Module 7: Quadrilaterals

Tuesday, March 26, 2024 11:14 PM

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Quadrilat...

Module 7: Quadrilaterals
Geometry

Content Objective

Students apply and prove theorems about the properties of parallelograms.

Students use the properties of rectangles to determine whether a parallelogram is a rectangle and to write proofs.

Students apply and prove the properties of rhombi and squares.

Students recognize and apply the properties of trapezoids and kites.

MA.912.GR.1.4
Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms.

MA.912.GR. 3.2
Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

100 80 80 100

MA.912.GR.3.3
Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

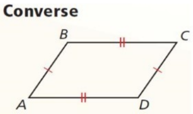
MA.912.GR.1.5

Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

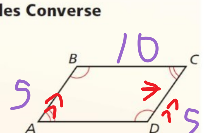


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram. and parallel

Proof Ex. 39, p. 383



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MA.912.GR. 3.2
Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

MA.912.GR.3.3
Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

MA.912.GR.1.5

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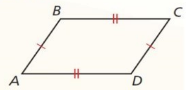

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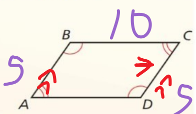

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its opposite angles are congruent.

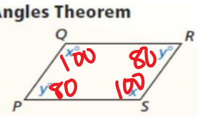
Theorems

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. 180°

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 373

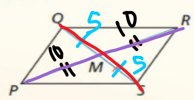


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 370



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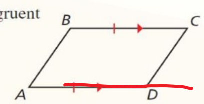
Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

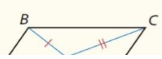
Proof Ex. 40, p. 383



and Rhombus

Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



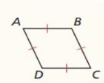
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

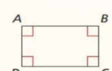


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396

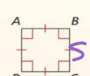


Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



5 5 5 10 5 5 5 90 90 90 90 5

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A **rhombus** is a A **rectangle** is a A **square** is a parallelogram a parallelogram.

Proof Ex. 40, p. 383

and Rhombus

Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

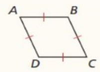
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

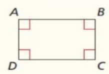


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396



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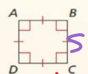
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



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A **rhombus** is a A **rectangle** is a A **square** is a parallelogram

Theorems

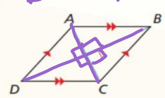
BOTH Rectangle

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular. 90°

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

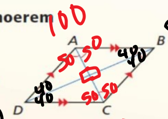


Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395



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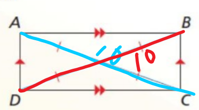
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$. 10 10

Proof Exs. 87 and 88, p. 396



Theorems: Conditions for Rhombi and Squares

Theorem 7.17

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a Square/Rhombus

Theorem 7.18

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a Square/Rhombus

Theorem 7.19

If two consecutive sides of a parallelogram are congruent, then the parallelogram is a Square/Rhombus

Theorem 7.20

If a quadrilateral is both a rectangle and a rhombus, then it is a SQUARE

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A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396

10 10

Theorems: **Conditions for Rhombi and Squares**

Theorem 7.17

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a Square/Rhombus

Theorem 7.18

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a Square/Rhombus

Theorem 7.19

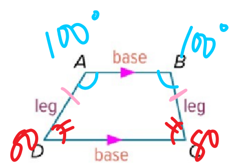
If two consecutive sides of a parallelogram are congruent, then the parallelogram is a Square/Rhombus

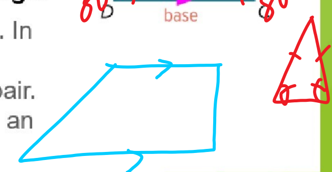
Theorem 7.20

If a quadrilateral is both a rectangle and a rhombus, then it is a SQUARE

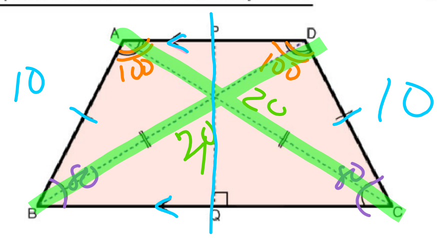
A **trapezoid** is a quadrilateral with at least one pair of parallel sides. In a trapezoid that is not a parallelogram, the parallel sides are called the **bases** and the nonparallel sides are called **legs**.

A **base angle** is formed by a base and a leg. In trapezoid *ABCD*, ∠*A* and ∠*B* are one pair of base angles, and ∠*C* and ∠*D* are the other pair. If the legs are congruent, then a trapezoid is an **isosceles trapezoid**.





