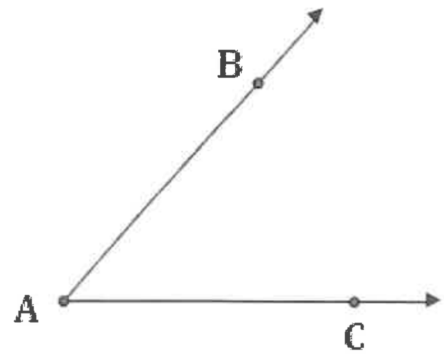
Geometry
Angles

Parts of an Angle

An **angle** consists of two rays with a common endpoint (or, initial point).

- Each ray is a **side** of the angle.
- The common endpoint is called the **vertex** of the angle.



Naming Angles

Angles can be named in one of two ways:

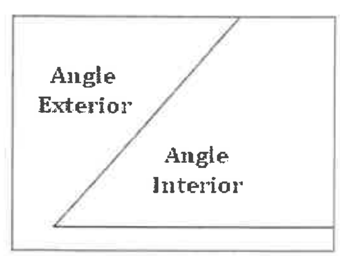
- **Point-vertex-point method.** In this method, the angle is named from a point on one ray, the vertex, and a point on the other ray. This is the most unambiguous method of naming an angle, and is useful in diagrams with multiple angles sharing the same vertex. In the above figure, the angle shown could be named $\angle BAC$ or $\angle CAB$.
- **Vertex method.** In cases where it is not ambiguous, an angle can be named based solely on its vertex. In the above figure, the angle could be named $\angle A$.

Measure of an Angle

There are two conventions for measuring the size of an angle:

- **In degrees.** The symbol for degrees is ^{o}. There are 360^{o} in a full circle. The angle above measures approximately **45^{o}** (one-eighth of a circle).
- **In radians**. There are 2π radians in a complete circle. The angle above measures approximately $\frac{1}{4}\pi$ **radians.**

Some Terms Relating to Angles



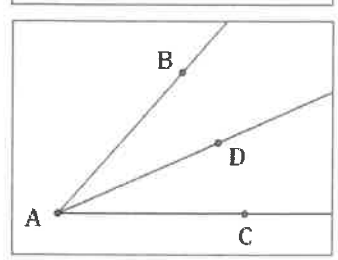
Angle interior is the area between the rays.

Angle exterior is the area not between the rays.

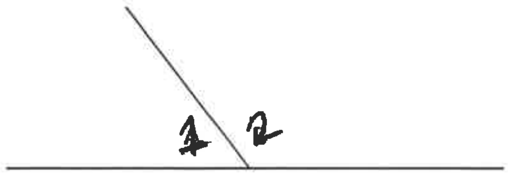
Adjacent angles are angles that share a ray for a side. $\angle BAD$ and $\angle DAC$ in the figure at right are adjacent angles.

Congruent angles area angles with the same measure.

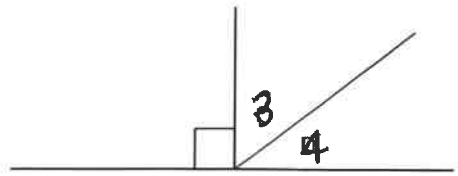
Angle bisector is a ray that divides the angle into two congruent angles. Ray $\overrightarrow{AD}$ bisects $\angle BAC$ in the figure at right.



Geometry
Types of Angles



Supplementary Angles



Complementary Angles

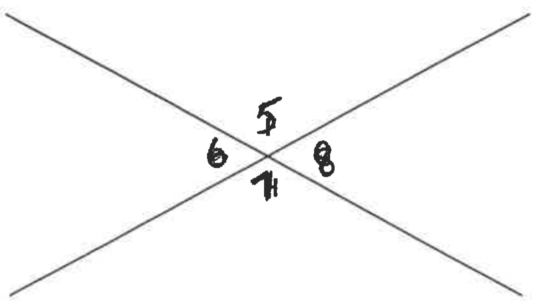
Angles 1 and 2 are supplementary.

