Lesson 3.8 Slope and Equations of Lines

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Lesson 3.8 Slope Equ...

Lesson 3.8 Slope and Equations of Lines

Workbook pages 191 - 196

MA.912.GR.3.3

Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

Content Objective

Students classify lines as parallel, perpendicular, or neither by using the slope criteria.

Learn

Slope Criteria for Parallel and Perpendicular Lines

Slope is the ratio of the change in the *y*-coordinate (rise) to the corresponding change in the *x*-coordinate (run) as you move from one point to another along a line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1},$$

$\frac{\text{Rise}}{\text{Run}}$ ↑ + ↓ − ⇒ + ⇐ −

Slopes of Parallel Lines

Two distinct lines have the same slope if the lines are parallel. $\frac{3}{4} \parallel \frac{3}{4}$

Slopes of Perpendicular Lines

Two lines are perpendicular if and only if the product of their slopes is −1

*also known as negative or opposite reciprocals.

$\frac{1}{2} \cdot \frac{-2}{1}$ $\quad -\frac{3}{4} \quad \frac{4}{3}$ $\quad \frac{-3}{1} \cdot \frac{1}{3}$

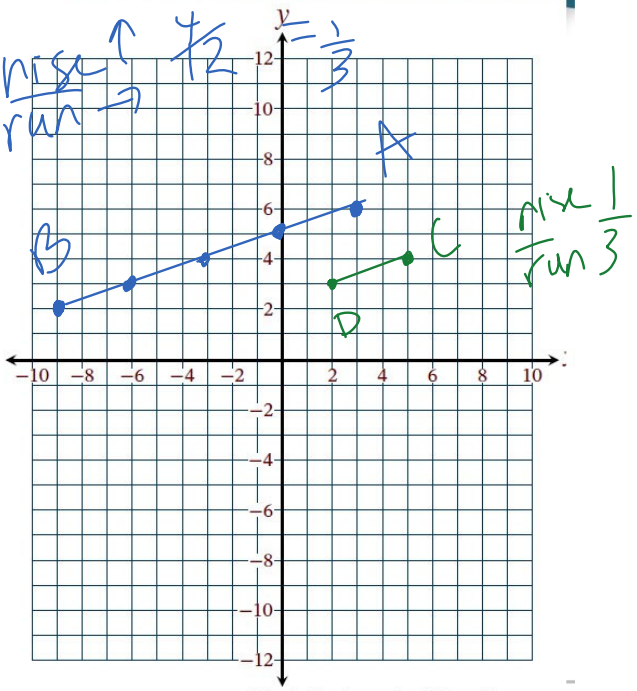
Example 1

Example 1
Determine Line Relationships When Giv

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for $A(3, 6)$, $B(-9, 2)$, $C(5, 4)$, and $D(2, 3)$. Graph each line to verify your answer.

slope $= \frac{y_2 - y_1}{x_2 - x_1}$, $\frac{2-6}{-9-3} = \frac{-4}{-12} = \frac{1}{3}$

$\frac{3-4}{2-5} = \frac{-1}{-3} = \frac{1}{3}$

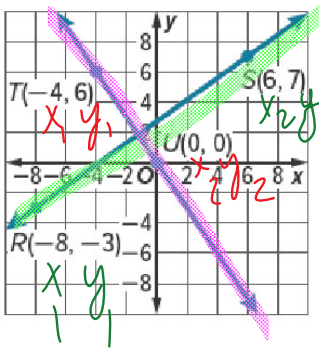


Example 2
Determine Line Relationships When Given Graphs

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

a. $\overline{RS}$ and $\overline{TU}$

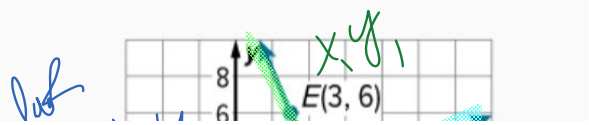
$\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{7+3}{6+8} = \frac{10}{14} = \frac{5}{7}$ RS



