Module 3: Logic & Line Relationships

Geometry

Inductive vs. Deductive Reasoning

Inductive Reasoning

Inductive reasoning uses observation to form a hypothesis or conjecture. The hypothesis can then be tested to see if it is true. The test must be performed in order to confirm the hypothesis.

Example: Observe that the sum of the numbers 1 to 4 is $(4 \cdot 5/2)$ and that the sum of the numbers 1 to 5 is $(5 \cdot 6/2)$. Hypothesis: the sum of the first n numbers is $(n * (n+1)/2)$. Testing this hypothesis confirms that it is true.

Deductive Reasoning

Deductive reasoning argues that if something is true about a broad category of things, it is true of an item in the category.

Example: All birds have beaks. A pigeon is a bird; therefore, it has a beak.

There are two key types of deductive reasoning of which the student should be aware:

- **Law of Detachment.** **Given that $p \to q$, if p is true then q is true.** In words, if one thing implies another, then whenever the first thing is true, the second must also be true.

  Example: Start with the statement: "If a living creature is human, then it has a brain." Then because you are human, we can conclude that you have a brain.

- **Syllogism.** **Given that $p \to q$ and $q \to r$, we can conclude that $p \to r$.** This is a kind of transitive property of logic. In words, if one thing implies a second and that second thing implies a third, then the first thing implies the third.

  Example: Start with the statements: "If my pencil breaks, I will not be able to write," and "if I am not able to write, I will not pass my test." Then I can conclude that "If my pencil breaks, I will not pass my test."

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Geometry
Conditional Statements

A conditional statement contains both a hypothesis and a conclusion in the following form:

If **hypothesis**, then **conclusion**.

For any conditional statement, it is possible to create three related conditional statements, as shown below. In the table, p is the hypothesis of the original statement and q is the conclusion of the original statement.

Statements linked below must be either both true or both false.

Type of Conditional Statement	Example Statement is:
Original Statement: If p, then q. $(p \rightarrow q)$ • Example: If a number is divisible by 6, then it is divisible by 3. • The original statement may be either true or false.	TRUE
Converse Statement: If q, then p. $(q \rightarrow p)$ • Example: If a number is divisible by 3, then it is divisible by 6. • The converse statement may be either true or false, and this does not depend on whether the original statement is true or false.	FALSE
Inverse Statement: If not p, then not q. $(\sim p \rightarrow \sim q)$ • Example: If a number is not divisible by 6, then it is not divisible by 3. • The inverse statement is always true when the converse is true and false when the converse is false.	FALSE
Contrapositive Statement: If not q, then not p. $(\sim q \rightarrow \sim p)$ • Example: If a number is not divisible by 3, then it is not divisible by 6. • The Contrapositive statement is always true when the original statement is true and false when the original statement is false.	TRUE

Note also that:

- When two statements must be either both true or both false, they are called logically **equivalent statements**.
  - The original statement and the contrapositive are equivalent statements.
  - The converse and the inverse are equivalent statements.
- If both the original statement and the converse are true, the phrase **"if and only if"** (abbreviated **"iff"**) may be used. For example, **"A number is divisible by 3 iff the sum of its digits is divisible by 3."**

3.2 Statements, Conditionals, and Biconditionals

- A STATEMENT is any sentence that is either TRUE or FALSE.

- TRUTH VALUE is the truth or falsity of a statement.

Example- statements are represented by lower case letters (p, q, t, ..)

p: A triangle has 3 sides. True

-NEGATION the opposite of the original statement and has the opposite truth value.

~p: A triangle does not have 3 sides. False

-Compound statements – statements joined by the words AND or OR.

-Conjunction (AND) $p \wedge q$ is true ONLY when both statements are true.

-Disjunction (OR) $p \vee q$ is ONLY false when both statements are false.

-CONDITIONALS

-Conditional – If ….., then …. $p \rightarrow q$

The statement following "if" is the hypothesis. The statement following "then" is the conclusion.

-Converse – switch the hypothesis and conclusion $q \rightarrow p$

-Inverse – negate both the hypothesis and conclusion $\sim p \rightarrow \sim q$

-Contrapositive – switch and negate $\sim q \rightarrow \sim p$

-Logically Equivalent statements – have the same truth value. Conditional and Contrapositive; Converse and Inverse

-Biconditionals - "if and only if" $p \leftrightarrow q$

True only when BOTH the conditional and converse are true.

Module 3

Geometry
Basic Properties of Algebra

Properties of Equality and Congruence.

