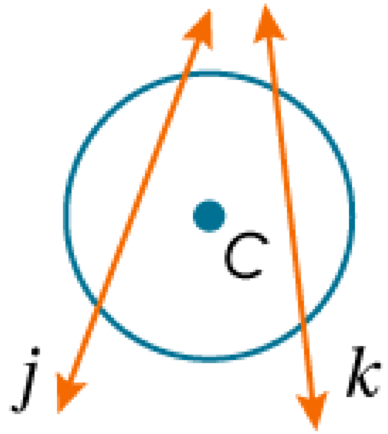
Tangents and Secants

Learn
Tangents, Secants, and Angle Measures

A **secant** is any line or ray that intersects a circle in exactly two points. Lines *j* and *k* are secants of ⊙*C*.

When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.



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Tangents, Secants, and Angle Measures

Theorems

10.14	If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
10.15	If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

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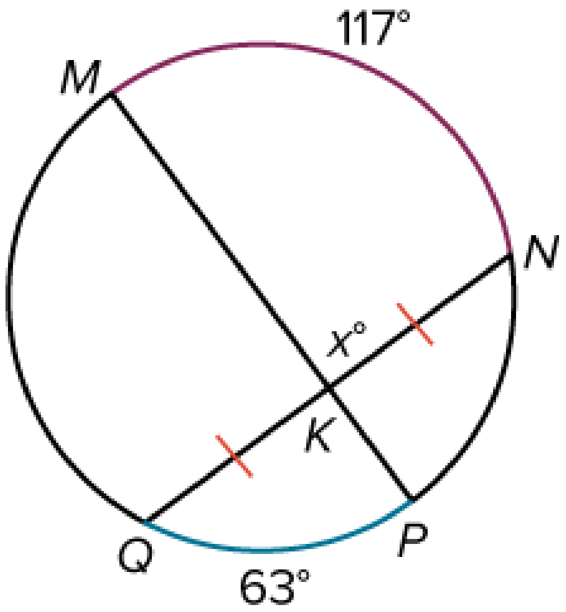
Theorems

10.16	If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.

Example 1
Intersecting Chords or Secants

Find the value of *x*.



Example 1
Intersecting Chords or Secants

$$m\angle MKN = \frac{1}{2}\left(m\,\widehat{QP} + m\,\widehat{MN}\right)$$

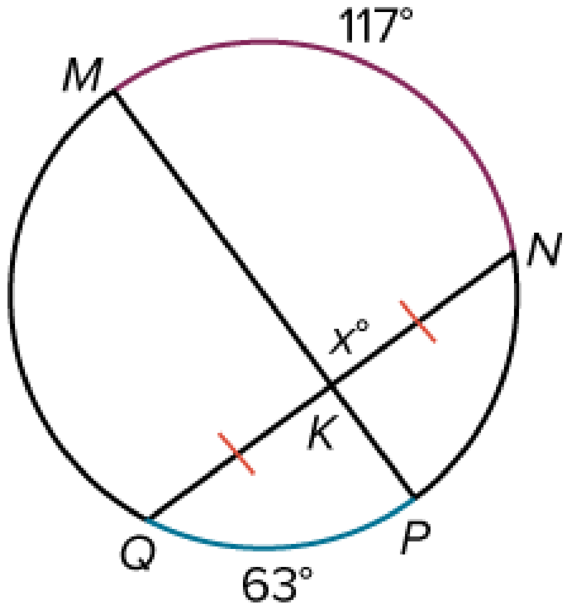
Theorem 10.14

$$x° = \frac{1}{2}(63° + 117°)$$

Substitution

$$x° = \frac{1}{2}(180°) \text{ or } 90°$$

Simplify.