Properties of an Isosceles Trapezoid



① Has one pair of parallel and unequal opposite sides (bases)

② Has one pair of congruent non-parallel sides (legs)

Theorem

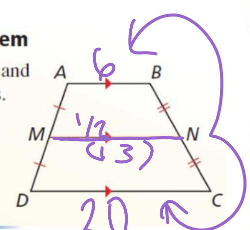
Theorem 7.17 **Trapezoid Midsegment Theorem**

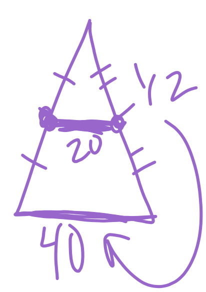
The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 406

$\frac{1}{2}(20+6)$





Theorems

Theorem 7.18 **Kite Diagonals Theorem**

If a quadrilateral is a kite, then its diagonals are

① Has one pair of parallel and unequal opposite sides (bases)

② Has one pair of congruent non-parallel sides (legs)

Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

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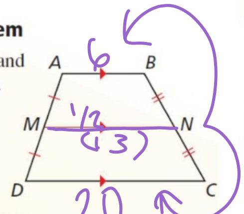

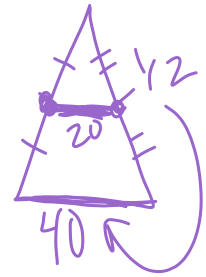

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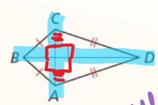
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular. 90

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

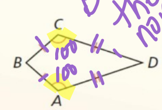


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.

Proof Ex. 47, p. 406



Between the non congruent sides

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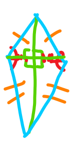


Kites

A **kite** is a convex quadrilateral with exactly two distinct pairs of adjacent congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

next

Theorems: Kites

Theorem 7.25

If a quadrilateral is a kite, then its diagonals are perpendicular. 90°

Theorem 7.26

If a quadrilateral is a kite, then exactly one pair of opposite

∠A and ∠D are vertex angles

∠B and ∠C are the non-vertex angles.

Lines AD and BC are diagonals and always meet at right angles

Line AD, the axis of symmetry, bisects diagonal BC, bisects ∠A and ∠D and bisects the kite into 2 congruent triangles: △ABD and △ACD

Side AB = side AC

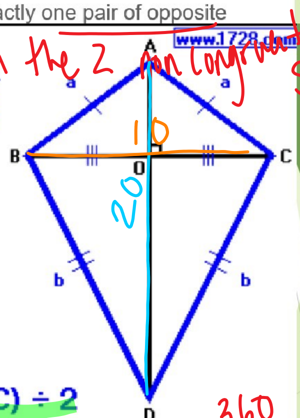
Side BD = side CD

Line OB = Line OC

Diagonal BC bisects the kite into 2 isoceles triangles

Kite Area = (AD × BC) ÷ 2

between the 2 non congruent sides



$\frac{20 \times 10}{2} = \frac{200}{2}$

100

360
− 200
160

100

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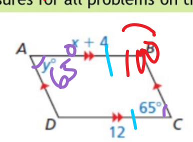
90 45 45 35 35 70

*Find all angles and side measures for all problems on this page!

Find the values of x and y.



$x + 4 = 12$
$-4 \quad -4$

If a quadrilateral is a kite, then exactly one pair of opposite

∠A and ∠D are vertex angles between the 2 non congruent sides

∠B and ∠C are the non-vertex angles.

Lines AD and BC are diagonals and always meet at right angles

Line AD, the axis of symmetry, bisects diagonal BC, bisects ∠A and ∠D and bisects the kite into 2 congruent triangles: △ABD and △ACD

Side AB = side AC

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Diagonal BC bisects the kite into 2 isoceles triangles

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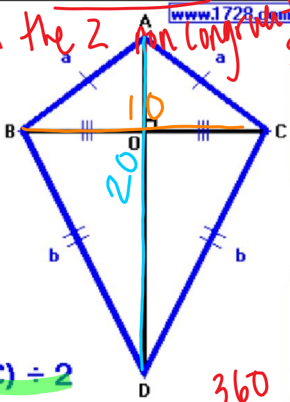

$$\frac{20 \times 10}{2} = \frac{200}{2}$$

100

360
− 200
160

100

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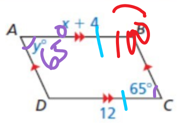
90 45 45 35 35 70

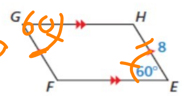
*Find all angles and side measures for all problems on this page!

Find the values of x and y.

