Module 10: Circles

Sunday, April 14, 2024 11:32 PM

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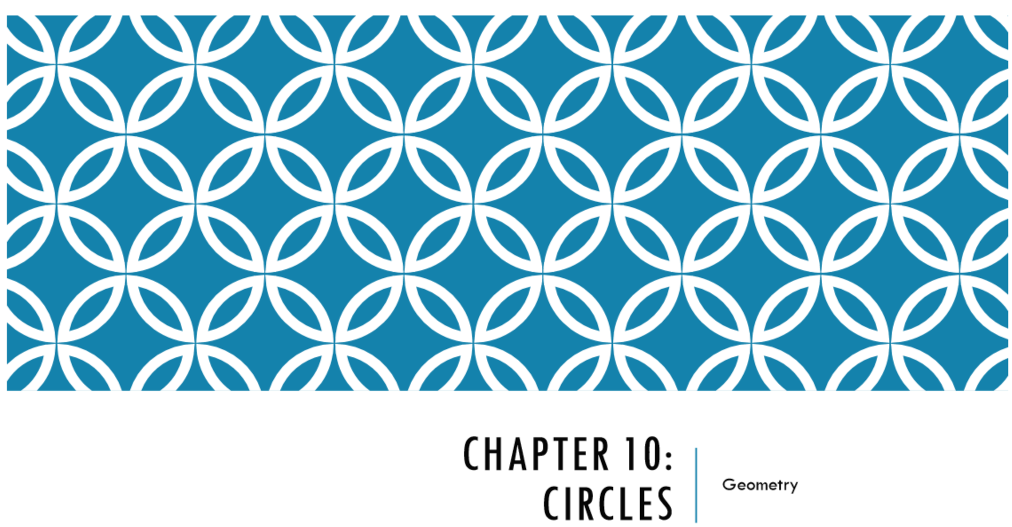
Geometry Module 10



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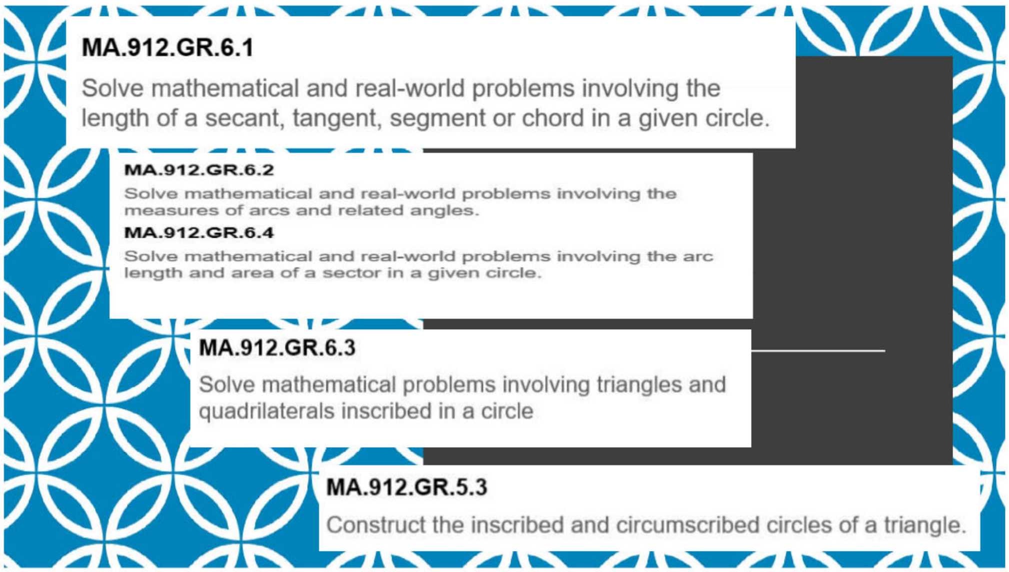
MA.912.GR.6.1
Solve mathematical and real-world problems involving the length of a secant, tangent, segment or chord in a given circle.
MA.912.GR.6.2
Solve mathematical and real-world problems involving the measures of arcs and related angles.
MA.912.GR.6.4
Solve mathematical and real-world problems involving the arc length and area of a sector in a given circle.
MA.912.GR.6.3
Solve mathematical problems involving triangles and quadrilaterals inscribed in a circle
MA.912.GR.5.3
Construct the inscribed and circumscribed circles of a triangle.