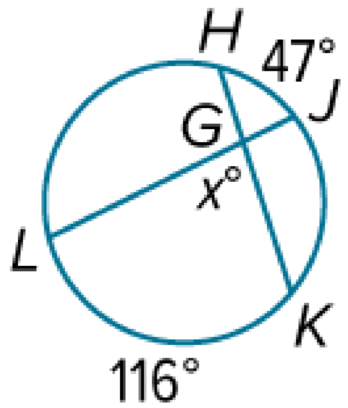


So, the value of *x* is 90, and $\overline{MP}$ is the perpendicular bisector of $\overline{QN}$.

Example 1
Intersecting Chords or Secants

Check

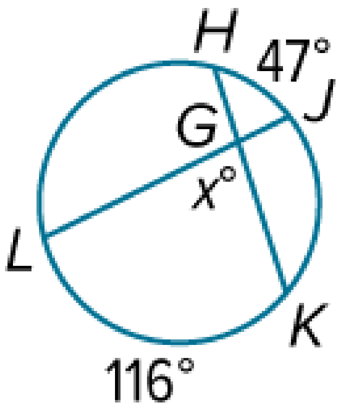
Find the value of *x*.



Example 1
Intersecting Chords or Secants

Check

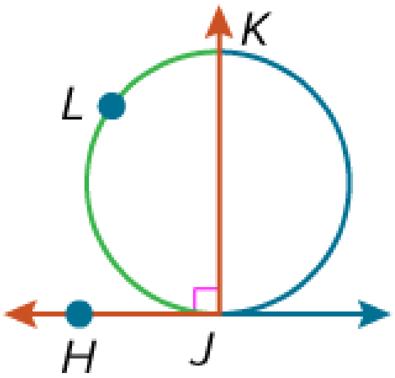
Find the value of *x*.



$x = 81.5$

Example 2
Secants and Tangents Intersecting on a Circle

Find $m\widehat{JLK}$.



Example 2
Secants and Tangents Intersecting on a Circle

$$m\angle HJK = \frac{1}{2} m\widehat{JLK}$$

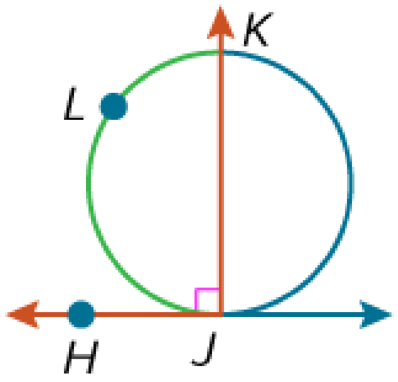
$$90° = \frac{1}{2} m\widehat{JLK}$$

$$180° = m\widehat{JLK}$$

Theorem 10.15

Substitution

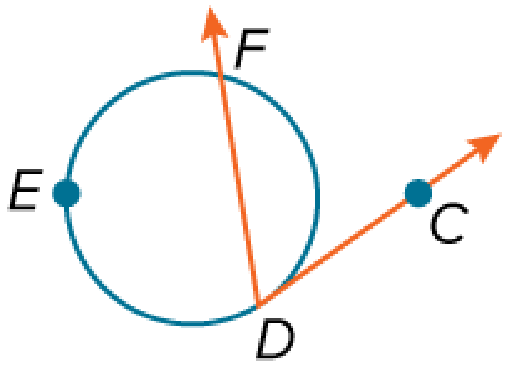
Solve.



Example 2
Secants and Tangents Intersecting on a Circle

Check

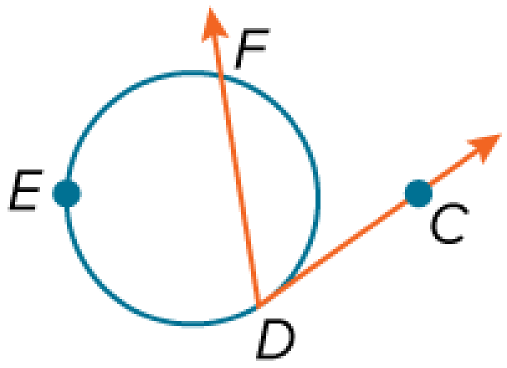
Find $m\widehat{DEF}$ if $m\angle FDC = 64°$.