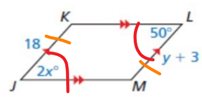


$x + 4 = 12$
$-4 \quad -4$
$x = 8$

1. Find FG and $m\angle G$.



2. Find the values of x and y.



$\frac{2x}{2} = \frac{50}{2}$

$x = 25$

$y + 3 = 18$
$-3 \quad -3$
$y = 15$

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Example 2

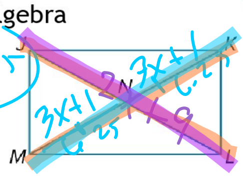
Use Properties of Rectangles and Algebra

3.5 + 9 =

Check $2(1.75) + 9 = 12.5$

Quadrilateral *JKLM* is a rectangle.

Part A $2x + 9$



If $MN = 3x + 1$ and $JL = 2x + 9$, find MK. Round to the nearest tenth if necessary.

$3(1.75) + 1 = 6.25$
$\times 2$
12.5

$\frac{4x}{4} = \frac{7}{4} = 1.75$

$6x + 2 = 2x + 9$
$-2x \quad -2x$
$4x + 2 = 9$
$-2 \quad -2$

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Example 2

Use Properties of Rectangles and Algebra

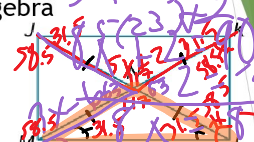
$\frac{8x}{8} = \frac{88}{8}$

180
− 63
117 / 2 = 58.5

$4\quad 3x + 3 + 5x - 1 = 90$

Check

Quadrilateral *JKLM* is a rectangle.



180
− 117
63 / 2
31.5

3(23) − 6

Part B

If $m\angle JNK = (5x + 2)^\circ$ and $m\angle JNM = (3x - 6)^\circ$,

$3x - 6 + 5x + 2 = 180$

$8x - 4 = 180$
$+4 \quad +4$

$\frac{8x}{8} = \frac{184}{8}$

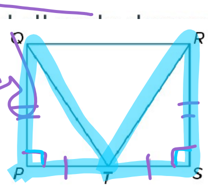
$x = 23$

Example 3

Prove Rectangular Relationships

Given: *PQRS* is a rectangle; $\overline{PT} \cong \overline{ST}$.

Prove: $\overline{QT} \cong \overline{RT}$



Statements	**Reasons**
1. *PQRS* is a rectangle; $\overline{PT} \cong \overline{ST}$	**1.** Given

$4x + 2 = 9$

Students, draw anywhere on this slide!

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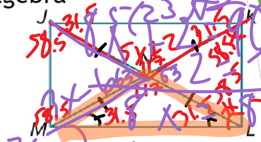
Example 2
Use Properties of Rectangles and Algebra

Check

Quadrilateral *JKLM* is a rectangle.



$3x + 3 + 5x - 1 = 90$

$\frac{8x}{8} = \frac{88}{8}$

$180 - 63 = 117 / 2 = 58.5$

$3(23) - 6$

$180 - 117 = 63 / 2 = 31.5$

$3x - 6 + 5x + 2 = 180$

$8x - 4 = 180$
$+4 \quad +4$

$\frac{8x}{8} = \frac{184}{8}$

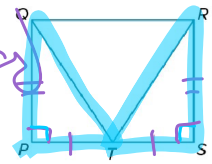
$x = 23$

Part B

If $m\angle JNK = (5x + 2)°$ and $m\angle JNM = (3x - 6)°$,

Example 3
Prove Rectangular Relationships

Given: *PQRS* is a rectangle; $\overline{PT} \cong \overline{ST}$.
Prove: $\overline{QT} \cong \overline{RT}$



Statements	Reasons
1. *PQRS* is a rectangle; $\overline{PT} \cong \overline{ST}$	1. Given
2. *PQRS* is a parallelogram	2. Definition of rectangle
3. PS = QR and QP = RS	3. Opp. sides of a ▱ are ≅.
4. ∠S and ∠P are Right ∠'s; 90 = 90	4. Definition of rectangle
5. ∠S ≅ ∠P	5. All right angles are congruent.
6. △QPT ≅ △RST	6. SAS
7. QT ≅ RT	7. CPCTC

Students, draw anywhere on this slide!

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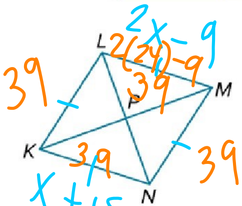
If *LM* = 2*x* − 9 and *KN* = *x* + 15 in rhombus *KLMN*, find the value of *x*.

Find all side lengths!



$2x - 9 = x + 15$
$-x \quad -x$

$x - 9 = 15$
$+9 \quad +9$

$x = 24$

Students, draw anywhere on this slide!