MA.912.GR.3.2

Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

MA.912.GR.3.3

Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

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MA.912.GR.7.2

Given a mathematical or real-world context, derive and create the equation of a circle using key features.

MA.912.GR.7.3

Graph and solve mathematical and real-world problems that are modeled with an equation of a circle. Determine and interpret key features in terms of the context.

Content Objective

Students find and apply the definitions of radius and diameter to find circumference and measures in intersecting circles.

Students find measures of angles and arcs in circles.

Students solve problems involving the relationship between arcs and chords, and radii that are perpendicular to chords.

Students solve problems using inscribed angles.

Students solve problems using relationships between circles and tangents and investigate constructions with circles.

Students solve problems using relationships between circles, tangents, and secants.

Students write and graph the equations of circles using key features.

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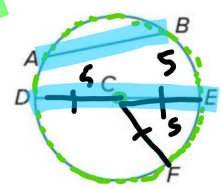
Parts of Circles

A **circle** is the set of all points in a plane that are the same distance from a given point called the **center of a circle**. The center of the circle below is C.

(5) A **radius of a circle** (plural radii) is a line segment from the center to a point on a circle. $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of circle C. Its measure is half the diameter.



A **chord of a circle** is a segment with endpoints on the circle. $\overline{AB}$ and $\overline{DE}$ are chords of circle C.

(10) A **diameter of a circle** is a chord that passes through the center of a circle. $\overline{DE}$ is a diameter of circle C. The measure is twice the radius.

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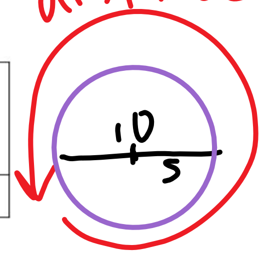
Parts of Circles

Key Concept: Circumference Formula

Words	If a circle has diameter *d* or radius *r*, the circumference C equals the diameter times pi or twice the radius times pi.
Symbols	$C = \pi d$ or $C = 2\pi r$

distance around the circle



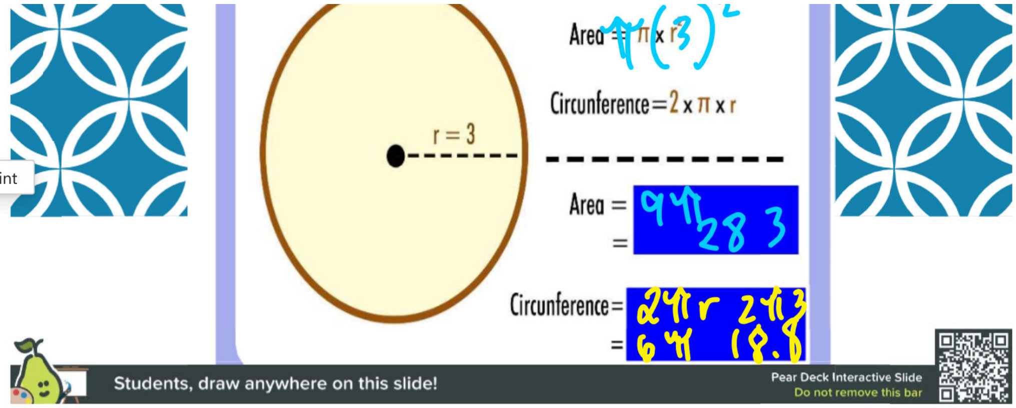
$C = 10\pi$

$C = 31.4$

$2\pi r$

$2(5)\pi$

$10\pi = 31.4$



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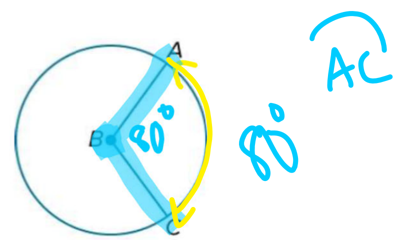
Measuring Angles and Arcs

A **central angle of a circle** is an angle with a vertex at the center of a circle and sides that are radii. $\angle ABC$ is a central angle of $\odot B$.