Example 2
Secants and Tangents Intersecting on a Circle

Check

Find $m\overset{\frown}{DEF}$ if $m\angle FDC = 64°$.

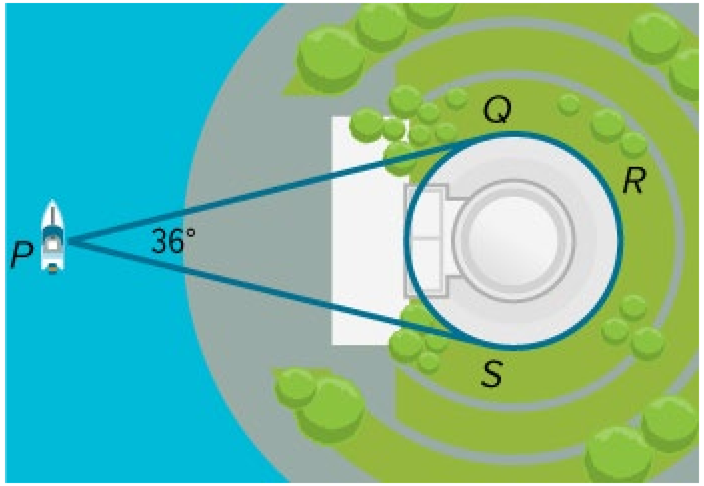


$m\overset{\frown}{DEF} = 232°$

Example 3

Tangents and Secants Intersecting Outside a Circle

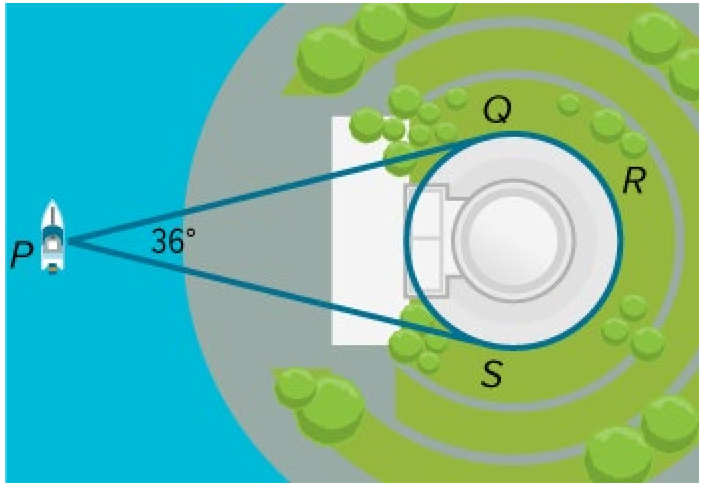
MEMORIALS A photographer is taking a photo of the Thomas Jefferson Memorial in Washington, D.C., from a boat in the Tidal Basin. The photographer's lines of sight are tangent to the memorial at points *Q* and *S*. If the camera's viewing angle measures 36°, what portion of the memorial will be visible in the photo?



Example 3

Tangents and Secants Intersecting Outside a Circle

Because the Thomas Jefferson Memorial can be modeled by a circle, the arc measure of the memorial is 360°. So, the portion of the memorial that will be visible in the photo is equal to $\frac{m\widehat{QS}}{360°}$.



Example 3
Tangents and Secants Intersecting Outside a Circle

Let $m\widehat{QS} = x°$. So, $m\widehat{QRS} = 360° - x°$.

$m\angle P = \frac{1}{2}\left(m\widehat{QRS} - m\widehat{QS}\right)$	Theorem 10.16
$36° = \frac{1}{2}[(360° - x°) - x°]$	Substitution
$36° = \frac{1}{2}(360° - 2x°)$	Simplify.
$72° = 360° - 2x°$	Multiply each side by 2.
$-288° = -2x°$	Subtract 360° from each side.
$144 = x$	Divide each side by –2°.