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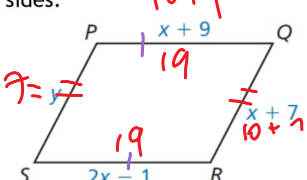
3. $RS \cong QP$
4. ∠S and ∠P are right angles.
6. $\triangle RST \cong \triangle$
7. $QT \cong RT$

For what values of *x* and *y* is quadrilateral *PQRS* a parallelogram?
Find the lengths of all the sides.



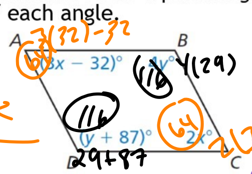
$x + 9 = 2x - 1$
$-x \quad -x$

$9 = x - 1$
$+1 \quad +1$

$10 = x$

$10 + 9$, 19, $17 = y$, $10 + 7 = 17$, $2(10) - 1$, 19

For what values of *x* and *y* is quadrilateral *ABCD* a parallelogram?
Determine the measures of each angle.



$3x - 32 = 2x$
$+32 \quad +32$

$3x = 2x + 32$
$-2x \quad -2x$

$3(32) - 32$, 64, 116, $y(29)$, $29 + 87$, $2(32)$

Students, draw anywhere on this slide!

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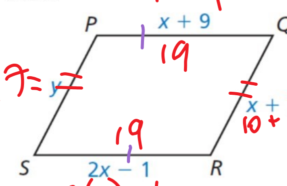
3. $RS \cong QP$
4. $\angle S$ and $\angle P$ are right angles.
6. $\triangle RST \cong \triangle$
7. $QT \cong RT$

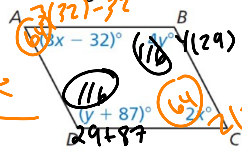
For what values of x and y is quadrilateral *PQRS* a parallelogram?
Find the lengths of all the sides.



$x + 9 = 2x - 1$
$-x \quad -x$
$9 = x - 1$
$+1 \quad +1$
$10 = x$

$10 + 9$
19
$17 = y$
$10 + 7 = 17$
19
$2(10) - 1$

For what values of x and y is quadrilateral *ABCD* a parallelogram?
Determine the measures of each angle.



$3(32) - 32$
64
116
116
64
$y(29)$
$29 + 87$
$2(32)$

$3x - 32 = 2x$
$+32 \quad +32$
$3x = 2x + 32$
$-2x \quad -2x$
$x = 32$

$4y = y + 87$
$-1y \quad -1y$
$3y = 87$
$y = 29$

diagonals bisect each other

$116 + 64 = 180$

Students, draw anywhere on this slide!

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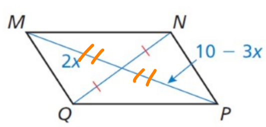
For what value of x is quadrilateral *MNPQ* a parallelogram?



diagonals bisect each other

$2x = 10 - 3x$
$+3x \quad +3x$
$5x = 10$
$x = 2$

Students, draw anywhere on this slide!

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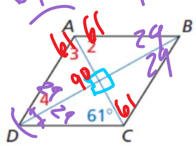
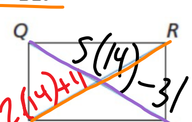
Find the measures of the numbered angles in rhombus *ABCD*.



$\angle F + 118 = 180$
$-118 \quad -118$
$\angle F = 62$
$\angle A + \angle D = 180$
$180 - 122 = 58$
$\frac{58}{2} = 29$

In rectangle *QRST*, $QS = 5x - 31$ and $RT = 2x + 11$.
Find the lengths of the diagonals of *QRST*.



diagonals of a rectangle congruent

$2x + 11 = 5x - 31$
$-2x \quad -2x$

$5(14) - 31 = QS = 39$
$2(14) + 11$
$RT = 39$

Find the measures of the numbered angles in rhombus *ABCD*.

$\angle F + 118 = 180$

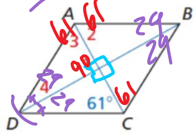
$-118 \quad -118$

$\angle F = 62$

$\angle A + \angle D = 180$

$180 - 122 = 58$

$\frac{58}{2} = 29$



In rectangle *QRST*, $QS = 5x - 31$ and $RT = 2x + 11$.
Find the lengths of the diagonals of *QRST*.

diagonals of a Rectangle congruent

$2x + 11 = 5x - 31$

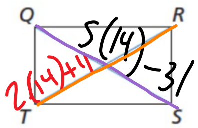
$-2x \quad -2x$

$+31 \quad +31$

$42 = 3x$

$\frac{42}{3} = \frac{3x}{3}$

$x = 14$

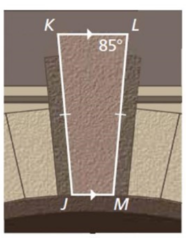


$5(14) - 31 = QS = 39$

$RT = 39$

Alt. Int

The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.

