# Special Right Triangles

Monday, February 26, 2024 10:07 PM

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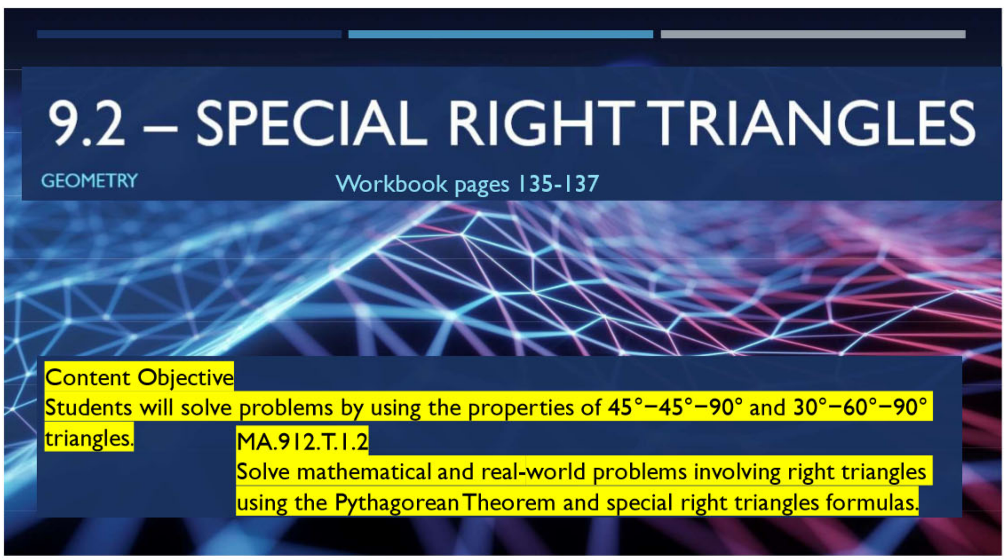
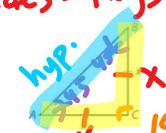
special right



Locker

## Explore 1 Investigating an Isosceles Right Triangle

Discover relationships that always apply in an isosceles right triangle. 2 sides = legs




C = 90
S = 180

(A) The figure shows an isosceles right triangle. Identify the base angles, and use the fact that they are complementary to write an equation relating their measures. 90°

$\angle A$ and $\angle B$ = 90°

(B) Use the Isosceles Triangle Theorem to write a different equation relating the base angle measures.

$x + x = 90$ $\quad \frac{2x}{2} = \frac{90}{2}$ $\quad x = 45°$

$$\begin{array}{r} 180 \\ -\ 90 \\ \hline 90 \end{array}$$

(C) What must the measures of the base angles be? Why?

45° same (isosceles)

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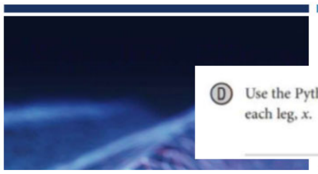
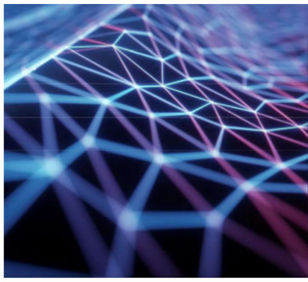


## Explore 1 Investigating an Isosceles Right Triangle

(D) Use the Pythagorean Theorem to find the length of the hypotenuse in terms of the length of each leg, $x$.



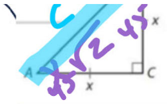
$a^2+b^2=c^2$

$1x^2+1x^2=c^2$

$\sqrt{2x^2}=\sqrt{c^2}$

$\sqrt{2\cdot x\cdot x}=c$

$x\sqrt{2}=c$



b $\sqrt{4\cdot 4}$

4

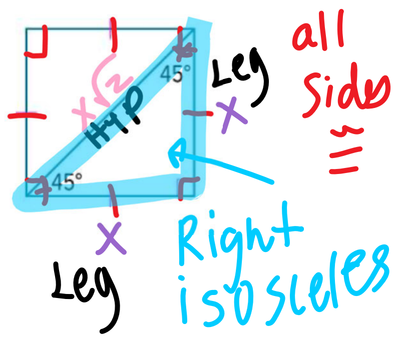
a $\sqrt{25}$

$\sqrt{5\cdot 5}=5$

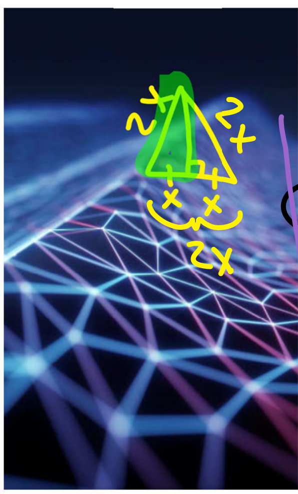
## Learn

$45° - 45° - 90°$ Triangles

The diagonal of a square forms two congruent isosceles right triangles. Because the base angles of an isosceles triangle are congruent, the measure of each acute angle is $90° \div 2$ or $45°$. Such a special right triangle is known as a $\mathbf{45° - 45° - 90°}$ **triangle.**




***In a** $45° - 45° - 90°$ triangle, the legs $\ell$ are congruent and the length of the hypotenuse $h$ is $\sqrt{2}$ times the length of a leg.




## Explore 2 Investigating Another Special Right Triangle

Discover relationships that always apply in a right triangle formed as half of an equilateral triangle.

all sides ≅ 90

Ⓐ $\triangle ABD$ is an equilateral triangle and $\overline{BC}$ is a perpendicular from $B$ to $\overline{AD}$. Determine all three angle measures in $\triangle ABC$.

