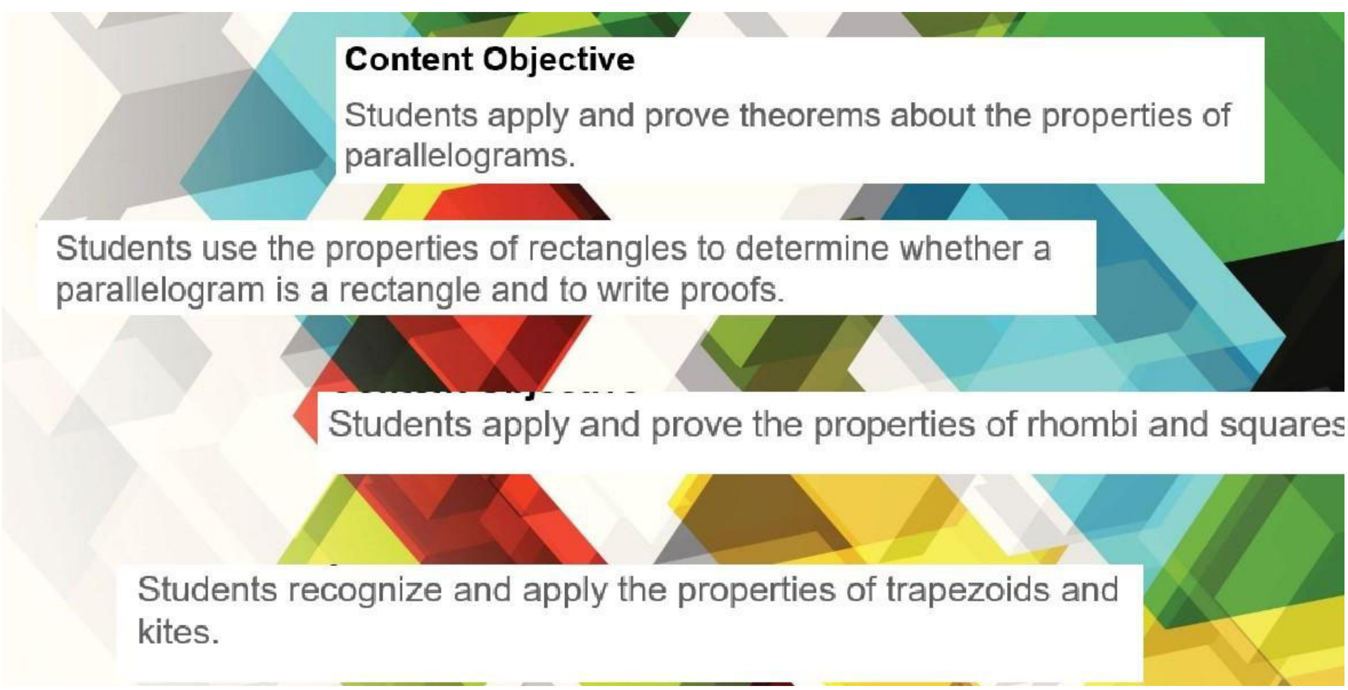
Quadrilat...





MA.912.GR.1.4
Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms.

MA.912.GR. 3.2
Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

MA.912.GR.3.3
Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

MA.912.GR.1.5
Prove relationships and theorems about trapezoids. Solve mathematical and real-world problems involving postulates, relationships and theorems of trapezoids.

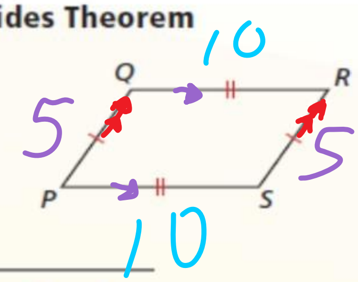
Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 368

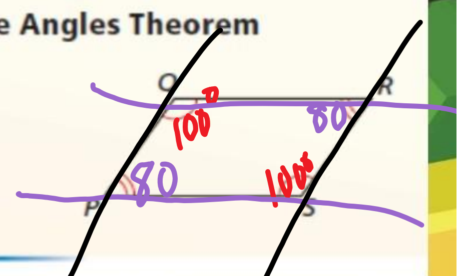


Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 373



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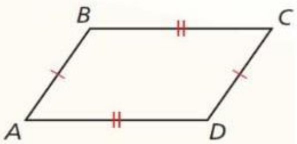
Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