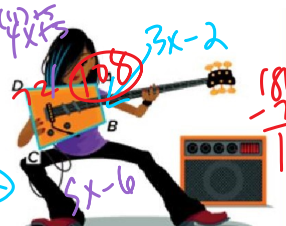


MUSIC The body of the guitar shown is a trapezoidal prism. The front face of the guitar is an isosceles trapezoid.
$AB = 3x - 2$, $CD = 3x + 9$,
$AD = 4x + 5$, and $BC = 5x - 6$.

Part A Prove $x = 11$.

Part B Find $m\angle A$ if $m\angle C = 72°$.

Part C Find the perimeter of the front face of the guitar in centimeters.



$3(11) + 9 = 42$

$5(11) - 6 = 49$

$108°$

171

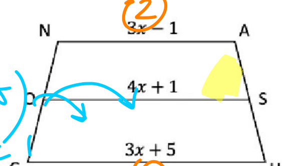
$4x + 5 = 5x - 6$

$-4x \quad -4x$

$5 = x - 6$

$11 = x$

$\overline{OS}$ is the median of trapezoid *NACHOS*, find the value of the median, given the following:



$4x + 1 = \frac{1}{2}(3x - 1 + 3x + 5)$

$4x + 1 = 3x + 2$

$-3x \quad -3x$

$1x + 1 = 2$

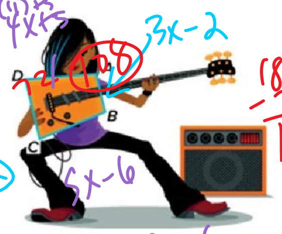
$x = 1$

MUSIC The body of the guitar shown is a trapezoidal prism. The front face of the guitar is an isosceles trapezoid.
AB = 3*x* − 2, *CD* = 3*x* + 9,
AD = 4*x* + 5, and *BC* = 5*x* − 6.

Part A Prove *x* = 11.

Part B Find *m∠A* if *m∠C* = 72°.

Part C Find the perimeter of the front face of the guitar in centimeters.



$3x-2$ $3x+9$ $5x-6$

31 42 49 49

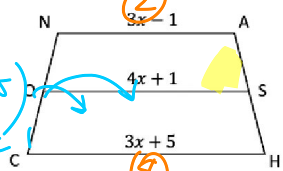
$108°$

$180 - 72 = 108$

171

$4x+5=5x-6$
$-4x \quad -4x$
$5=x-6$
$11=x$

$\overline{OS}$ is the median of trapezoid *NACHOS*, find the value of the median, given the following:



$4x+1=\frac{1}{2}(3x-1+3x+5)$

$4(1)+1=5$

$4x+1=\frac{1}{2}(6x+4)$

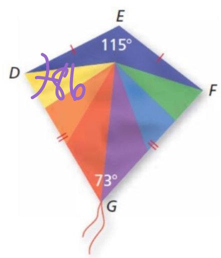
$4x+1=3x+2$
$-3x \quad -3x$
$1x+1=2$
$-1 \quad -1$

$x=1$

$mN=\frac{1}{2}(12+28)$

Find *m* ∠*D* in the kite shown.

Find m<F in the kite.



$115+73=188$

$360-188=\frac{172}{2}$

$D \,\&\, F = 86°$

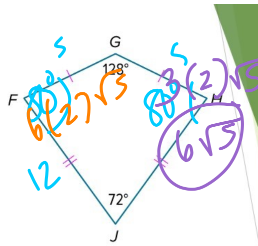
Example 6

Find Angle Measures in Kites

Check

If *FGHJ* is a kite, find *m∠F*.

Also find m<H

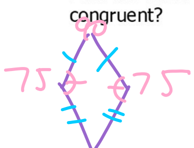


$128+72=200$

$360-200=160$

$\frac{160}{2}=80$

In a kite, the measures of the angles are 3*x* °, 75°, 90°, and 120°. Find the value of *x* . What are the measures of the angles that are congruent?



