

Lesson 7.1 Angles of Polygons

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Lesson 7.1
Angles Of



Lesson 7.1 Angles of Polygons

Content Objective

Students use theorems about the interior and exterior angles of polygons to solve problems and then prove these theorems.

MA.912.GR.1.3

Prove relationships and theorems about triangles.

Solve mathematical and real-world problems involving postulates, relationships and theorems of triangles.



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Learn

Interior Angles of Polygons

Polygon Interior Angles Sum Theorem

The sum of the interior angle measures of an n -sided polygon is $180^\circ(n-2)$.

Angles in polygons

We can work out the **angle sum of any polygon** by splitting it into triangles. Remember that the angles in a triangle = 180° .

3 Triangle	Quadrilateral	5 Pentagon	$\frac{20}{2} \times 180^\circ$
$1 \times 180^\circ = 180^\circ$	180° 180° $2 \times 180^\circ = 360^\circ$	180° 180° $3 \times 180^\circ = 540^\circ$	
6 Hexagon	7 Heptagon	8 Octagon	If the polygon has n sides, there will be $(n - 2)$ triangles inside.
180° 180° 180°	180°	180°	Miss

$4 \times 180^\circ = 720^\circ$

$5 \times 180^\circ = 900^\circ$

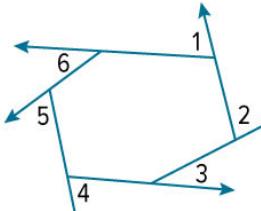
$6 \times 180^\circ = 1080^\circ$

Learn

Exterior Angles of Polygons



Polygon Exterior Angles Sum Theorem

Words	The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360° .
Example	$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$ 



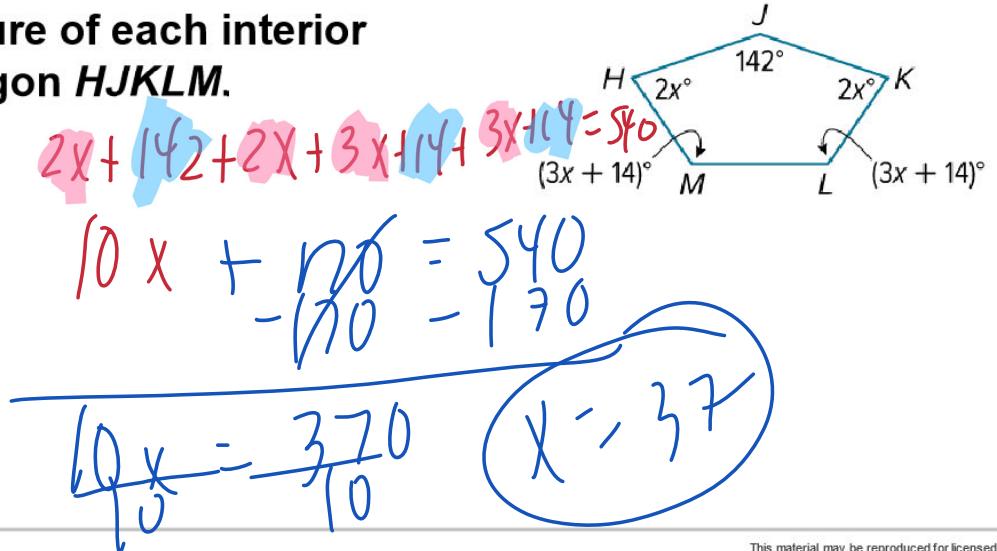
Example 1

Find the Interior Angles Sum of a Polygon

Find the measure of each interior angle of pentagon $HJKLM$.

$$\begin{aligned}180(n-2) \\180(5-2) \\180(3) \\540\end{aligned}$$

$$\begin{aligned}2x + 142 + 2x + 3x + 14 + 3x + 14 = 540 \\10x + 170 = 540 \\-170 -170 \\8x = 370 \\x = 37\end{aligned}$$



Example 1

Find the Interior Angles Sum of a Polygon

Step 1 Find the sum.

