Lesson 10.3 and 10.4 Arcs, Chords, Angles

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Lesson 10.3 and

Lesson 10.3 Arcs and Chords
Workbook pages 209-212
Lesson 10.4 Inscribed Angles
Workbook pages 215-220

Content Objective
Students solve problems involving the relationship between arcs and chords, and radii that are perpendicular to chords.

Students solve problems using inscribed angles.

Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.6.1

Solve mathematical and real-world problems involving the length of a secant, tangent, segment or chord in a given circle.

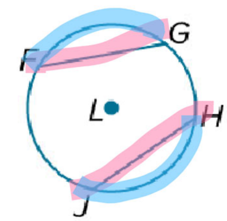
MA.912.GR.6.2

Solve mathematical and real-world problems involving the measures of arcs and related angles.

Learn
Arcs and Chords

A chord is a segment with endpoints on a circle. If a chord is not a diameter, then its endpoints divide the circle into a major and minor arc.

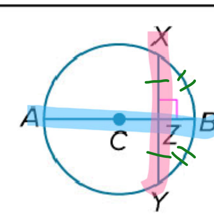
Theorem 10.3

Words	In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.	
Example	$\overset{\frown}{FG} \cong \overset{\frown}{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.	

Learn
Arcs and Chords

If a line, segment, or ray divides an arc into two congruent arcs, then it bisects the arc.

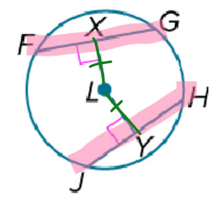
Theorem 10.4

Words	If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.
Example	If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\overset{\frown}{XB} \cong \overset{\frown}{BY}$. 

Learn
Arcs and Chords

In addition to Theorem 10.3, you can use the following theorem to determine whether two chords in a circle are congruent.

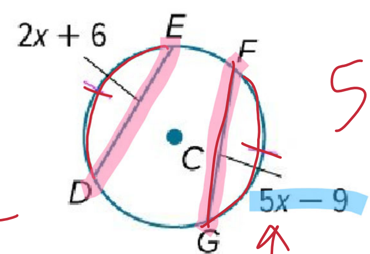
Theorem 10.6

Words	In the same circle or in congruent circles, chords are congruent if and only if they are equidistant from the center.	
Example	$\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.	

Example 2

Use Congruent Arcs to Find Chord Length

In $\odot C$, $\overset{\frown}{DE} \cong \overset{\frown}{FG}$. Find FG.



$2x + 6 = 5x - 9$

$-2x \qquad -2x$

$6 = 3x - 9$

$+9 \qquad +9$

$\frac{15}{3} = \frac{3x}{3}$

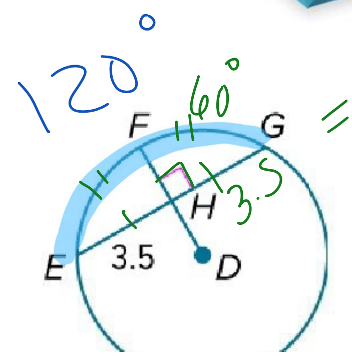
$x = 5$

$5(5) - 9 = 16$

Example 3

Use a Radius Perpendicular to a Chord

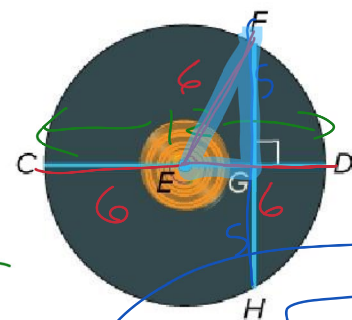
In $\odot D$, $m\overset{\frown}{EFG} = 120°$. Find $m\overset{\frown}{FG}$ and EG.