$3x+75+90+120=360$

$3x+285=360$

3(25)

$a^2+b^2=c^2$

$6^2+12^2=c^2$

$36+144=c^2$

$180=c^2$

$\sqrt{180}=c$

$6\sqrt{5}$

$12^2+24^2=c^2$

$144+576=c^2$

$720=c^2$

$12\sqrt{5}$

Find Angle Measures in Kites

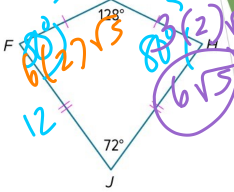
$128 + 72 = 200$

Check

$360 - 200 = 160$

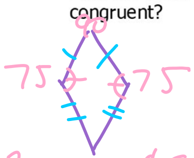
If *FGHJ* is a kite, find $m\angle F$.

Also find m<H



$\frac{160}{2} = 80$

In a kite, the measures of the angles are $3x$°, 75°, 90°, and 120°. Find the value of x. What are the measures of the angles that are congruent?



$3x + 75 + 90 + 120 = 360$

$3x + 285 = 360$

$a^2 + b^2 = c^2$

$6^2 + 12^2 = c^2$

$36 + 144 = c^2$

$180 = c^2$

$\sqrt{180} = c$

$6\sqrt{5}$

$12^2 + 24^2 = c^2$

$144 + 576 = c^2$

$720 = c^2$

$12\sqrt{5}$

$6\sqrt{5} + 6\sqrt{5} + 12\sqrt{5} + 12\sqrt{5}$

Students, draw anywhere on this slide!

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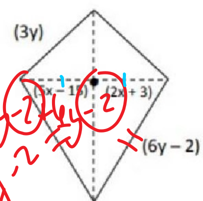
Kite's Perimeter = 86 ft. Determine the value of x and y.



$5x - 15 = 2x + 3$

$-2x \quad -2x$

$3x - 15 = 3$

$+15 \quad +15$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

$3y + 3y + 6y - 2 + 6y - 2$

$18y - 4 = 86$

$+4 \quad +4$

$\frac{18y}{18} = \frac{90}{18}$

$y = 5$

Students, draw anywhere on this slide!

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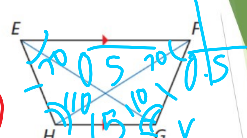
If *EG* = *FH*, is trapezoid *EFGH* isosceles? yes



If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid *EFGH* isosceles?

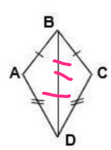
$15 + 9 = 24\left(\frac{1}{2}\right)$

12 ✓

If a quadrilateral is a kite, it has one diagonal forming two congruent triangles.

Given: kite ABCD

Prove: $\Delta BAD \cong \Delta BCD$



Proof:

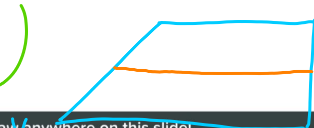
Statements	Reasons
1. kite ABCD	1. Given
2. $\overline{AD} \cong$ ___; $\overline{AB} \cong$ ___	2. A kite has two distinct sets of adjacent, congruent sides.
3. ___	3. Reflexive property.
4. $\Delta BAD \cong \Delta BCD$	4. ___

$12 = \frac{1}{2}(x + 9)$

$12 = 0.5x + 4.5$

$-4.5 \quad -4.5$

$7.5 = 0.5x$



Students, draw anywhere on this slide!

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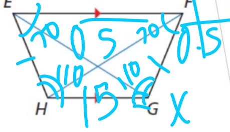
If $m\angle HEF = 70°$ and $m\angle FGH = 110°$,
is trapezoid $EFGH$ isosceles?



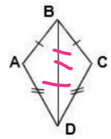
$15 + 9 = 24\left(\frac{1}{2}\right)$

12 ✓

If a quadrilateral is a kite, it has one diagonal forming two congruent triangles.

Given: kite ABCD

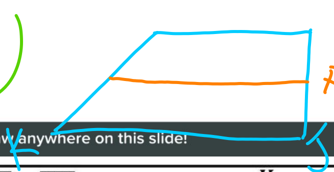
Prove: $\Delta BAD \cong \Delta BCD$



Proof:

Statements	Reasons
1. kite ABCD	1. Given
2. $\overline{AD} \cong$ ___; $\overline{AB} \cong$ ___	2. A kite has two distinct sets of adjacent, congruent sides.
3. ___	3. Reflexive property.
4. $\Delta BAD \cong \Delta BCD$	4. ___

$12 = \frac{1}{2}(x + 9)$



$12 = 0.5x + 4.5$

$-4.5 \quad -4.5$

$7.5 = 0.5x$

Students, draw anywhere on this slide!

