

Special Right Triangles

Monday, February 26, 2024 10:07 PM

Click Link Below to Open the Interactive Pear Deck PowerPoint

<https://app.peardeck.com/student/twobctvds>



special
right

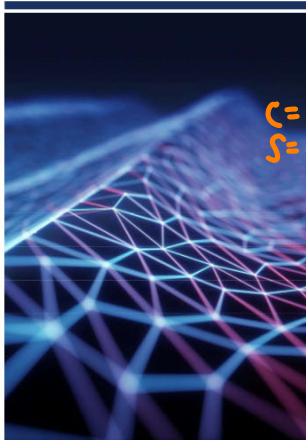
The image shows the cover of a worksheet titled "9.2 – SPECIAL RIGHT TRIANGLES". The background is a dark blue grid of glowing lines. The title is in large white letters. Below the title, it says "GEOMETRY" and "Workbook pages 135-137". A yellow box contains the "Content Objective" and a description of the problems to be solved. Another yellow box contains the standard "MA.912.T.1.2" and a description of the tasks.

9.2 – SPECIAL RIGHT TRIANGLES

GEOMETRY Workbook pages 135-137

Content Objective
Students will solve problems by using the properties of $45^\circ-45^\circ-90^\circ$ and $30^\circ-60^\circ-90^\circ$ triangles.

MA.912.T.1.2
Solve mathematical and real-world problems involving right triangles using the Pythagorean Theorem and special right triangles formulas.



$C = 90$
 $S = 180$

Explore 1 Investigating an Isosceles Right Triangle

Discover relationships that always apply in an isosceles right triangle.

- (A) The figure shows an isosceles right triangle. Identify the base angles, and use the fact that they are complementary to write an equation relating their measures.

$$\angle A \text{ and } \angle B = 90^\circ$$

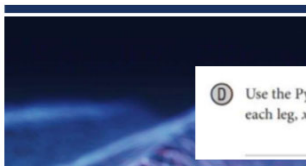
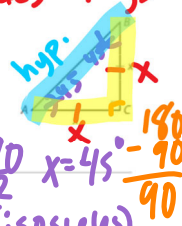
- (B) Use the Isosceles Triangle Theorem to write a different equation relating the base angle measures.

$$x + x = 90$$

- (C) What must the measures of the base angles be? Why?

45°

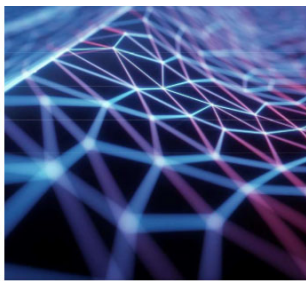
same (isosceles)



Explore 1 Investigating an Isosceles Right Triangle

- (D) Use the Pythagorean Theorem to find the length of the hypotenuse in terms of the length of each leg, x .





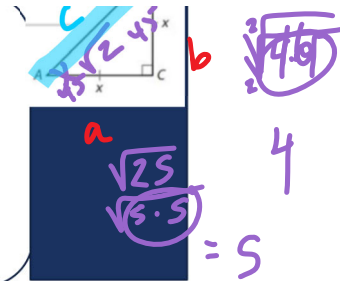
$$a^2 + b^2 = c^2$$

$$1x^2 + 1x^2 = c^2$$

$$\sqrt{2x^2} = \sqrt{c^2}$$

$$\sqrt{2 \cdot x \cdot x} = c$$

$$x\sqrt{2} = c$$



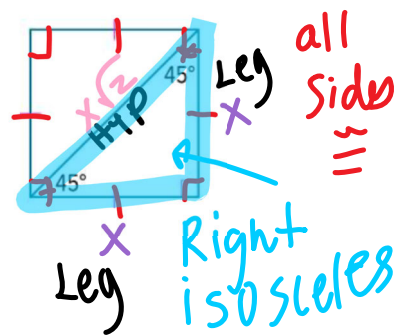
1.

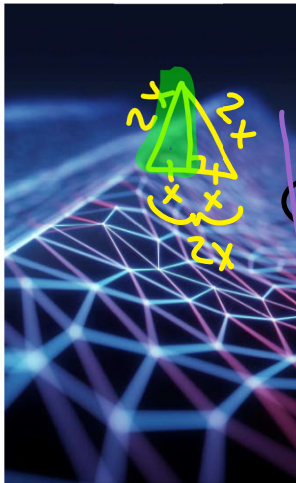
Learn

45° – 45° – 90° Triangles

The diagonal of a square forms two congruent isosceles right triangles. Because the base angles of an isosceles triangle are congruent, the measure of each acute angle is $90^\circ \div 2$ or 45° . Such a special right triangle is known as a **45° – 45° – 90° triangle**.

*In a 45° – 45° – 90° triangle, the legs ℓ are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.





Explore 2 Investigating Another Special Right Triangle

Discover relationships that always apply in a right triangle formed as half of an equilateral triangle.

- A $\triangle ABD$ is an equilateral triangle and BC is a perpendicular from B to AD . Determine all three angle measures in $\triangle ABC$.

- B Explain why $\triangle ABC \cong \triangle DBC$.

SSS ASA AAS

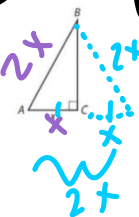
- C Let the length of AC be x . What is the length of AB , and why?