Angles 1 and 2 form a linear pair.

$m\angle 1 + m\angle 2 = 180^0$

Angles 3 and 4 are complementary.

$m\angle 3 + m\angle 4 = 90^0$



Vertical Angles

Angles which are opposite each other when two lines cross are vertical angles.

Angles 5 and 7 are vertical angles.
Angles 6 and 8 are vertical angles.

$m\angle 5 = m\angle 7 \quad and \quad m\angle 6 = m\angle 8$

In addition, each angle is supplementary to the two angles adjacent to it. For example:

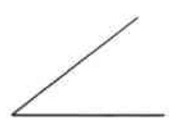
Angle 5 is supplementary to Angles 6 and 8.

An acute angle is one that is less than 90°. In the illustration above, angles 6 and 8 are acute angles.

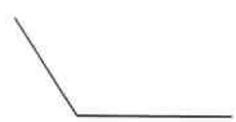
A right angle is one that is exactly 90°.

An obtuse angle is one that is greater than 90°. In the illustration above, angles F and H are obtuse angles.

A straight angle is one that is exactly 180°.



Acute



Obtuse

Right

Straight

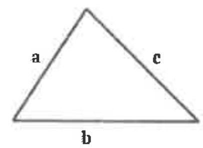
Geometry

Perimeter and Area of a Triangle

Perimeter of a Triangle

The perimeter of a triangle is simply the sum of the measures of the three sides of the triangle.

$$P = a + b + c$$

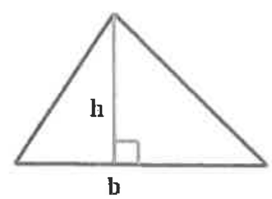


Area of a Triangle

There are two formulas for the area of a triangle, depending on what information about the triangle is available.

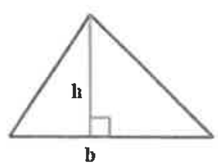
Formula 1: The formula most familiar to the student can be used when the base and height of the triangle are either known or can be determined.

$$A = \frac{1}{2}bh$$

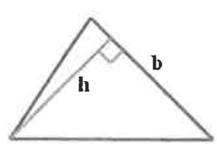


where, b is the length of the base of the triangle.
h is the height of the triangle.

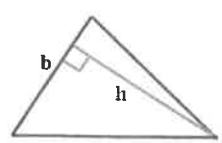
Note: The base can be any side of the triangle. The height is the measure of the altitude of whichever side is selected as the base. So, you can use:



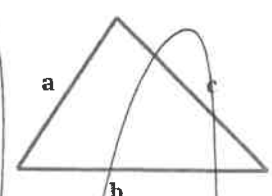
or



or



Formula 2: Heron's formula for the area of a triangle can be used when the lengths of all of the sides are known. Sometimes this formula, though less appealing, can be very useful.



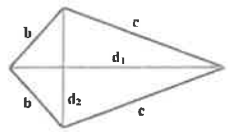
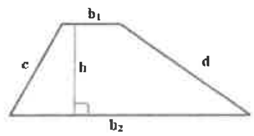
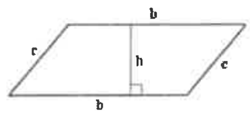
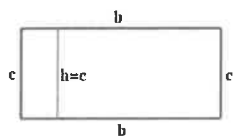
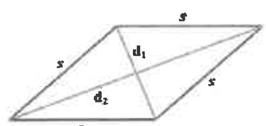
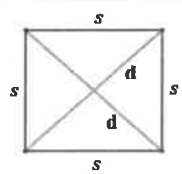
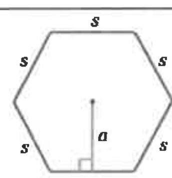
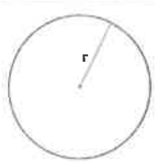
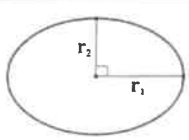
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where, $s = \frac{1}{2}P = \frac{1}{2}(a+b+c)$. **Note:** s is sometimes called the semi-perimeter of the triangle.

a, b, c are the lengths of the sides of the triangle.