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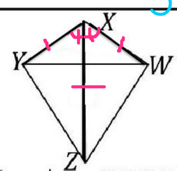
Given: $\overline{YX} \cong \overline{WX}$
$\overline{XZ}$ bisects $\angle YXW$

Prove: $\overline{YZ} \cong \overline{WZ}$



STATEMENT	REASON
1. $\overline{YX} \cong \overline{WX}$, $\overline{XZ}$ bisects $\angle YXW$	1.
2. <YXZ = <WXZ	2. Definition of bisector
3. $\overline{XZ} \cong \overline{XZ}$	3. Reflexive Property
4. $\Delta YXZ \cong$ WXZ	4. SAS
5. YZ=WZ	5. CPCTC

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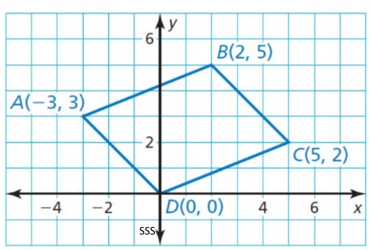
Show that quadrilateral *ABCD* is a parallelogram.

CD CB

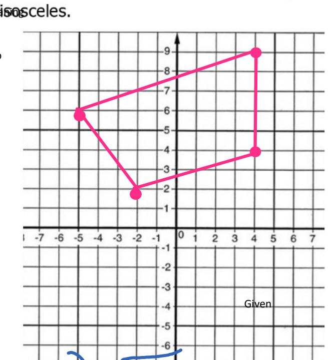
BD = BD



The points $A(-5, 6)$, $B(4, 9)$, $C(4, 4)$, and $D(-2, 2)$ form the vertices of a quadrilateral. Show that *ABCD* is a trapezoid. Then decide whether it is isosceles.

Slopes of AD and BC are congruent because
They are parallel -1/1

Slopes of AB and DC are congruent also
Parallel 2/5



$3\sqrt{5}$

Students, draw anywhere on this slide!

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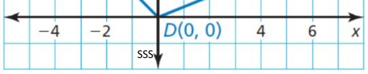
Distance Formula from point Y (4,8) and (7,2)
7-4 squared plus 2-8 squared
3 squared plus -6 squared
9 + 36
45

So square root of 45

Three vertices of ▱*ABCD* are $A(2, 4)$, $B(5, 2)$, and $C(3, -1)$. Find the coordinates of vertex D.

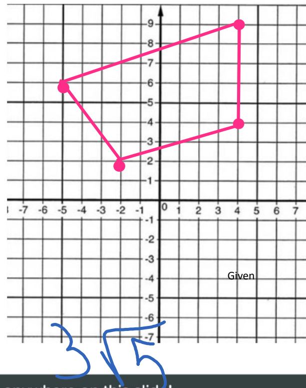
BD = BD



The points $A(-5, 6)$, $B(4, 9)$, $C(4, 4)$, and $D(-2, 2)$ form the vertices of a quadrilateral. Show that $ABCD$ is a trapezoid. Then decide whether it is isosceles.

Slopes of AD and BC are congruent meaning
They are parallel -1/1

Slopes of AB and DC are congruent also
Parallel 2/5



Students, draw anywhere on this slide!

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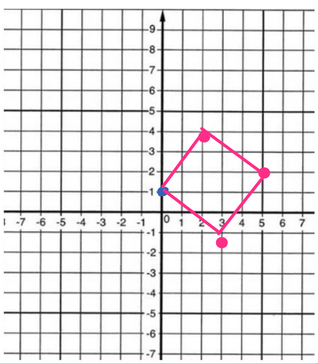
Distance Formula from point Y (4,8) and (7,2)
7-4 squared plus 2-8 squared
3 squared plus -6 squared
9 + 36
45

So square root of 45

Square root of 9 times square root of 5

Three vertices of ▱$ABCD$ are $A(2, 4)$, $B(5, 2)$, and $C(3, -1)$. Find the coordinates of vertex D.

(0,1)



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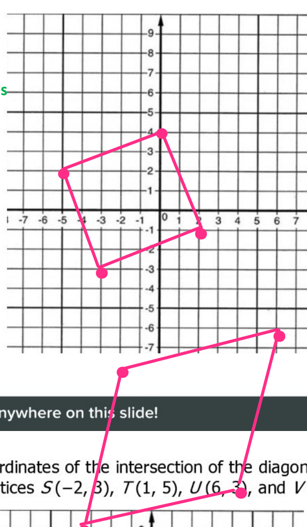
Decide whether ▱$PQRS$ with vertices $P(-5, 2)$, $Q(0, 4)$, $R(2, -1)$, and $S(-3, -3)$ is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

This is not an isosceles trapezoid
Since
The
Legs
Are
Not
Congruent

The slopes are -5/2 and 2/5
They are negative reciprocals
Therefore all angles are 90 degrees
Being perpendicular.