A **degree** is $\frac{1}{360}$ of the circular rotation about a point. This leads to the following relationship.

360



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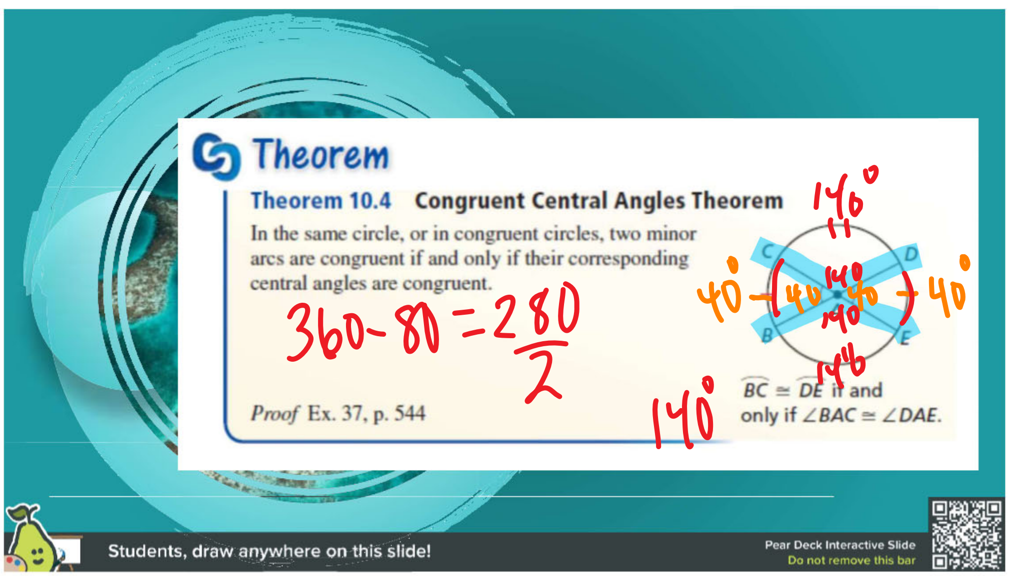

Theorem
Theorem 10.4 Congruent Central Angles Theorem
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.
360-80=280/2
140°
Proof Ex. 37, p. 544
140°
40°
40°
C
D
B
E
BC ≅ DE if and only if ∠BAC ≅ ∠DAE.
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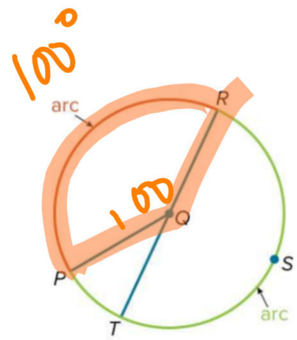
Measuring Angles and Arcs

An **arc** is part of a circle that is defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.

A **minor arc** has a measure less than 180°. The measure of a minor arc is equal to the measure of its related central angle.



$m\,\widehat{PR} = m\angle PQR.$

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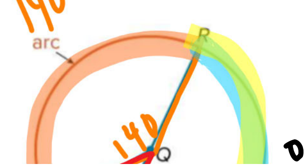
Measuring Angles and Arcs

minoR arc 140° (less than 180)

A **major arc** has a measure greater than 180°. The measure of a major arc is equal to 360° minus the measure of the minor arc with the same endpoints. $m\,\widehat{PSR} = 360° - m\,\widehat{PR}$.



PR

360
-140
220°

PSR

260

A **semicircle** is an arc that measures exactly 180°. The endpoints of a semicircle lie on a diameter. $m\overset{\frown}{RST} = 180°$.

Congruent arcs are arcs in the same or congruent circles that have the same measure.

