Lesson 10.7 Equations of Circles

Workbook pages 247-251

Content Objective

Students write and graph the equations of circles using key features.

Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.3.2

Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

MA.912.GR.3.3

Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.7.2

Given a mathematical or real-world context, derive and create the equation of a circle using key features.

MA.912.GR.7.3

Graph and solve mathematical and real-world problems that are modeled with an equation of a circle. Determine and interpret key features in terms of the context.

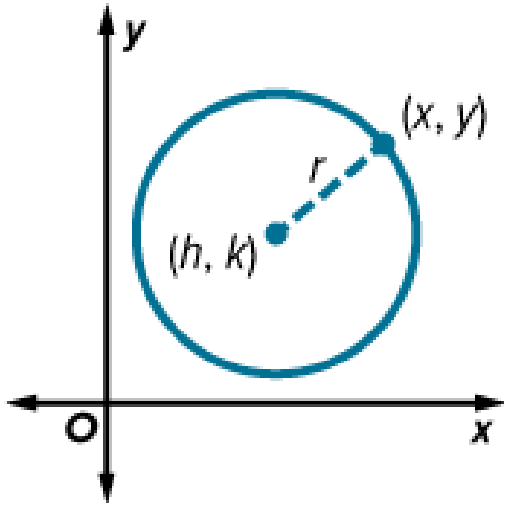
Learn

Equations of Circles

Key Concept: Equation of a Circle in Standard Form

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

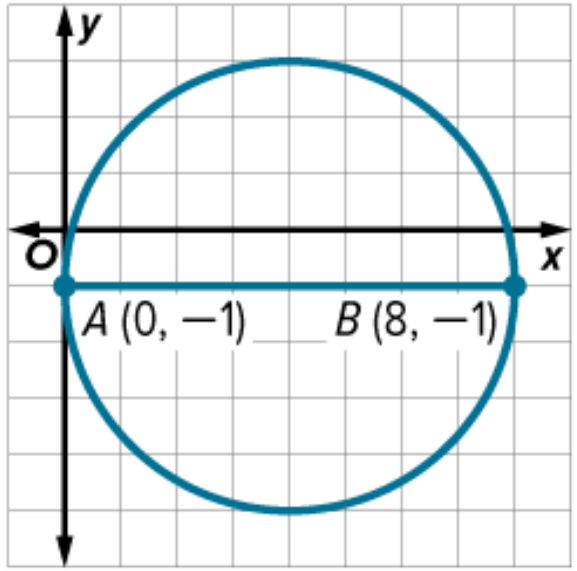
The standard form of the equation of a circle is also called the *center-radius* form.



Example 1

Write an Equation by Using the Center and Radius

$\overline{AB}$ is a diameter of the circle. Write the equation of the circle.



Example 1

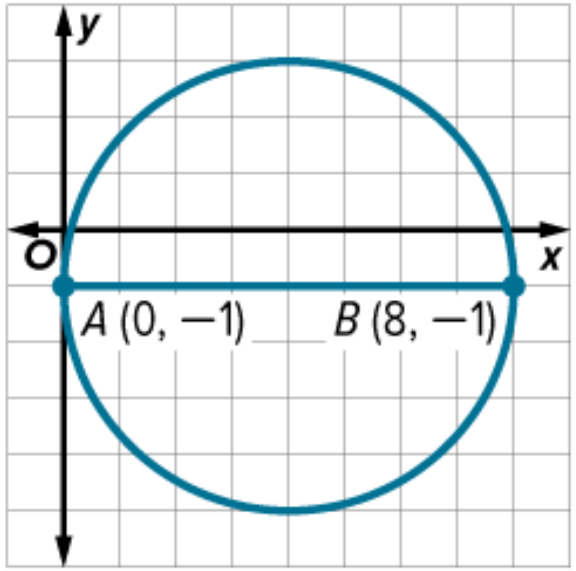
Write an Equation by Using the Center and Radius

Because $\overline{AB}$ is a diameter of the circle, the center of the circle is the midpoint of $\overline{AB}$.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$= \left(\frac{0+8}{2}, \frac{-1+(-1)}{2}\right) \quad \text{Substitution}$$

$$= (4, -1) \quad \text{Simplify.}$$



Example 1

Write an Equation by Using the Center and Radius

So, the center is at $(4, -1)$ and the radius is 4.

$$(x - h)^2 + (y - k)^2 = r^2$$ Equation of a circle

$$(x - 4)^2 + \big(y - (-1)\big)^2 = 4^2$$ Substitution

$$(x - 4)^2 + (y + 1)^2 = 16$$ Simplify.

Example 2

Write an Equation by Using the Center and a Point

Write the equation of the circle with center at (−3, −5) that passes through (0, 0).

Example 2

Write an Equation by Using the Center and a Point

Step 1 Find the length of the radius.

Find the distance between the points to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula

$$= \sqrt{[0 - (-3)]^2 + [0 - (-5)]^2}$$ $(x_1, y_1) = (-3, -5)$ and $(x_2, y_2) = (0, 0)$

$$= \sqrt{34}$$ Simplify.

Example 2

Write an Equation by Using the Center and a Point

Step 2 Write the equation of the circle.

Write the equation using $h = -3$, $k = -5$, and $r = \sqrt{34}$.

$$(x - h)^2 + (y - k)^2 = r^2$$ Equation of a circle

$$[x - (-3)]^2 + [y - (-5)]^2 = \left(\sqrt{34}\right)^2$$ Substitution

$$(x + 3)^2 + (y + 5)^2 = 34$$ Simplify.

Example 3

Graph a Circle

The equation of a circle is $x^2 + y^2 + 8x - 14y + 40 = 0$. State the coordinates of the center and the measure of the radius. Then graph the equation and determine the domain and range.

Example 3
Graph a Circle

Step 1 Complete the squares.

Write the equation in standard form by completing the squares.

$x^2 + y^2 + 8x - 14y + 40 = 0$	Original equation
$x^2 + y^2 + 8x - 14y = -40$	Subtract 40 from each side.
$x^2 + 8x + y^2 - 14y = -40$	Group terms.
$x^2 + 8x + 16 + y^2 - 14y + 49 = -40 + 16 + 49$	Complete the squares.
$(x + 4)^2 + (y - 7)^2 = 25$	Factor and simplify.

Example 3
Graph a Circle

Step 2 Identify *h*, *k*, and *r*.

$(x + 4)^2 + (y - 7)^2 = 25$ can also be written as $[x - (-4)]^2 + (y - 7)^2 = 5^2$.

Now you can identify *h*, *k*, and *r*.

$h = -4$, $k = 7$, and $r = 5$

The center is at $(-4, 7)$, and the radius is 5.

Example 3
Graph a Circle

Step 3 Graph the circle.

Plot the center and four points that are 5 units from the center. Sketch the circle through these four points.

Step 4 Describe the domain and range.

The domain is $-9 \leq x \leq 1$. The range is $2 \leq y \leq 12$.