A pentagon has 5 sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

$$m\angle H + m\angle J + m\angle K + m\angle L + m\angle M$$

$$= 180(n - 2)$$

Polygon Interior Angles Sum Thm.

$$= 180^\circ (5-2)$$

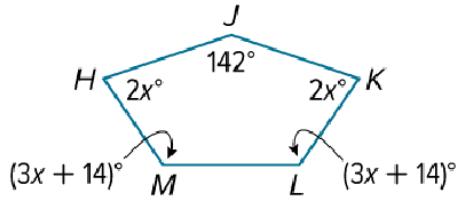
Substitute.

$$= 180 (3)$$

$$= 540^\circ$$

Solve.

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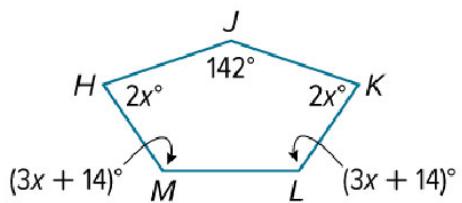
Example 1

Find the Interior Angles Sum of a Polygon



Step 2 Find the value of x.

Use the sum of the interior angle measures to determine the value of x.



$$2x^\circ + 2x^\circ + (3x + 14)^\circ + (3x + 14)^\circ + 142^\circ = 540^\circ \text{ Write an equation.}$$

$$x = 37 \quad \text{Solve.}$$

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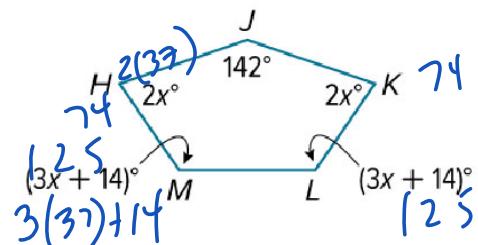
Example 1

Find the Interior Angles Sum of a Polygon



Step 3 Find the measure of each angle.

Use the value of x to find the measure of each angle.



$$m\angle J = 142^\circ \quad m\angle K = 2(37)^\circ \text{ or } 74^\circ \quad m\angle L = [3(37) + 14]^\circ \text{ or } 125^\circ$$

$$m\angle M = [3(37) + 14]^\circ \text{ or } 125^\circ$$

$$m\angle H = 2x^\circ = 2(37)^\circ \text{ or } 74^\circ$$

Example 1

Find the Interior Angles Sum of a Polygon

Check

Find the measure of all the angles.

$$\begin{aligned} & 180(n-2) \\ & 180(6-2) \\ & 180(4) \\ & \underline{\underline{720}} \end{aligned}$$

The diagram shows a hexagon with vertices A, B, C, D, E, F. The interior angles are labeled as follows: $(x+2)^\circ$, $(x-8)^\circ$, $(x+7)^\circ$, $(x-3)^\circ$, $(x+6)^\circ$, and $(x-4)^\circ$. The angle at vertex A is $(x+2)^\circ$, at B is $(x-8)^\circ$, at C is $(x+7)^\circ$, at D is $(x-3)^\circ$, at E is $(x+6)^\circ$, and at F is $(x-4)^\circ$.

$$\begin{aligned} & (x+2) + (x-8) + (x+7) \\ & + (x-3) + (x+6) + (x-4) = 720 \\ & \frac{6x}{6} = \underline{\underline{720}} \\ & x = 120 \end{aligned}$$

Example 2

Identify the Polygon Given the Sum of the Interior Angle Measures

Let n = the number of sides in the polygon. If the sum of the interior angle measures is 1440° , what kind of polygon is it? By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $180(n-2)$.

$$\begin{aligned} 1440 &= 180(n-2) \\ 1440 &= 180n - 360 \\ +360 & \qquad +360 \\ 1800 &= 180n \end{aligned}$$

$$\begin{aligned} \frac{1800}{180} &= \frac{180n}{180} \\ 10 &= n \end{aligned}$$

The polygon has 10 sides, so it is a regular decagon.