Example 3
Tangents and Secants Intersecting Outside a Circle

So, $m\widehat{QS} = 144°$, and the portion of the memorial that will be visible in the photo is $\frac{144}{360}$ or 40%.

Learn
Tangents, Secants, and Segment Lengths

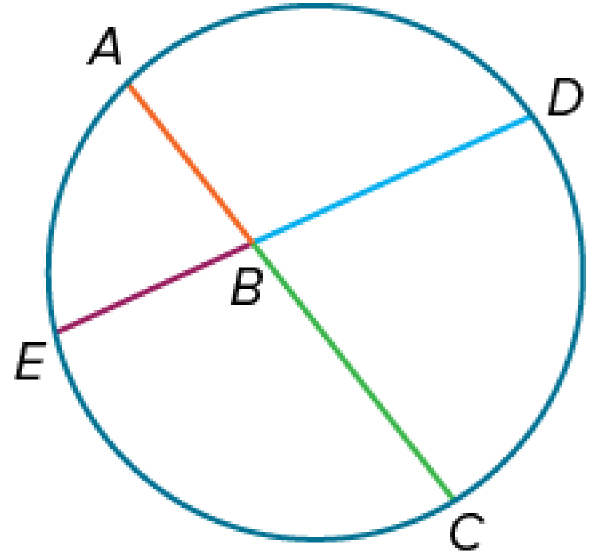
Special relationships also exist for the segment lengths of two chords when the chords intersect inside a circle, or when secants and tangents intersect outside of a circle.

When two chords intersect inside a circle, each chord is divided into segments, called **chord segments**. A **secant segment** is a segment of a secant line that has exactly one endpoint on the circle. An **external secant segment** is a secant segment that lies in the exterior of the circle. A **tangent segment** is the segment of a tangent with one endpoint on the circle.

Learn

Tangents, Secants, and Segment Lengths

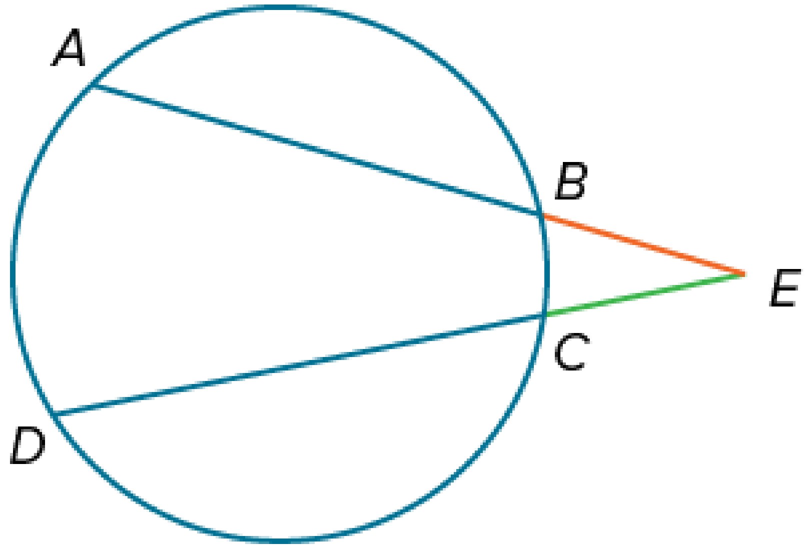
Theorem 10.17 Segments of Chords Theorem

Words	If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.	
Example	$AB \cdot BC = DB \cdot BE$	