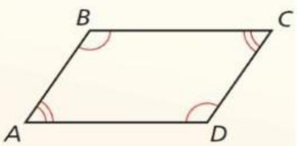


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383



Theorems

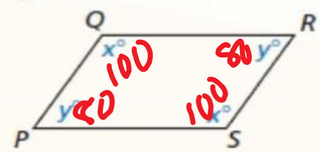
Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 373

100+80=180

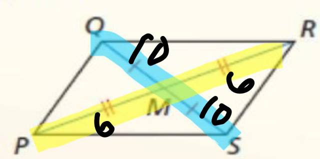


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 370

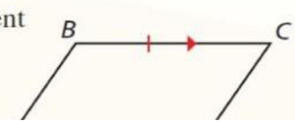


Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

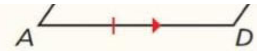
If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is



a parallelogram.

Proof Ex. 40, p. 383

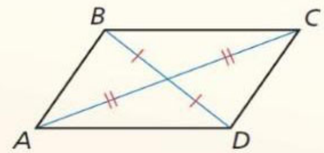


Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

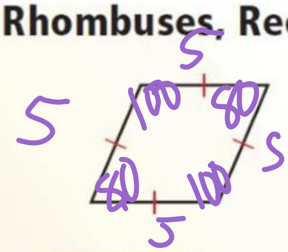
If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

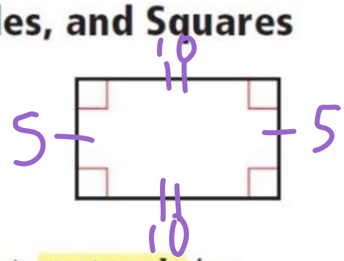
Proof Ex. 41, p. 383

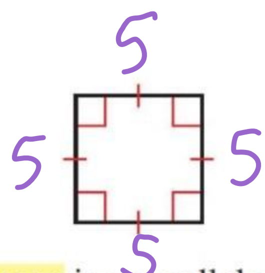


Core Concept

Rhombuses, Rectangles, and Squares







A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.

A **square** is a parallelogram with four congruent sides and four right angles.

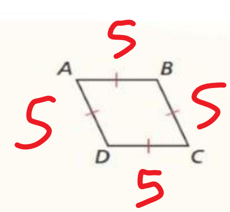
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

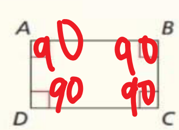


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396

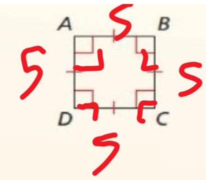


Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



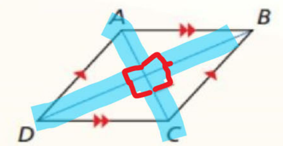
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular. 90°

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

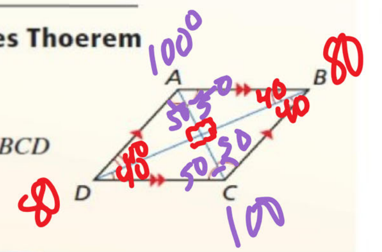


Theorem 7.12 Rhombus Opposite Angles Thoerem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395



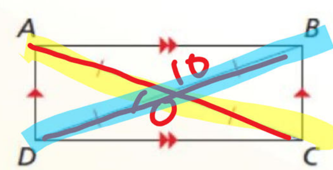
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent. 10 10

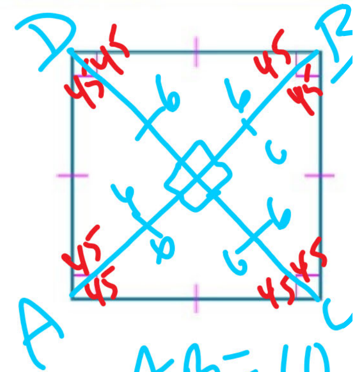
$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



Rhombus
Rectangle

A **square** is a parallelogram with all four sides and all four angles congruent. All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).