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Arc Length and Radian Measure

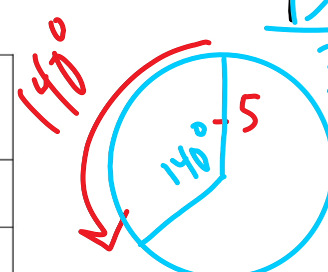
$2\pi 5 \cdot \frac{140}{360} =$

$\frac{4398.2}{360}$

Key Concept: Arc Length in Degrees

Words	The ratio of the length of an arc ℓ to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360°.
Proportion	$\frac{\ell}{2\pi r} = \frac{x}{360°}$
Equation	$\ell = \frac{x}{360°} \cdot 2\pi r$

$C = 2\pi r \cdot \frac{140}{360}$



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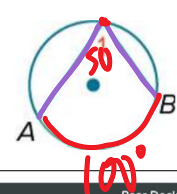
Inscribed Angles

For each of these cases, the following theorem holds true.

Theorem 10.7: Inscribed Angle Theorem

Words	If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.
Example	$m\angle 1 = \frac{1}{2} m\,\widehat{AB}$ and $m\,\widehat{AB} = 2m\angle 1$ Central $\angle$ 50 

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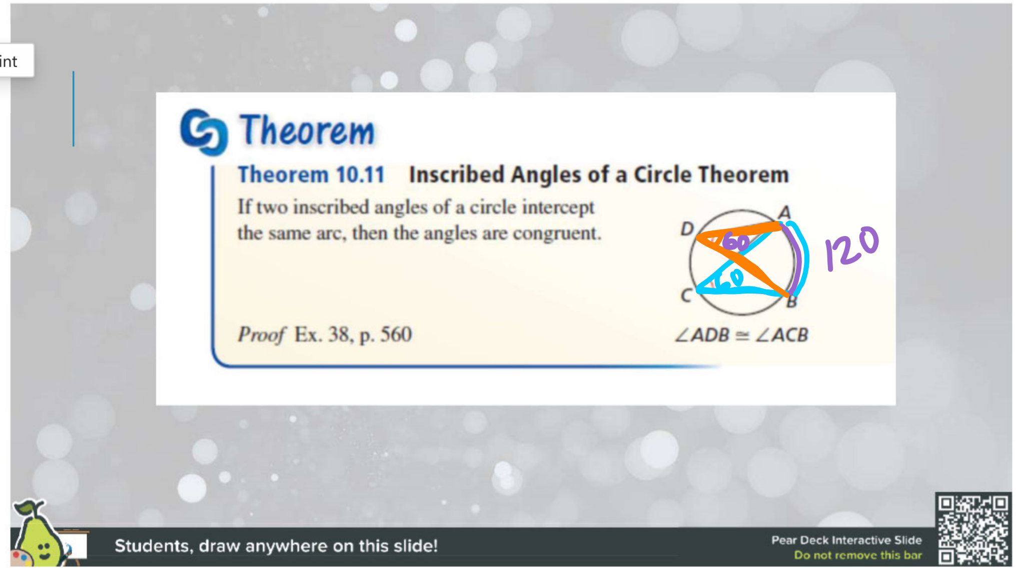
Theorem
Theorem 10.11 Inscribed Angles of a Circle Theorem
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.
D
A
C
B
60
60
120
Proof Ex. 38, p. 560
∠ADB ≅ ∠ACB
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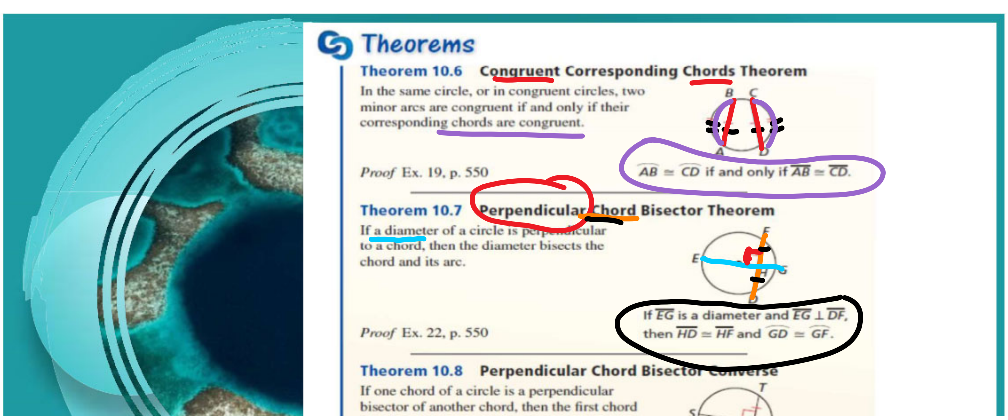
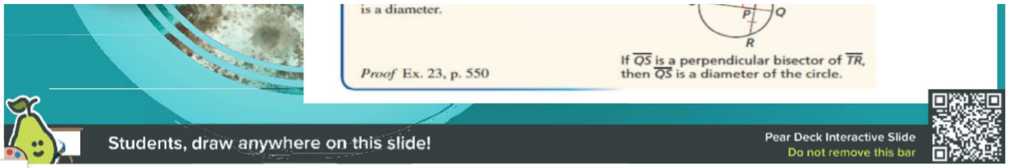