Learn

Tangents, Secants, and Segment Lengths

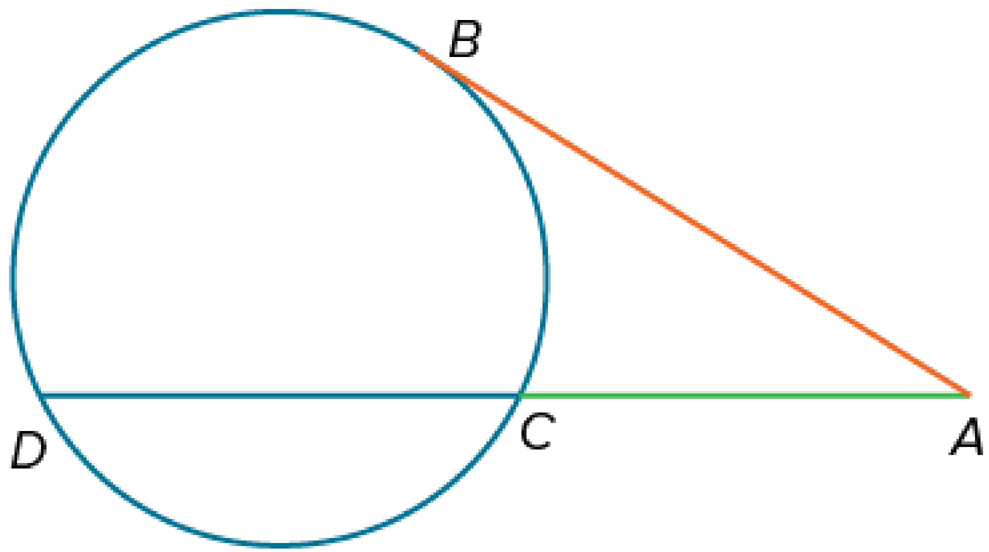
Theorem 10.18 Secant Segments Theorem

Words	If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.	
Example	$AE \cdot BE = DE \cdot CE$	

Learn

Tangents, Secants, and Segment Lengths

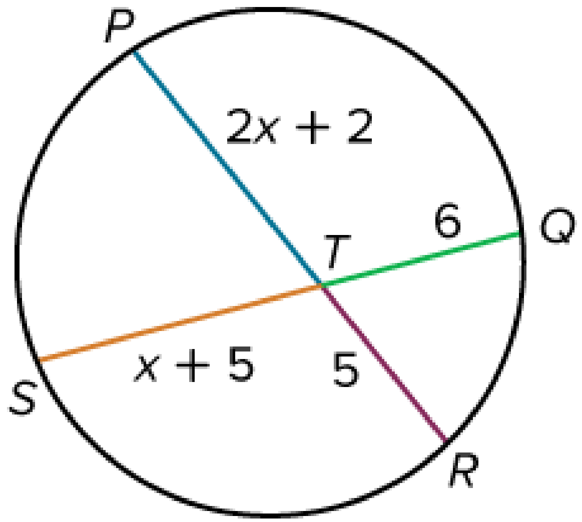
Theorem 10.19

Words	If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.	
Example	$AB^2 = AC \cdot AD$	

Example 4

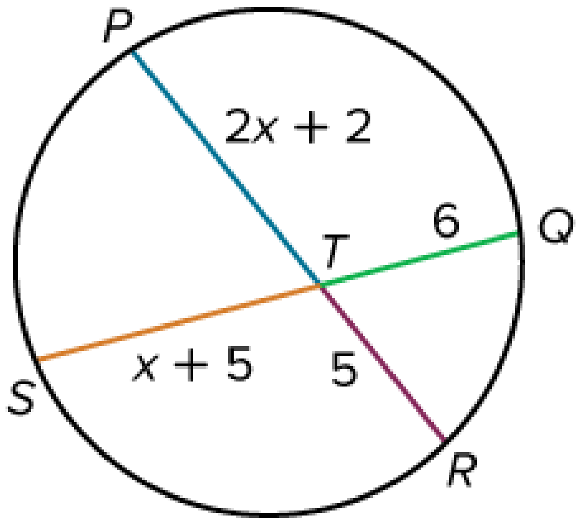
Use the Intersection of Two Chords

Find *PT*.