$3.5 + 3.5 = 7$

Example 4
Use a Diameter Perpendicular to a Chord

RECORDS The record shown can be modeled by a circle. Diameter $\overline{CD}$ is 12 inches long, and chord $\overline{FH}$ is 10 inches long. Find *EG*.



c 6 $5 = a$

$x = b$

$a^2 + b^2 = c^2$

$5^2 + b^2 = 6^2$

$\cancel{25} + b^2 = 36$

$-\cancel{25} \quad -25$

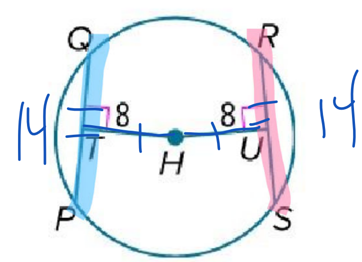
$b^2 = 11$

$b = \sqrt{11}$

$b = 3.3$

Example 5
Chords Equidistant from the Center

In $\odot H$, $PQ = 3x - 4$ and $RS = 14$. Find x.



$3x - \cancel{4} = 14$

$+\cancel{4} \quad +4$

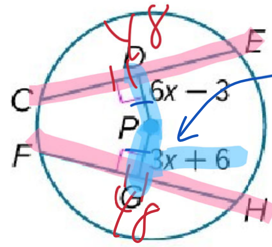
$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

Example 5
Chords Equidistant from the Center

Check

In $\odot P$, $\overline{CE} = \overline{FH} = 48$. Find PG.



$3(3) + 6$
$9 + 6 = 15$

$6x - 3 = 3x + 6$
$\;\;+3 \qquad\;\; +3$

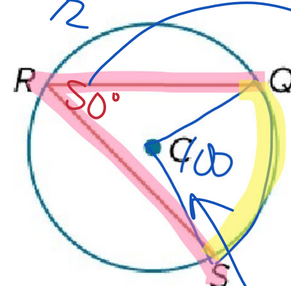
$6x = 3x + 9$
$-3x \;\; -3x$

$3x = 9$
$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

Learn
Inscribed Angles

An **inscribed angle** is an angle that has its vertex on a circle and sides that contain chords of the circle. In $\odot C$, $\angle QRS$ is an inscribed angle, and minor arc $\overset{\frown}{QS}$ is intercepted by $\angle QRS$.



$\frac{1}{2}$

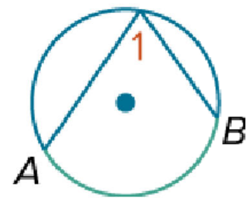
100

central angle

Learn
Inscribed Angles

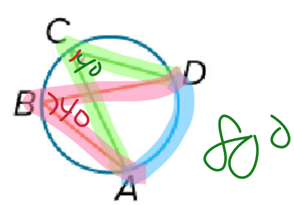
For each of these cases, the following theorem holds true.

Theorem 10.7: Inscribed Angle Theorem

Words	If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.
Example	$m\angle\_ = -m\widehat{AB}$ and $m\widehat{AB} = \_ m\angle\_$ 

Learn
Inscribed Angles

Theorem 10.8

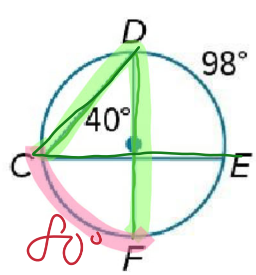
Words	If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.
Example	$\angle B$ and $\angle C$ both intercept $\widehat{AD}$. So, $\angle B \cong \angle C$. 

Example 1
Use Inscribed Angles to Find Measures

Find each measure.

a. $m\widehat{CF}$ 80°

b. $m\angle C$ 49°



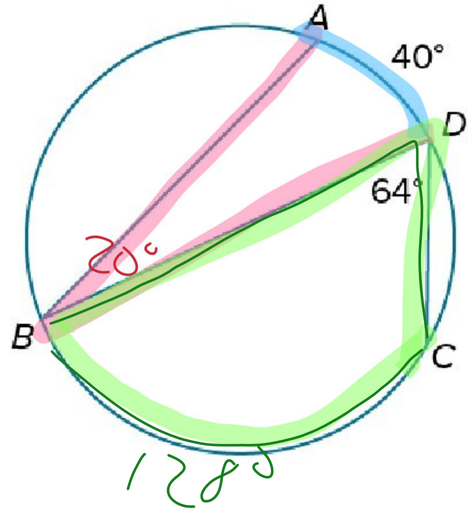
Example 1
Use Inscribed Angles to Find Measures