Theorems
Theorem 10.6 Congruent Corresponding Chords Theorem
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
B
C
A
D
Proof Ex. 19, p. 550
AB ≅ CD if and only if AB ≅ CD.
Theorem 10.7 Perpendicular Chord Bisector Theorem
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
F
E
H
G
D
Proof Ex. 22, p. 550
If EG is a diameter and EG ⊥ DF, then HD ≅ HF and GD ≅ GF.
Theorem 10.8 Perpendicular Chord Bisector Converse
If one chord of a circle is a perpendicular bisector of another chord, then the first chord
T
S

1



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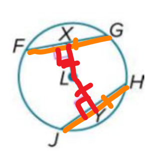
Arcs and Chords

In addition to Theorem 10.3, you can use the following theorem to determine whether two chords in a circle are congruent.

Theorem 10.6

Words	In the same circle or in congruent circles, chords are congruent if and only if they are equidistant from the center.	
Example	$\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.	

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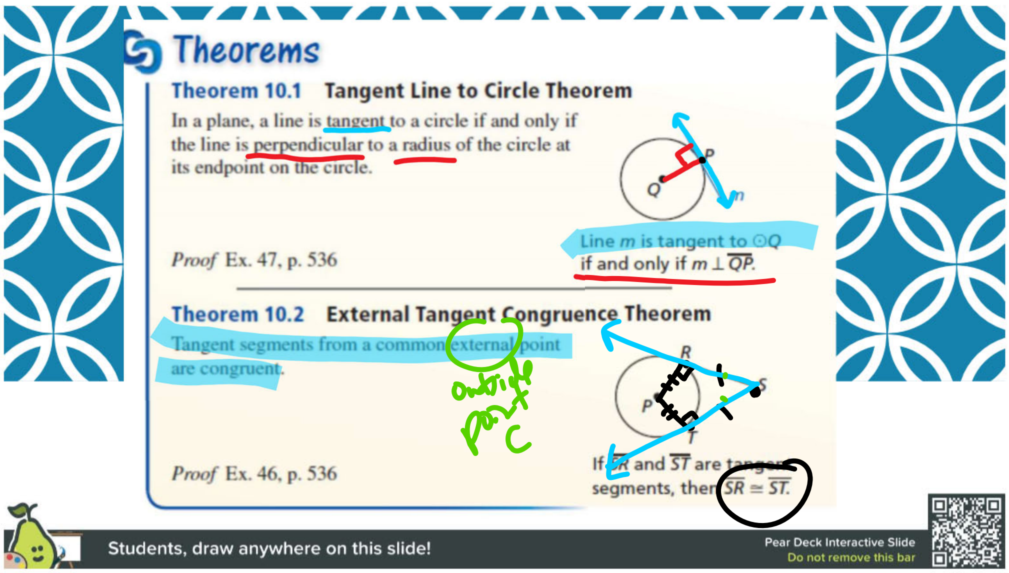
Theorems
Theorem 10.1 Tangent Line to Circle Theorem
In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.
Proof Ex. 47, p. 536
Line m is tangent to ⊙Q if and only if m ⊥ QP.
Theorem 10.2 External Tangent Congruence Theorem
Tangent segments from a common external point are congruent.
Proof Ex. 46, p. 536
If SR and ST are tangent segments, then SR ≅ ST.
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Theorem

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.

Proof Ex. 33, p. 568

minor arc <180

$m\angle 1 = \frac{1}{2}m\widehat{AB}$ $\quad m\angle 2 = \frac{1}{2}m\widehat{BCA}$

$50 = \frac{1}{2}(100)$ $\quad 130 = \frac{1}{2}(260)$

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Theorems

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.