Property	Definition for Equality	Definition for Congruence
	For any real numbers **a**, **b**, and **c**:	For any geometric elements **a**, **b** and **c**. (e.g., segment, angle, triangle)
Reflexive Property	$a = a$	$a \cong a$
Symmetric Property	$If\ a = b, then\ b = a$	$If\ a \cong b, then\ b \cong a$
Transitive Property	$If\ a = b\ and\ b = c, then\ a = c$	$If\ a \cong b\ and\ b \cong c, then\ a \cong c$
Substitution Property	If $a = b$, then either can be substituted for the other in any equation (or inequality).	

More Properties of Equality. For any real numbers **a**, **b**, and **c**:

Property	Definition for Equality
Addition Property	$If\ a = b, then\ a + c = b + c$
Subtraction Property	$If\ a = b, then\ a - c = b - c$
Multiplication Property	$If\ a = b, then\ a \cdot c = b \cdot c$
Division Property	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$

Properties of Addition and Multiplication. For any real numbers **a**, **b**, and **c**:

Property	Definition for Addition	Definition for Multiplication
Commutative Property	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative Property	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive Property	$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$	

TWO COLUMN PROOFS

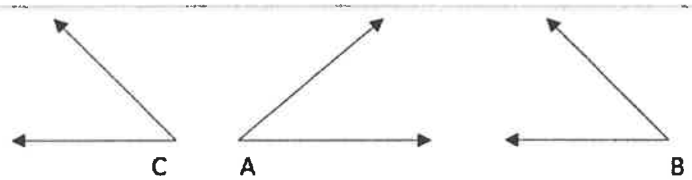
When writing your own two-column proof, keep these things in mind:

- **Number each step.**
- **Start with the given information.**
- **Statements with the same reason can be combined into one step.** It is up to you.
- **Draw a picture and mark it with the given information.**
- **You must have a reason for EVERY statement.**
- **The order of the statements in the proof is not always fixed, but make sure the order makes logical sense.**
- **Reasons will be definitions, postulates, properties and previously proven theorems.** "Given" is only used as a reason if the information in the statement column was given in the problem.
- **Use symbols and abbreviations for words within proofs.** For example, ≅ can be used in place of the word *congruent*. You could also use ∠ for the word angle.
- **What the proof asks you to prove will always be the last statement.**

EXAMPLE 1

Given: $\angle A$ and $\angle C$ are complementary
$\angle B$ and $\angle C$ are complementary

Prove: $\angle A \cong \angle B$



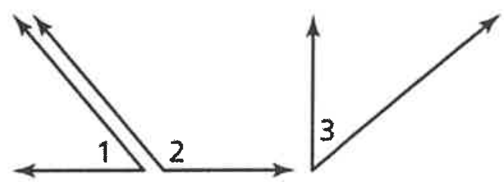
Statements	Reasons
1. $\angle A$ and $\angle C$ are complementary $\angle B$ and $\angle C$ are complementary	1. Given
2. $\angle A + \angle C = 90°$ $\angle B + \angle C = 90°$	2. Definition of Complementary Angles
3. $\angle A + \angle C = \angle B + \angle C$	3. Transitive Property
4. $\angle A = \angle B$	4. Subtraction Property
5. $\angle A \cong \angle B$	5. Definition of Congruent Angles

FLOW CHART PROOFS

Flowchart proofs are organized with boxes and arrows; each "statement" is inside the box and each "reason" is underneath each box. Each statement in a proof allows another subsequent statement to be made. In flowchart proofs, this progression is shown through arrows. Flowchart proofs are useful because it allows the reader to see how each statement leads to the conclusion.

EXAMPLE 2 Proving a Case of Congruent Supplements Theorem

Use the given two-column proof to write a flowchart proof that proves that two angles supplementary to the same angle are congruent.



Given $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 2$ are supplementary.

Prove $\angle 1 \cong \angle 3$

Flowchart Proof

$\boxed{\angle 1 \text{ and } \angle 2 \text{ are supplementary.}}$ → $\boxed{m\angle 1 + m\angle 2 = 180^\circ}$ → $\boxed{m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2}$

Given — Definition of supplementary angles

$\boxed{\angle 3 \text{ and } \angle 2 \text{ are supplementary.}}$ → $\boxed{m\angle 3 + m\angle 2 = 180^\circ}$ → $\boxed{m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2}$