AB = 10
CD = 10

Theorems: Conditions for Rhombi and Squares

Theorem 7.17

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a Square & rhombus

Theorem 7.18

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a Square & rhombus

Theorem 7.19

If two consecutive sides of a parallelogram are congruent, then the parallelogram is a Square / rhombus

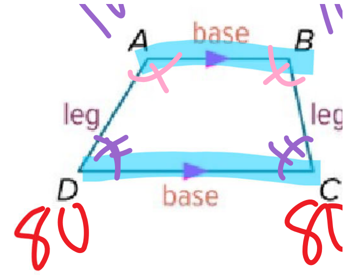
Theorem 7.20

If a quadrilateral is both a rectangle and a rhombus, then it is a Square & parallelogram

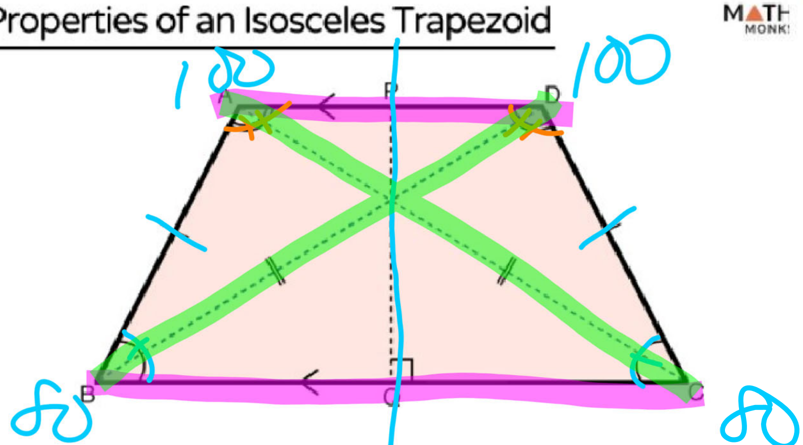
A **trapezoid** is a quadrilateral with at least one

A trapezoid is a quadrilateral with at least one pair of parallel sides. In a trapezoid that is not a parallelogram, the parallel sides are called the **bases** and the nonparallel sides are called **legs**.

A **base angle** is formed by a base and a leg. In trapezoid *ABCD*, ∠*A* and ∠*B* are one pair of base angles, and ∠*C* and ∠*D* are the other pair. If the legs are congruent, then a trapezoid is an **isosceles trapezoid**.



Properties of an Isosceles Trapezoid



① Has one pair of parallel and unequal opposite sides (bases)
② Has one pair of congruent non-parallel sides (legs)
③ Lower base angles & upper base angles are congruent
④ Diagonals are congruent
⑤ Any lower base angle is supplementary to any upper base angle
⑥ Has one line of symmetry connecting the bases at their midpoints

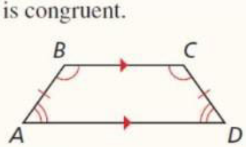
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then ∠*A* ≅ ∠*D* and ∠*B* ≅ ∠*C*.

Proof Ex. 39, p. 405

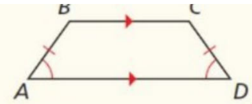


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

Proof Ex. 40, p. 405

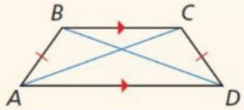


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



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Theorem

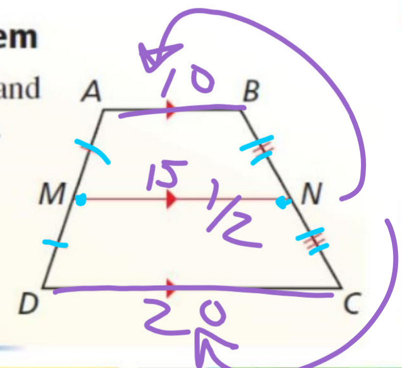
Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 406

$\frac{1}{2}(10+20)$



$\frac{1}{2}(30) = 15$

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Theorems

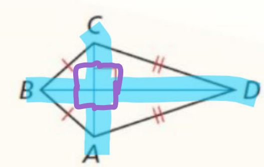
Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

90°

If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

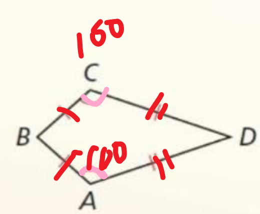


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406



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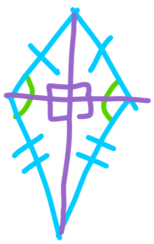

Kites

A **kite** is a convex quadrilateral (4) with exactly two distinct pairs of adjacent congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

next to

Theorems: Kites

Theorem 7.25

If a quadrilateral is a kite, then its diagonals are perpendicular.