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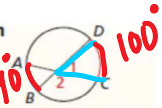


Proof Ex. 35, p. 568

$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

$\angle 1 = \frac{1}{2}(100+40)$

$\frac{1}{2}(140)$

$\angle 1 = 70°$

Theorem 10.16 Angles Outside the Circle Theorem

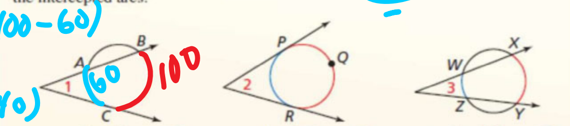
If a tangent and a secant, two tangents, *or* two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.

−

$\frac{1}{2}(100-60)$

$\frac{1}{2}(40)$

$\angle 1 = 20$



$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$ $\quad m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$ $\quad m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

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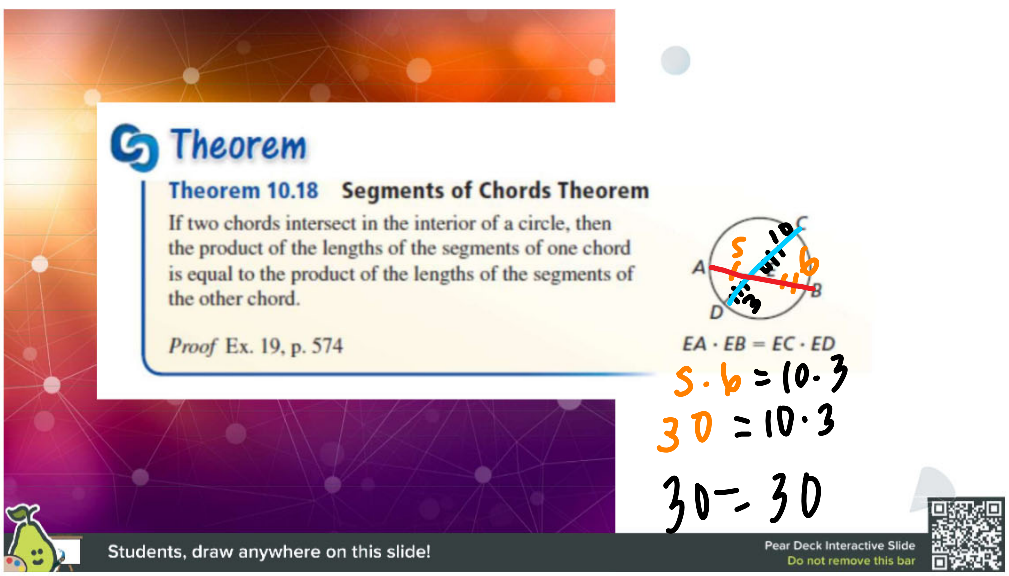
Theorem
Theorem 10.18 Segments of Chords Theorem
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.
Proof Ex. 19, p. 574
A
B
C
D
5
6
10
3
EA · EB = EC · ED
5 · 6 = 10 · 3
30 = 10 · 3
30 = 30
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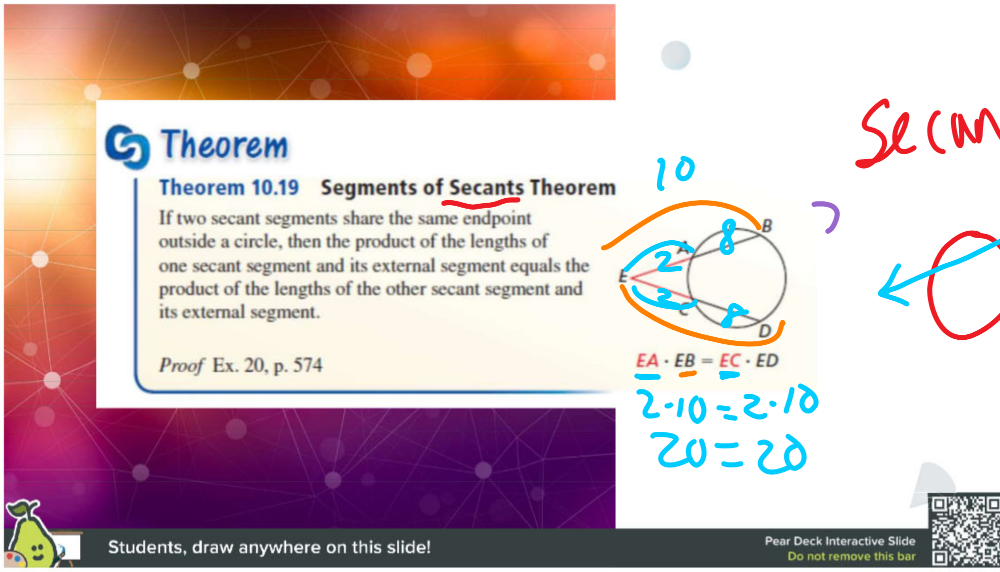
Theorem
Theorem 10.19 Segments of Secants Theorem
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.
Proof Ex. 20, p. 574
10
2
8
2
8
A
B
C
D
E
$EA \cdot EB = EC \cdot ED$
$2 \cdot 10 = 2 \cdot 10$
$20 = 20$
Secant
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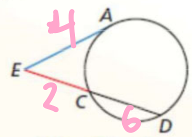
Theorem

Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

Proof Exs. 21 and 22, p. 574



$EA^2 = EC \cdot ED$

$4^2 = 2 \cdot 8$

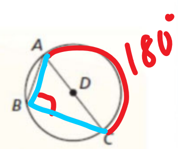
$16 = 16$

Theorems

Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

Proof Ex. 39, p. 560



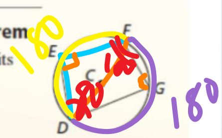
$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

$$\begin{array}{r} 360 \\ -180 \\ \hline 180 \end{array}$$

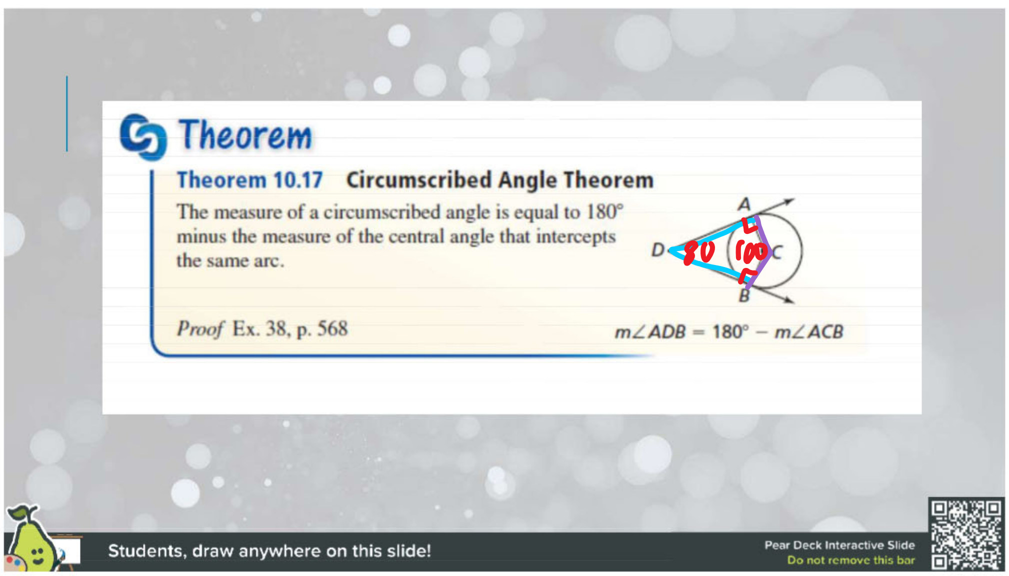
Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

Proof Ex. 40, p. 560



D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.