Geometry

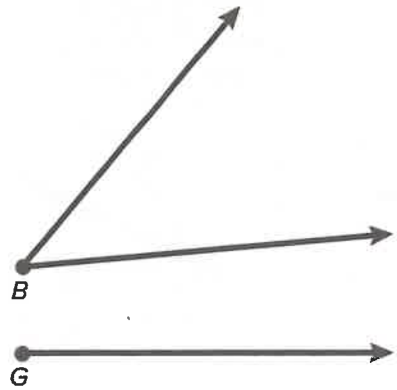
Summary of Perimeter and Area Formulas – 2D Shapes

Shape	Figure	Perimeter	Area
Kite		$P = 2b + 2c$ $b, c = sides$	$A = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Trapezoid		$P = b_1 + b_2 + c + d$ $b_1, b_2 = bases$ $c, d = sides$	$\mathrm{A} = \frac{1}{2}(\mathrm{b}_1 + \mathrm{b}_2)\mathrm{h}$ $\mathrm{b}_1, \mathrm{b}_2 = \mathrm{bases}$ $\mathrm{h} = \mathrm{height}$
Parallelogram		$P = 2b + 2c$ $b, c = sides$	$\mathrm{A} = \mathrm{bh}$ $b = base$ $h = height$
Rectangle		$P = 2b + 2c$ $b, c = sides$	$\mathrm{A} = \mathrm{bh}$ $b = base$ $h = height$
Rhombus		$P = 4s$ $s = side$	$A = bh = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Square		$P = 4s$ $s = side$	$A = s^2 = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Regular Polygon		$P = ns$ $n = number\ of\ sides$ $s = side$	$A = \frac{1}{2}\ a \cdot P$ $a = apothem$ $P = perimeter$
Circle		$C = 2\pi r = \pi d$ $r = radius$ $d = diameter$	$A = \pi r^2$ $r = radius$
Ellipse		$P \approx 2\pi\sqrt{\frac{1}{2}(r_1{}^2 + r_2{}^2)}$ $r_1 = major\ axis\ radius$ $r_2 = minor\ axis\ radius$	$A = \pi r_1 r_2$ $r_1 = major\ axis\ radius$ $r_2 = minor\ axis\ radius$

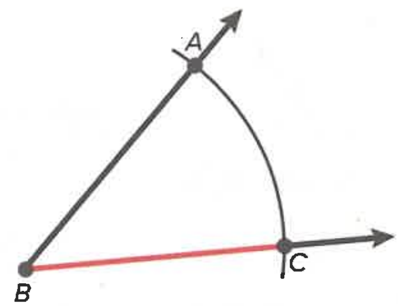
You can use various tools to copy an angle. To use string, start by loosely tying the end of the string around a pencil to ensure the length of the portion of the string being used doesn't change each time. Use a thumbtack to fix the other end of the string to a point.

CONSTRUCTION • Copy an Angle

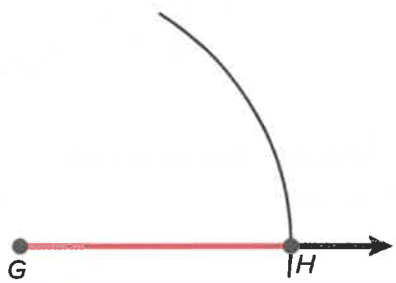
STEP 1 You are given ∠*B*. Use a straightedge to draw a ray on your paper. Label its endpoint G.