Example 2: Check

Identify the Polygon Given the Sum of the Interior Angle Measures

$$180(8-2) = 1080$$

Let n = the number of sides in the polygon. If the sum of the interior angle measures is 1080°, what kind of polygon is it?

$$\begin{aligned} 1080 &= 180(n-2) \\ 1080 &= 180n - 360 \\ +360 & \quad +360 \\ \hline 1440 &= 180n \end{aligned}$$

$$\begin{aligned} \frac{1440}{180} &= \frac{180n}{180} \\ 8 &= n \\ \text{octagon} \end{aligned}$$

Example 3

Identify the Polygon Given the Interior Angle Measure

The measure of an interior angle of a regular polygon is 150°.

Find the number of sides in the polygon.

a! sides : < is are =
Let n = the number of sides in the polygon. Because all angles of a regular polygon are congruent, the sum of the interior angle measures is $150n$ °. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $180(n-2)$.

$$\begin{aligned} 150n &= 180(n-2) \\ 150n &= 180n - 360 \\ -180n & \quad -180n \\ -30n &= -360 \end{aligned}$$

$$\begin{aligned} -30n &= -360 \\ -30 & \quad -30 \\ n &= 12 \end{aligned}$$

$$\begin{aligned} 180(12-2) &= 180 \\ 180(10) &= 1800 \end{aligned}$$

The polygon has 12 sides, so it is a regular dodecagon

Example 3: Check

Identify the Polygon Given the Interior Angle Measure

The measure of an interior angle of a regular polygon is 120°.

Find the number of sides in the polygon.

$$120n - 180(n-2)$$

$$180(6-2)$$



$$\begin{aligned}
 180n - 180(n-1) &= 180 \\
 120n = 180n - 360 & \\
 -180n &= -180n \\
 -60n = -360 & \\
 -60 &= -60 \\
 n = 6 &
 \end{aligned}$$

$$\begin{aligned}
 180(4) &= 720 \\
 \frac{720}{6} &= 120^\circ
 \end{aligned}$$

Example 4

Find Missing Values

Find the value of x .

$$\begin{aligned}
 139 + 6x + 9x + 2x &= 360 \\
 17x + 139 &= 360 \\
 -139 & \\
 \hline
 17x &= 221 \\
 \frac{17x}{17} &= 13 \\
 x &= 13
 \end{aligned}$$

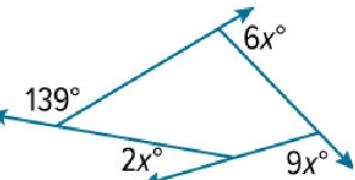
Example 4

Find Missing Values

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x .

$$\begin{aligned}
 6x^\circ + 9x^\circ + 2x^\circ + 139^\circ &= 360^\circ \\
 x &= 13
 \end{aligned}$$

Write an equation.
Solve.



Example 4

Find Missing Values

Check

Find the value of x .

$$52 + 2x + 88 + x + 10 + x + 2 = 360 \quad \therefore 360$$

$$\begin{array}{r} 4x + 152 = 360 \\ -152 \\ \hline 4x = 208 \\ \frac{4x}{4} = \frac{208}{4} \\ x = 52 \end{array}$$

Example 5

Find Exterior Angle Measures of a Polygon

$$\rightarrow 360$$

Find the measure of each exterior angle of a regular dodecagon.

$$\frac{\text{all sides}}{\text{angles}} =$$

$$\frac{360}{12} = 30$$

$$30 \times 12 = 360$$

Example 5

Find Exterior Angle Measures of a Polygon

FIND EXTERIOR ANGLE MEASURES OF A POLYGON

A regular dodecagon has 12 congruent sides and 12 congruent interior angles. The exterior angles are also congruent, because angles supplementary to congruent angles are congruent.

Let n = the measure of each exterior angle and write and solve an equation.

$$12n = 360^\circ$$

Polygon Exterior Angles Sum Theorem

$$n = 30^\circ$$

Solve.

The measure of each exterior angle of a regular dodecagon is 30° .



Example 5

Find Exterior Angle Measures of a Polygon

Check

What is the measure of each exterior angle of a regular octagon?

$$\begin{array}{r} 45 \\ \times 8 \\ \hline 360 \end{array}$$

$$360$$

$$\frac{360}{8} = 45^\circ$$



$$\begin{array}{l} 8 \\ \swarrow \\ \text{all sides} \\ \therefore \angle's = \end{array}$$