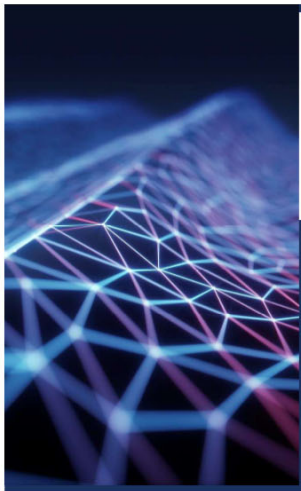
$2x$ all sides are $2x$



SAS

$BC = BC$
Reflexive Prop





Explore 2 Investigating Another Special Right Triangle

① Using the Pythagorean Theorem, find the length of $\overline{BC}$.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 x^2 + b^2 &= (2x)^2 \\
 x^2 + b^2 &= 4x^2 \\
 -x^2 & \quad -x^2 \\
 \hline
 b^2 &= 3x^2 \\
 b &= \sqrt{3x^2} = x\sqrt{3}
 \end{aligned}$$



Learn

$30^\circ - 60^\circ - 90^\circ$ Triangles

A $30^\circ - 60^\circ - 90^\circ$ triangle is a special right triangle or right triangle with side lengths that share a special relationship. You can use an equilateral triangle to find this relationship.

When an altitude is drawn from any vertex of an

$\overline{BD}$
is a
perp.



Equilateral
all sides

equilateral triangle, two congruent $30^\circ - 60^\circ - 90^\circ$ triangles are formed. In the figure,

$\triangle ABD \cong \triangle CBD$, so $\overline{AD} \cong \overline{CD}$. If $AD = x$, then $CD = x$ and $AC = 2x$. Because $\triangle ABC$ is equilateral, $AB = 2x$ and $BC = 2x$.

(continued on the next slide)



also
bisects

2x

each angle
each $\angle = 60^\circ$

Learn

$30^\circ - 60^\circ - 90^\circ$ Triangles

Use the Pythagorean Theorem to find a , the length of the altitude $\overline{BD}$, which is also the longer leg of $\triangle BDC$.

$$a^2 + x^2 = (2x)^2$$

Pythagorean Theorem

$$a^2 + x^2 = 4x^2$$

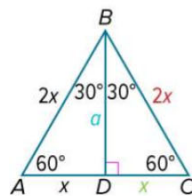
Simplify.

$$a^2 = 3x^2$$

Subtract x^2 from each side.

$$a = x\sqrt{3}$$

Simplify.

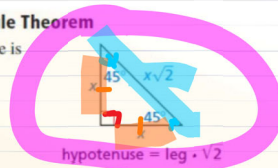


Theorem

Theorem 9.4 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

Proof Ex. 19, p. 476



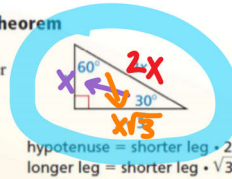
$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$

Theorem

Theorem 9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

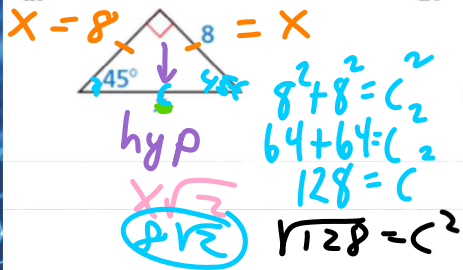
Proof Ex. 21, p. 476



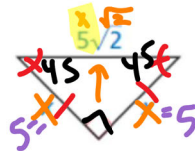
$$\begin{aligned} \text{hypotenuse} &= \text{shorter leg} \cdot 2 \\ \text{longer leg} &= \text{shorter leg} \cdot \sqrt{3} \end{aligned}$$

Find the value of x . Write your answer in simplest form.

a.



b.



$$\sqrt{64} \cdot \sqrt{2} = 8\sqrt{2}$$

Find the values of x and y . Write your answer in simplest form.

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{9}{\sqrt{3}}$$

$$x = \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3}$$

$$x = 3\sqrt{3}$$

$x = 3\sqrt{3}$



The road sign is shaped like an equilateral triangle.
 Estimate the area of the sign by finding the area of the equilateral triangle.

36 in.

YIELD

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(36)(18\sqrt{3})$

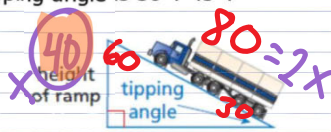
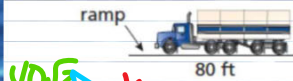
$A = (18)(18)\sqrt{3}$

$A = 324\sqrt{3}$

Handwritten calculations and diagrams show the derivation of the area of the equilateral triangle road sign. The sign is labeled with a side length of 36 in. and the word "YIELD". The calculations use the formula $A = \frac{1}{2}bh$ and the height of an equilateral triangle, $h = \frac{\sqrt{3}}{2}s$, to find the area.

A tipping platform is a ramp used to unload trucks. How high is the end of an 80-foot ramp when the tipping angle is 30° ? 45° ?

$$x = 40 \quad \frac{2x}{2} = \frac{80}{2}$$



$$\frac{x\sqrt{3}}{2} = \frac{80}{2} \quad \text{circled } 40\sqrt{3}$$

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{80}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{80\sqrt{2}}{2} = \text{circled } 40\sqrt{2}$$