90°

Theorem 7.26

If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Between the 2 non congruent sides

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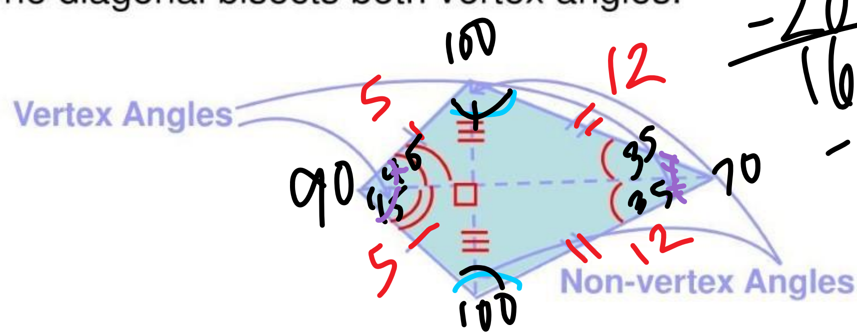
Properties of Kites and Trapezoids

Kite:

- *2 distinct pairs of consecutive congruent sides.*
- One diagonal is the ⊥ bisector of the other.
- Non-vertex angles are congruent.
- One diagonal bisects both vertex angles.



$$\begin{array}{r} 360 \\ -200 \\ \hline 160 \\ -90 \\ \hline 70 \end{array}$$

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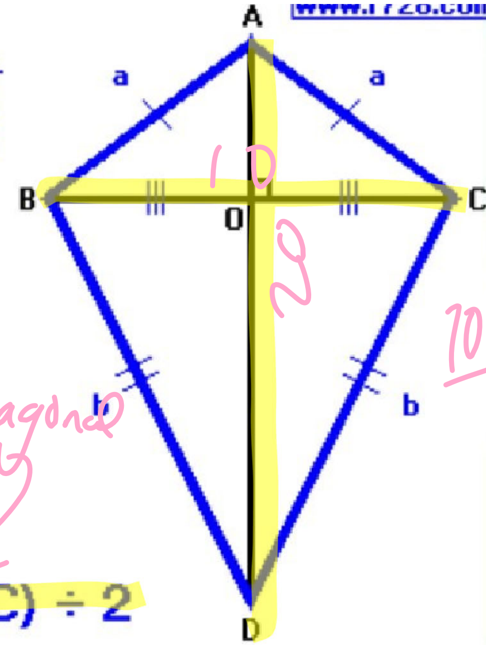
∠A and ∠D are vertex angles.
∠B and ∠C are the non-vertex angles.
Lines AD and BC are diagonals and always meet at right angles
Line AD, the axis of symmetry, bisects diagonal BC, bisects ∠A and ∠D and bisects the kite into 2 congruent triangles:
△ABD and △ACD

Side AB = side AC
Side BD = side CD
Line OB = Line OC
Diagonal BC bisects the kite into 2 isoceles triangles

Kite Area = (AD × BC) ÷ 2



diagonal

$\frac{10 \cdot 20}{2} =$

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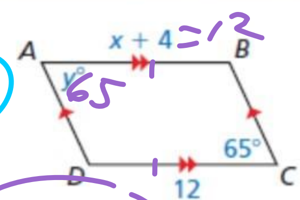
*Find all angles and side measures for all problems on this page!

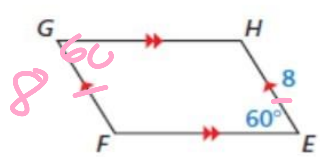
Find the values of x and y.



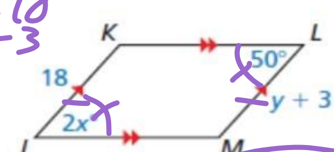
$x + 4 = 12$
$-4 \quad -4$
$x = 8$

$y = 65$

1. Find FG and $m\angle G$.



2. Find the values of x and y.



$y + 3 = 18$
$-3 \quad -3$
$y = 15$

$\frac{2x}{2} = \frac{50}{2}$

$x = 25$

Students, draw anywhere on this slide!

Example 2
Use Properties of Rectangles and Algebra

Check