$\frac{10}{14} = \frac{5}{7}$ $\frac{0-6}{0+4} = \frac{-6}{4} = \frac{-3}{2}$

Example 2
Determine Line Relationships When Given Graphs

b. $\overleftrightarrow{EF}$ and $\overleftrightarrow{DG}$

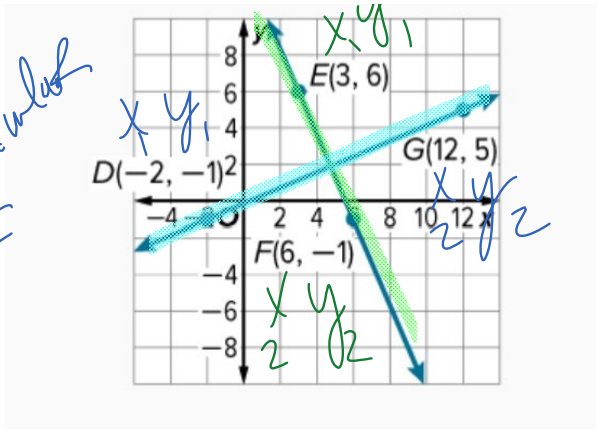


b. $\overline{EF}$ and $\overline{DG}$

$\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{-1-6}{6-3} = \frac{-7}{3}$

$\frac{5+1}{12+2} = \frac{6}{14} = \frac{3}{7}$

perpendicular

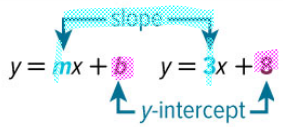
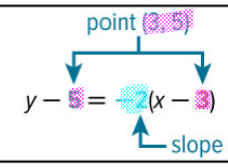


Learn
Equations of Lines

An equation of a nonvertical line can be written in different but equivalent forms.

Standard Form = $Ax + By = C$

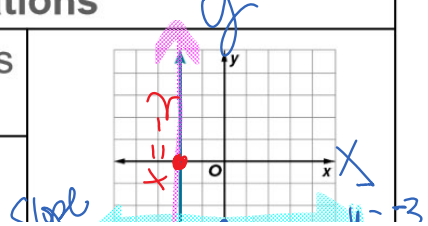
Slope = $-A/B$ y-int C/B

Key Concept: Nonvertical Line Equations	
The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept.	
The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is any point on the line and m is the slope of the line.	

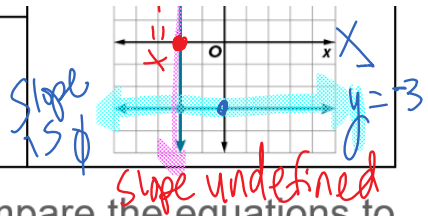
Learn
Equations of Lines

The equations of horizontal and vertical lines involve only one variable.

perpendicular

Key Concept: Horizontal and Vertical Line Equations	
The equation of a horizontal line is $y = b$, where b is the y-intercept of the line. $y = -3$	
The equation of a vertical line is $x = a$, where a is the x-intercept of the line.	

The equation of a vertical line is $x = a$, where a is the x-intercept of the line.



When given the equations of two lines, you can compare the equations to determine the relationship between the lines.