Given — Definition of supplementary angles

$\boxed{m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2}$

↓ Transitive Property of Equality

$\boxed{m\angle 1 = m\angle 3}$

↓ Subtraction Property of Equality

$\boxed{\angle 1 \cong \angle 3}$

Definition of congruent angles

PARAGRAPH PROOFS

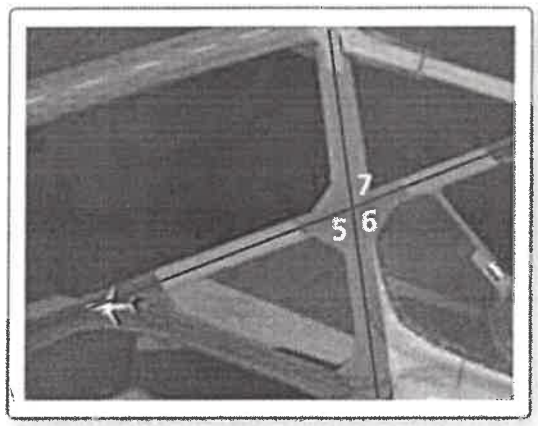
Another proof format is a paragraph proof, which presents the statements and reasons of a proof as sentences in a paragraph. It uses words to explain the logical fl ow of the argument.

EXAMPLE 3 Proving the Vertical Angles Congruence Theorem

Use the given paragraph proof to write a two-column proof of the Vertical Angles Congruence Theorem.

Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$



Paragraph Proof

$\angle 5$ and $\angle 7$ are vertical angles formed by intersecting lines. As shown in the diagram, $\angle 5$ and $\angle 6$ are a linear pair, and $\angle 6$ and $\angle 7$ are a linear pair. Then, by the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are supplementary and $\angle 6$ and $\angle 7$ are supplementary. So, by the Congruent Supplements Theorem, $\angle 5 \cong \angle 7$.

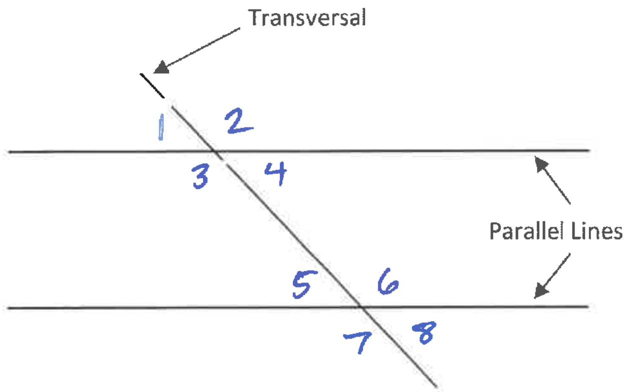
EXAMPLE 4 Algebraic 2-Column Proof

Given: $5(x + 2) = -3x - 6$ Prove: $x = -2$

$$5(x+2) = -3x-6$$
$$5x+10 = -3x-6$$
$$+3x \qquad +3x$$
$$8x+10 = -6$$
$$-10 \qquad -10$$
$$\frac{8x}{8} = \frac{-16}{8}$$
$$x = -2$$

Statements	Reasons
1. $5(x + 2) = -3x - 6$	1. Given
2. $5x + 10 = -3x - 6$	2. Distributive Property
3. $8x + 10 = -6$	3. Addition Property of Equality
4. $8x = -16$	4. Subtraction Property of Equality
5. $x = -2$	5. Division Property of Equality

Geometry
Parallel Lines and Transversals



Alternate: refers to angles that are on opposite sides of the transversal.

Consecutive: refers to angles that are on the same side of the transversal.

Interior: refers to angles that are between the parallel lines.

Exterior: refers to angles that are outside the parallel lines.

Corresponding Angles

Corresponding Angles are angles in the same location relative to the parallel lines and the transversal. For example, the angles on top of the parallel lines and left of the transversal (i.e., top left) are corresponding angles.

Angles 1 and 5 **(top left)** are **Corresponding Angles**. So are angle pairs 2 and 6 **(top right)**, 3 and 7 **(bottom left)**, and 4 and 8 **(bottom right)**. Corresponding angles are congruent.

Alternate Interior Angles

Angles 3 and 6 are **Alternate Interior Angles**. Angles 4 and 5 are also alternate interior angles. Alternate interior angles are congruent.

Alternate Exterior Angles

Angles 1 and 8 are **Alternate Exterior Angles**. Angles 2 and 7 are also alternate exterior angles. Alternate exterior angles are congruent.

Consecutive Interior Angles

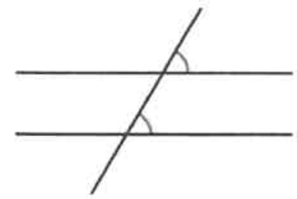
Angles 3 and 5 are **Consecutive Interior Angles**. Angles 4 and 6 are also consecutive interior angles. Consecutive interior angles are supplementary.