Check

Find each measure.

a. $m\angle B$ 20°

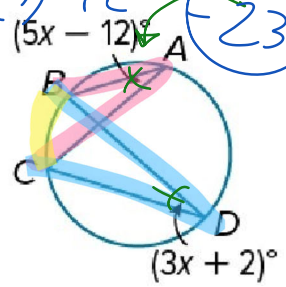
b. $m\overset{\frown}{BC}$ 128°



Example 2
Find Measures of Congruent and Inscribed Angles

Find $m\angle A$.

$5(7)-12$ 23°



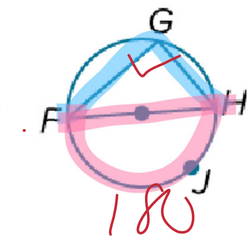
$3(7)+2$
23°

$$5x - \cancel{12} = 3x + 2$$
$$\cancel{+12} \qquad +12$$
$$5x = 3x + 14$$
$$-3x \quad -3x$$
$$\frac{2x}{2} = \frac{14}{2}$$
$$x = 7$$

Learn
Inscribed Polygons

In an **inscribed polygon**, all of the vertices of the polygon lie on a circle. Inscribed triangles and quadrilaterals have special properties.

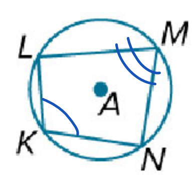
Theorem 10.9

Words	An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.	
Example	If $\widehat{FJH}$ is a semicircle, then $m\angle G = 90°$. If $m\angle G = 90°$, then $\widehat{FJH}$ is a semicircle and $\overline{FH}$ is a diameter.	

Learn
Inscribed Polygons

While many different types of triangles, including right triangles, can be inscribed in a circle, only certain quadrilaterals can be inscribed in a circle.

Theorem 10.10

Words	If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	
Example	If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary. 180	

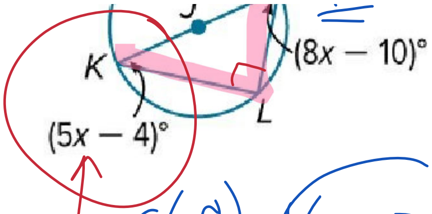
Example 4
Find Angle Measures in Inscribed Triangles

Find $m\angle K$.

$5x - 4 + 8x - 10 = 90$

$8(8) - 10 =$

$13x - 14 = 90$



$+14 \quad +14$

$13x = 104$, $\frac{13x}{13} = \frac{104}{13}$

$x = 8$

$5(8) - 4 = 36°$

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Example 5

Find Angle Measures

STAINED GLASS Luca is creating a collection of stained glass ornaments. The ornament shown uses a quadrilateral inscribed in a circle. Find $m\angle F$ and $m\angle G$.

$3x - 16 + x - 4 = 180$

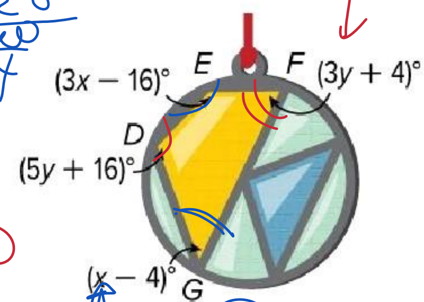
$4x - 20 = 180$

$+20 \quad +20$

$\frac{4x}{4} = \frac{200}{4}$

$x = 50$

$3(20) + 4 =$



$5y + 16 + 3y + 4 = 180$

$8y + 20 = 180$

$-20 \quad -20$

$\frac{8y}{8} = \frac{160}{8}$

$y = 20$

$64°$

$46°$

$50 - 4 = 46$

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