Example 3

Determine Line Relationships When Given Equations

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

a. $y = 3x - 2$; $y - 0 = -\frac{1}{3}(x - 2)$

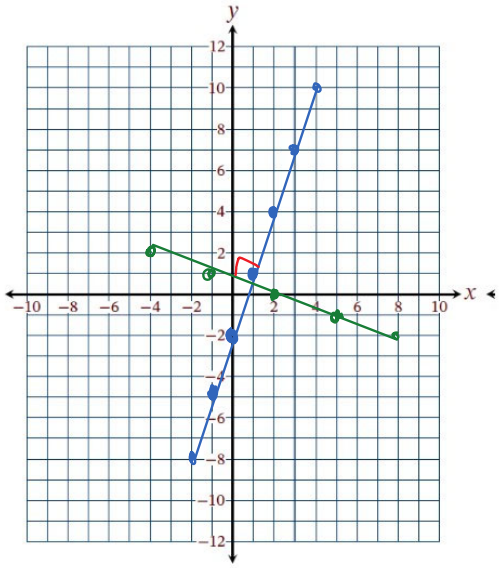
slope-intercept form | point-slope form

$y = 3x - 2$ $\qquad$ $y - 0 = -\frac{1}{3}(x - 2)$

slope

$\frac{3}{1}$ $\qquad$ $-\frac{1}{3}$

negative reciprocals (perpendicular)



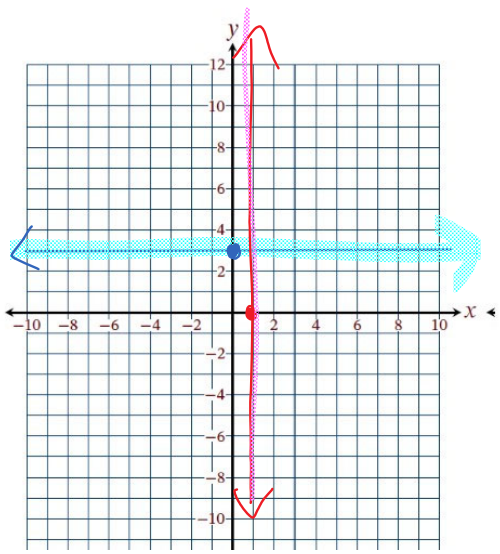
Example 3

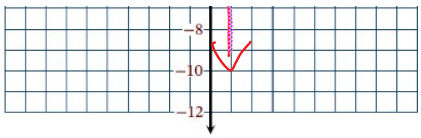
Determine Line Relationships When Given Equations

b. $y = 3$; $x = 1$

slope 0 $\qquad$ slope: undefined

perpendicular





Example 3

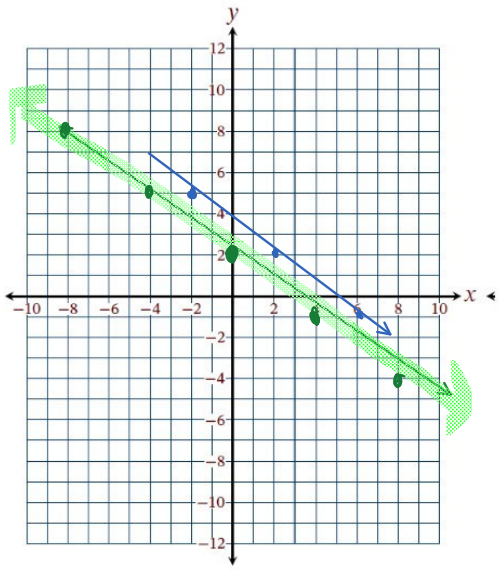
Determine Line Relationships When Given Equations

c. $y - 5 = -\frac{3}{4}(x + 2)$; $y = -\frac{3}{4}x + 2$

point-slope form: $y - 5 = -\frac{3}{4}(x + 2)$

slope-intercept form: $y = -\frac{3}{4}x + 2$

slope



Example 3

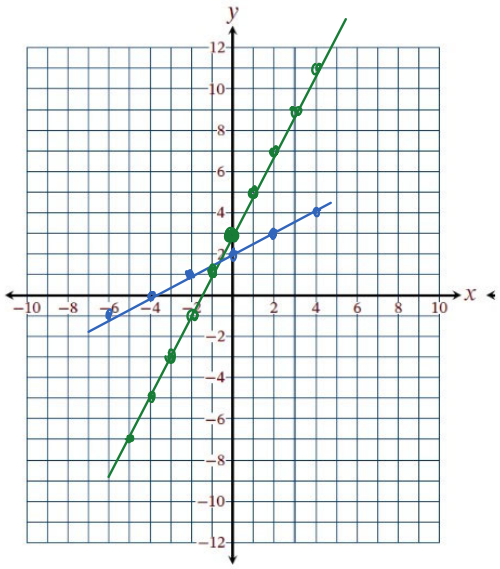
Determine Line Relationships When Given Equations