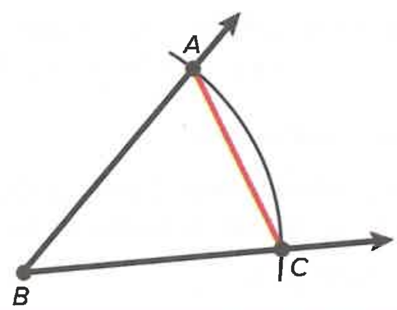
STEP 2 Place the thumbtack at the vertex of ∠*B* and draw a large arc that intersects both sides of ∠*B*. Label the points of intersection *A* and *C*.



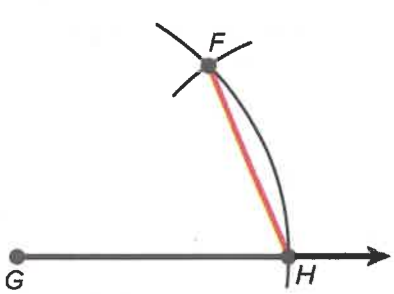
STEP 3 Without changing the length of the string, place the thumbtack on *G* and draw an arc that starts above the ray and intersects the ray. Label the point of intersection *H*.



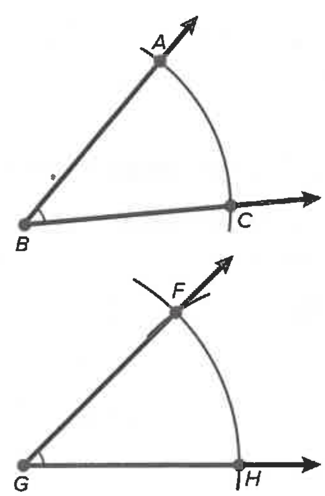
STEP 4 Place the thumbtack on *C* and adjust your string so that the pencil tip is on *A*.



STEP 5 Without changing the length of the string, place the thumbtack on *H* and draw an arc to intersect the larger arc you drew in **Step 3.** Label the point of intersection *F*.

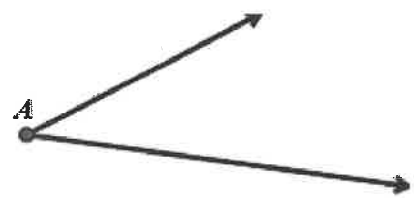


STEP 6 Use a straightedge to draw $\overrightarrow{GF}$. $\angle ABC \cong \angle FGH$.

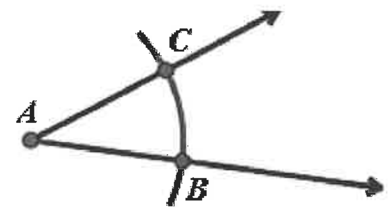


GO ONLINE You may want to complete the construction activity using other tools.

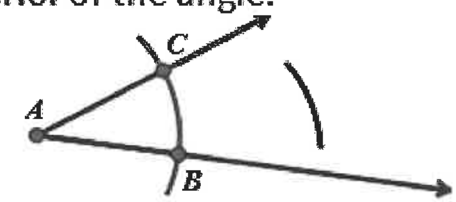
(a) Given an angle.



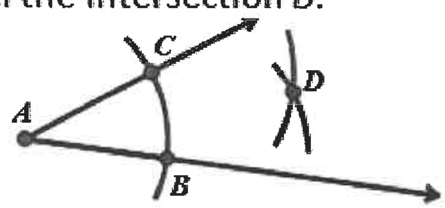
(b) Create an arc of any size, such that it intersects both rays of the angle. Label those points B and C.



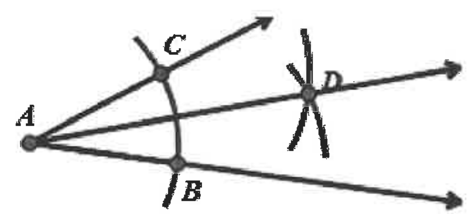
(c) Leaving the compass the same measurement, place your pointer on point B and create an arc in the interior of the angle.



(d) Do the same as step (c) but placing your pointer at point C. Label the intersection D.



(e) Create $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the angle bisector.



(f) $\overrightarrow{AD}$ is the angle bisector.