Each of the sides are congruent.

Therefore the parallelogram is
A rectangle, rhombus, and
Square!



Slopes: 4/1 and 1/4
The sides are congruent,
but the slopes
Are not negative reciprocals so
the angles
Are not 90 degrees.

Therefore the parallelogram is
A rhombus only.

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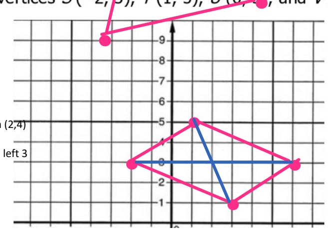
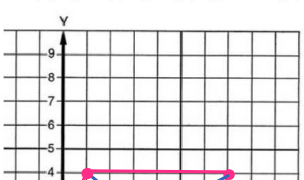
Find the coordinates of the intersection of the diagonals of ▱$STUV$ with vertices $S(-2, 3)$, $T(1, 5)$, $U(6, 3)$, and $V(3, 1)$.



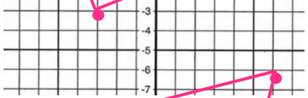
The slope is -2/3 so follow the slope from (2,4)
Go
Down 3 and left 2 or from (3,-1) up 2 and left 3

(2,3)

Find the coordinates of the intersection of the diagonals of ▱$LMNO$ with vertices $L(1, 4)$, $M(7, 4)$, $N(6, 0)$, and $O(0, 0)$.



(3.5,2)



Slopes: 4/1 and 1/4
The sides are congruent,
but the slopes
Are not negative reciprocals so
the angles
Are not 90 degrees.

Therefore the parallelogram is
A rhombus only.

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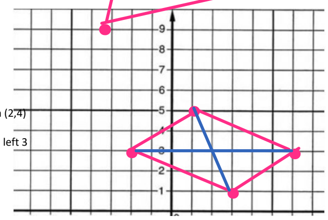
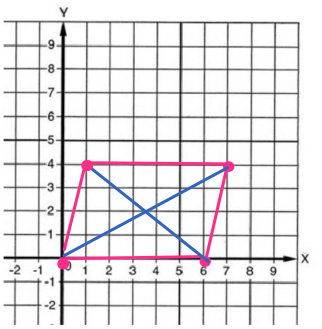
Find the coordinates of the intersection of the diagonals of ▱*STUV* with vertices *S*(−2, 3), *T*(1, 5), *U*(6, 3), and *V*(3, 1).



The slope is -2/3 so follow the slope from (2,4)
Go
Down 3 and left 2 or from (3,-1) up 2 and left 3

(2,3)

Find the coordinates of the intersection of the diagonals of ▱*LMNO* with vertices *L*(1, 4), *M*(7, 4), *N*(6, 0), and *O*(0, 0).



(3 5,2)

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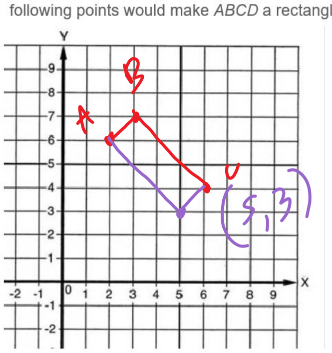
Check

A quadrilateral has vertices *A*(2, 6), *B*(3, 7), and *C*(6, 4). Which of the following points would make *ABCD* a rectangle?



(5,3)

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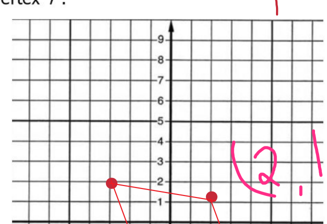
Three vertices of ▱*WXYZ* are *W*(−1, −3), *X*(−3, 2), and *Z*(4, −4). Find the coordinates of vertex *Y*.



Follow the slope up 5 and left two
(2,1) is the missing vertex.

(2,1)

Answer *true* or *false*.

1. All rectangles are parallelograms. T
2. All squares are rectangles. T
3. All rhombi are squares. F
4. All squares are parallelograms. T
5. All rhombi are parallelograms. T

T

Three vertices of ▱$WXYZ$ are $W(-1, -3)$, $X(-3, 2)$, and $Z(4, -4)$. Find the coordinates of vertex Y.

Follow the slope up 5 and left two
(2,1) is the missing vertex.

(2,1)

Answer *true* or *false*.

1. All rectangles are parallelograms. T T
2. All squares are rectangles. T
3. All rhombi are squares. F
4. All squares are parallelograms. T T
5. All rhombi are parallelograms. F T

T

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