Example 4
Use the Intersection of Two Chords

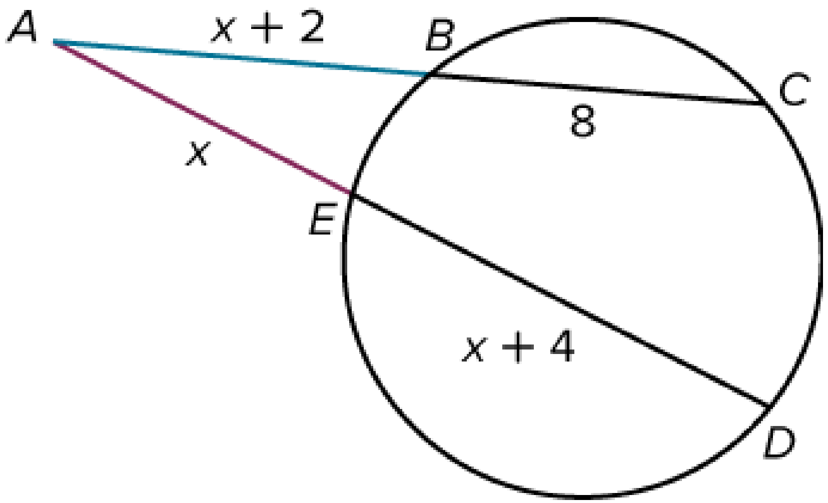
$PT \cdot TR$	$= ST \cdot TQ$	Theorem 10.17
$(2x + 2) \cdot 5$	$= (x + 5) \cdot 6$	Substitute.
$10x + 10$	$= 6x + 30$	Multiply.
x	$= 5$	Solve.



So, $PT = 2(5) + 2$ or 12.

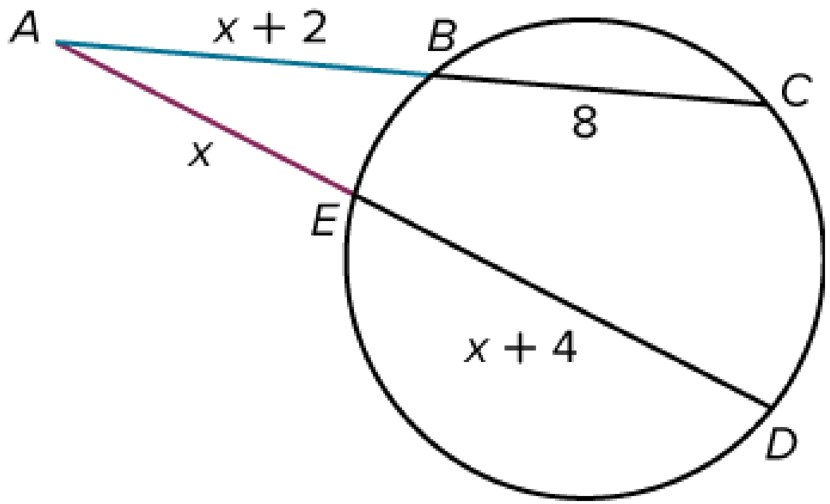
Example 5
Use the Intersection of Two Secants

Find *AB*.