$x+x+x=180$

$3x=180$

$\frac{3x}{3}=\frac{180}{3}$

$x=60$


Ⓑ Explain why $\triangle ABC \cong \triangle DBC$.

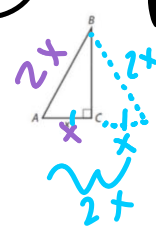
SSS ASA
SAS
AAS

SAS

BC=BC
Reflexive
Prop

Ⓒ Let the length of $\overline{AC}$ be $x$. What is the length of $\overline{AB}$, and why?

2x all sides are 2x

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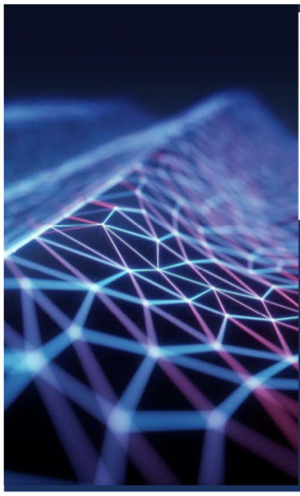


## Explore 2 Investigating Another Special Right Triangle

(D) Using the Pythagorean Theorem, find the length of $\overline{BC}$.

$a^2+b^2=c^2$

$x^2+b^2=(2x)^2$

$2x \cdot 2x$

$x^2+b^2=4x^2$

$-x^2 \qquad -1x^2$

$\sqrt{b^2}=\sqrt{3x^2}$

$b=\sqrt{3 \cdot x \cdot x} = x\sqrt{3}$

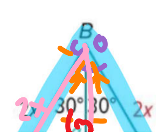

### Learn

30° – 60° – 90° Triangles

A **30° – 60° – 90° triangle** is a special right triangle or right triangle with side lengths that share a special relationship. You can use an equilateral triangle to find this relationship.

When an altitude is drawn from any vertex of an

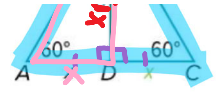
$\overline{BD}$ is a perp. bisect




Equilateral all sides

equilateral triangle, two congruent $30° - 60° - 90°$ triangles are formed. In the figure,

bisect also < bisector




2x

≅ angles each ≅ 60°

$\triangle ABD \cong \triangle CBD$, so $\overline{AD} \cong \overline{CD}$. If $AD = x$, then $CD = x$ and $AC = 2x$. Because $\triangle ABC$ is equilateral, $AB = 2x$ and $BC = 2x$.

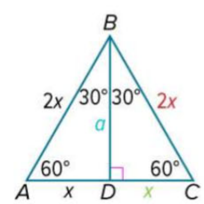
(continued on the next slide)

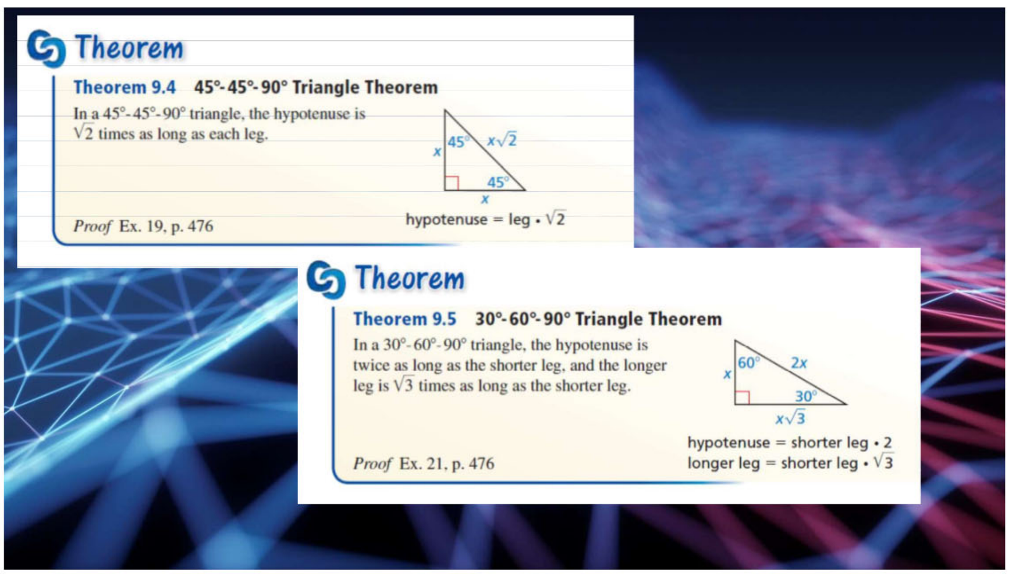
**Learn**

$30° - 60° - 90°$ Triangles

Use the Pythagorean Theorem to find $a$, the length of the altitude $\overline{BD}$, which is also the longer leg of $\triangle BDC$.

| | |
|---|---|
| $a^2 + x^2 = (2x)^2$ | Pythagorean Theorem |
| $a^2 + x^2 = 4x^2$ | Simplify. |
| $a^2 = 3x^2$ | Subtract $x^2$ from each side. |
| $a = x\sqrt{3}$ | Simplify. |





## Theorem

**Theorem 9.4 45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



hypotenuse = leg • $\sqrt{2}$

*Proof* Ex. 19, p. 476

## Theorem

**Theorem 9.5 30°-60°-90° Triangle Theorem**

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



hypotenuse = shorter leg • 2

longer leg = shorter leg • $\sqrt{3}$

*Proof* Ex. 21, p. 476