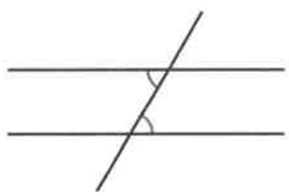
Geometry
Proving Lines are Parallel

The properties of parallel lines cut by a transversal can be used to prove two lines are parallel.

Corresponding Angles

If two lines cut by a transversal have congruent corresponding angles, then the lines are parallel. Note that there are 4 sets of corresponding angles.



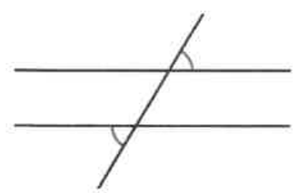


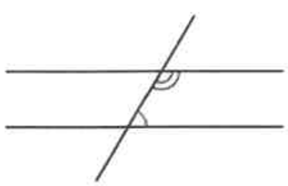
Alternate Interior Angles

If two lines cut by a transversal have congruent alternate interior angles congruent, then the lines are parallel. Note that there are 2 sets of alternate interior angles.

Alternate Exterior Angles

If two lines cut by a transversal have congruent alternate exterior angles, then the lines are parallel. Note that there are 2 sets of alternate exterior angles.





Consecutive Interior Angles

If two lines cut by a transversal have supplementary consecutive interior angles, then the lines are parallel. Note that there are 2 sets of consecutive interior angles.

Geometry

Parallel and Perpendicular Lines in the Coordinate Plane

Parallel Lines

Two lines are parallel if their slopes are equal.

- In $y = mx + b$ form, if the values of m are the same.

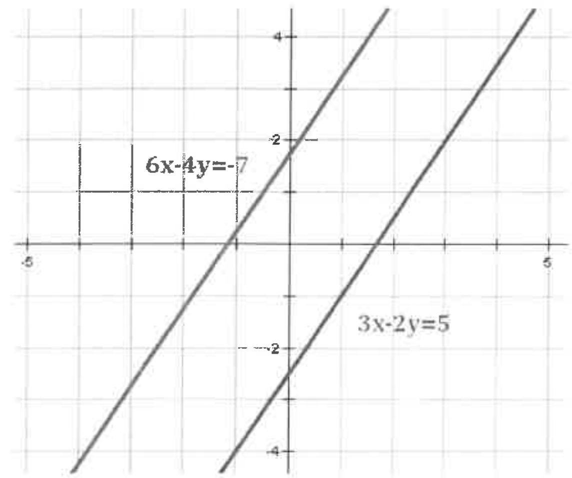
  Example: $y = 2x - 3$ and
  $y = 2x + 1$

- In Standard Form, if the coefficients of x and y are proportional between the equations.

  Example: $3x - 2y = 5$ and
  $6x - 4y = -7$

- Also, if the lines are both vertical (i.e., their slopes are undefined).

  Example: $x = -3$ and
  $x = 2$



$\perp$ m opposite reciprocal

Perpendicular Lines

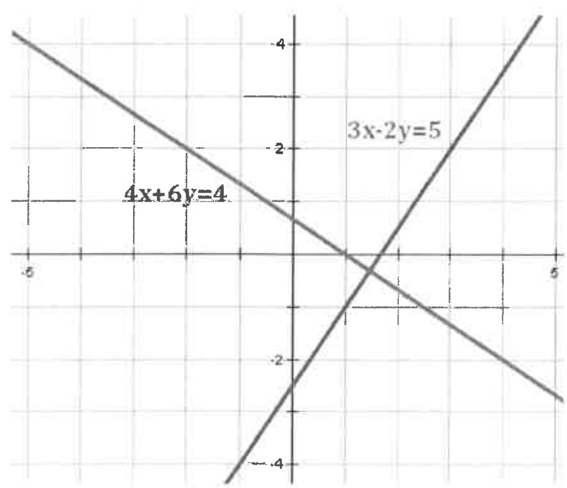
Two lines are perpendicular if the product of their slopes is -1. That is, if the slopes have different signs and are multiplicative inverses.

- In $y = mx + b$ form, the values of m multiply to get -1..

  Example: $y = 6x + 5$ and
  $y = -\frac{1}{6}x - 3$

- In Standard Form, if you add the product of the x-coefficients to the product of the y-coefficients and get zero.

  Example: $4x + 6y = 4$ and
  $3x - 2y = 5$ because $(4 \cdot 3) + (6 \cdot (-2)) = 0$



- Also, if one line is vertical (i.e., m is undefined) and one line is horizontal (i.e., $m = 0$).

  Example: $x = 6$ and
  $y = 3$

point-slope form $y - y_1 = m(x - x_1)$ slope $m = \frac{y_2 - y_1}{x_2 - x_1}$