Example 5
Use the Intersection of Two Secants

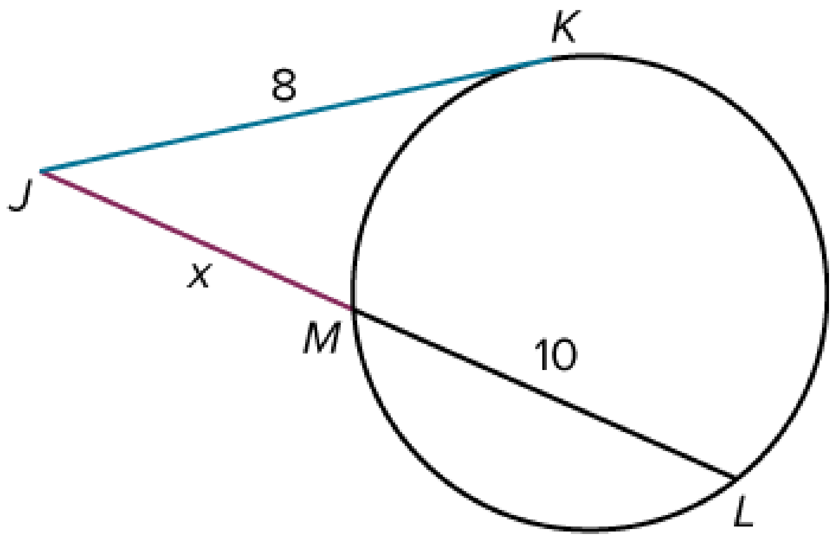
$AB \cdot AC = AE \cdot AD$	Theorem 10.18
$(x+2)(x+10) = x(2x+4)$	Substitute.
$x^2 + 12x + 20 = 2x^2 + 4x$	Multiply.
$0 = x^2 - 8x - 20$	Simplify..
$0 = (x-10)(x+2)$	Factor.
$x = 10 \text{ or } -2$	Zero Product Property



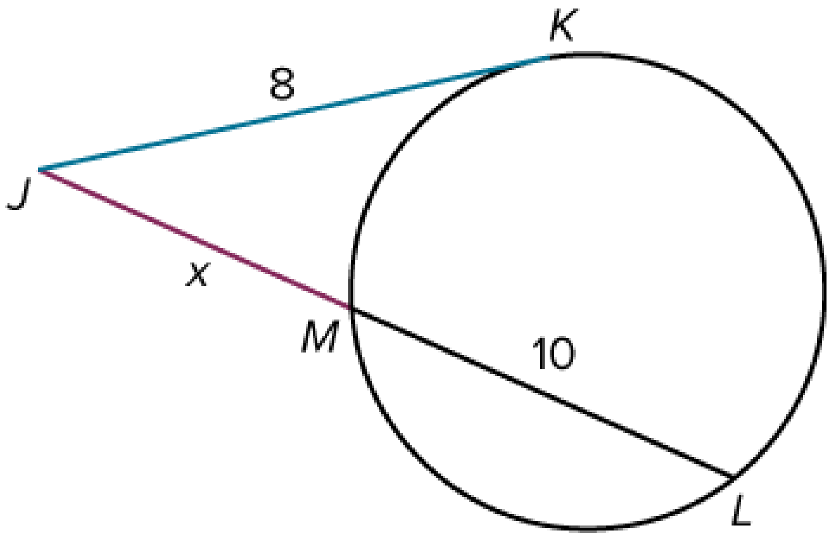
The length of a segment must be positive, so *AB* = 10 + 2 or 12.

Example 6
Use the Intersection of a Secant and a Tangent

$\overline{JK}$ is tangent to the circle. Find *JL*. Round to the nearest tenth.



Example 6
Use the Intersection of a Secant and a Tangent



$JK^2 = JM \cdot JL$	Theorem 10.19
$8^2 = x(x + 10)$	Substitute.
$64 = x^2 + 10x$	Multiply.
$0 = x^2 + 10x - 64$	Subtract 64 from each side.

Example 6

Use the Intersection of a Secant and a Tangent

The expression is not factorable, so use the Quadratic Formula to solve for *x*.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-10 \pm \sqrt{10^2 - 4(1)(-64)}}{2(1)}$	$a = 1$, $b = 10$, and $c = -64$
$= \frac{-10 \pm \sqrt{356}}{2}$	Simplify.
≈ 4.4 or -14.4	Use a calculator.

Because lengths cannot be negative, $x \approx 4.4$ and $JL \approx 10 + 4.4$ or 14.4.