d. $y = 2x + 3$; $y - 1 = \frac{1}{2}(x + 2)$

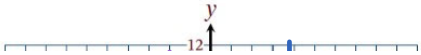
slope-intercept form: $y = 2x + 3$

point-slope form: $y - 1 = \frac{1}{2}(x + 2)$

slope



Determine Line Relationships When Given Equations

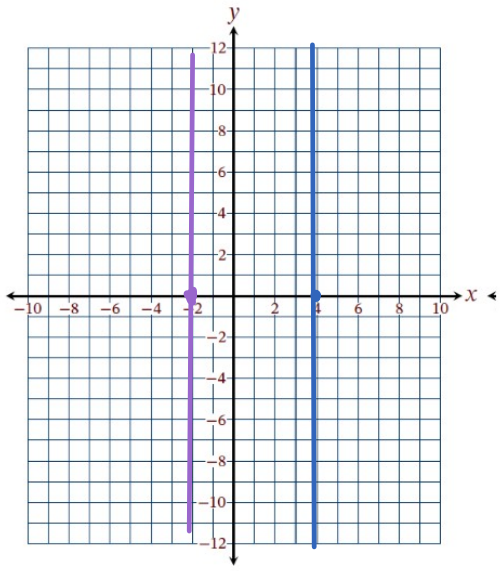


Determine Line Relationships When Given Equations

e. $x = -2$, $x = 4$

X-axis X-axis

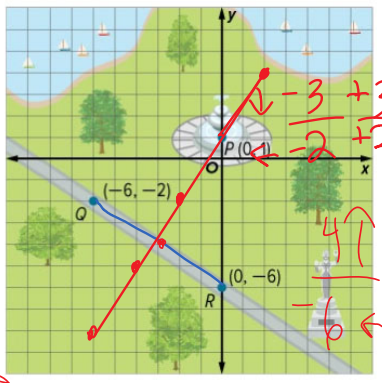
parallel lines



Example 4

Use Slope to Graph a Line

DESIGN Valentina is designing a park using grid paper. She wants to build a sidewalk that connects with the fountain at $P(0, 1)$ and is perpendicular to the existing sidewalk that passes through points $Q(-6, -2)$ and $R(0, -6)$. Graph the line that represents the new sidewalk.



x_1, y_1 x_2, y_2

$-3 + 3\uparrow$ $-2 + 2\rightarrow$

$4\uparrow$ Rise

$-6\leftarrow$ Run

new slope (opposite (neg.) reciprocals) $+\frac{3}{2}$

$\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{-6+2}{0+6} = \frac{-4\downarrow}{-6\leftarrow} = \frac{-2}{3}$ $\frac{2\uparrow}{-3\leftarrow}$

Example 5

Write Equations of Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line parallel to $y = -\frac{3}{4}x + 3$ containing $(-3, 6)$.

Same Slope

x_1, y_1

$y = mx + b$ Slope-intercept form

$6 = \frac{-3}{4}(-3) + b$

$b = 3.75$

$y = -\frac{3}{4}x + 3.75$

$6 = \frac{-3}{4}(-3) + b$

$6 = \frac{9}{4} + b$

$6 = 2.25 + b$

$-2.25 \quad -2.25$

$b = 3.75$

$y = \frac{-3}{4}x + 3.75$

$(y - y_1) = m(x - x_1)$

Point-slope form

$y - 6 = -\frac{3}{4}(x + 3)$

$y - 6 = -\frac{3}{4}x - \frac{9}{4}(-2.25)$

$+6 \quad +6$

$y = \frac{-3}{4}x + 3.75$