Theorem
Theorem 10.17 Circumscribed Angle Theorem
The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.
A
D
80
100
C
B
Proof Ex. 38, p. 568
m∠ADB = 180° − m∠ACB
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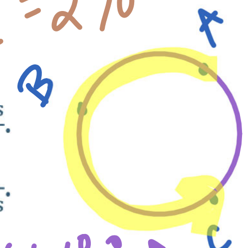
Arc Length and Radian Measure

$\frac{270}{1} \times \frac{\pi}{180} = \frac{270\pi}{180}$ $\frac{3\pi}{2}$

$\overset{\frown}{ABC} = 270°$



Key Concept: Degree and Radian Conversion Rules

1. To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180°}$.

2. To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

$\frac{4}{5}\pi \cdot \frac{180}{\pi} = \frac{720\cancel{\pi}}{5\cancel{\pi}} = 144°$

$\frac{4\pi}{5}$

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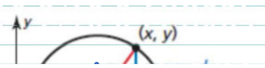
Core Concept

Standard Equation of a Circle

Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean



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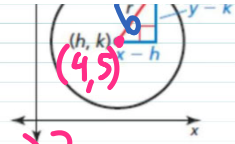
Theorem (Theorem 9.1),

$(x - h)^2 + (y - k)^2 = 6^2$.

This is the **standard equation of a circle** with center (h, k) and radius r.



$(x-4)^2+(y-5)^2 = 36$

Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of ⊙ C.

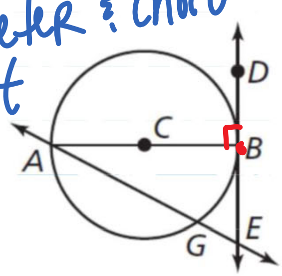
a. $\overline{AC}$ Radius

b. $\overline{AB}$ diameter & chord

c. $\overrightarrow{DE}$ Tangent

d. $\overleftrightarrow{AE}$ secant

e. AG Chord



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Example 3

Find Circumference and Area

TRAFFIC CIRCLES Traffic circles, also known as roundabouts, are circular roadways that reflow traffic in one direction around an island. A car enters a traffic circle and is 18 meters from the center of the island. If the car drives around the traffic circle until it is back to its original position, what is the circumference of the car's path? What is the area of the roundabout, including the island in the middle?

radius = 18

$C = 2\pi r$

$C = 2\pi(18)$

$C = 36\pi = 113.1$

$A = \pi r^2$

$A = \pi(18)^2$

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$A = 324\pi = 1017.9$

Example 4

Find Diameter and Radius

Find the diameter and radius of a circle to the nearest hundredth if the circumference of the circle is 77.8 centimeters.

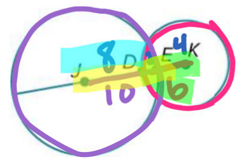
$C = 2\pi r$ or πd

$\frac{77.8}{2} = \frac{2\pi r}{2}$

$\frac{38.9}{\pi} = \frac{\pi r}{\pi}$

$\boxed{r = 12.38 \quad d = 24.76}$

The diameter of $\odot K$ is 12 units, the diameter of $\odot J$ is 20 units, and JD = 8 units. Find EK. 4



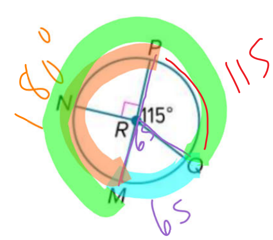
Example 2

Classify Arcs and Find Arc Measures

$\overline{PM}$ is a diameter of $\odot R$. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.

a. $\overset{\frown}{MQ}$ central angle minor arc
65°

b. $\overset{\frown}{MNP}$ semi circle
180



180

c. $\overset{\frown}{MNQ}$ major arc $180° + 115° = 295°$

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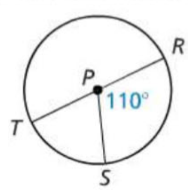
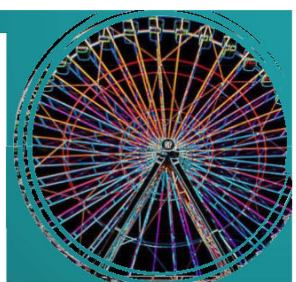
Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.

a. $\overset{\frown}{RS}$

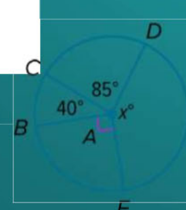
b. $\overset{\frown}{RTS}$

c. $\overset{\frown}{RST}$





Find the value of x.



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