Lesson 7.1 Angles of Polygons

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Content Objective

Students use theorems about the interior and exterior angles of polygons to solve problems and then prove these theorems.

MA.912.GR.1.3

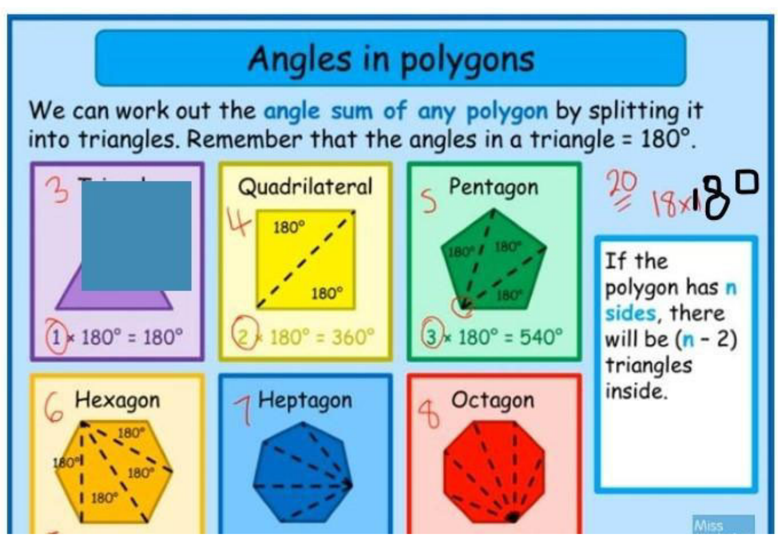
Prove relationships and theorems about triangles.
Solve mathematical and real-world problems involving postulates, relationships and theorems of triangles.

Learn

Interior Angles of Polygons

Polygon Interior Angles Sum Theorem

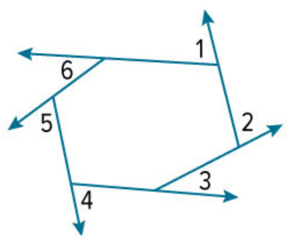
The sum of the interior angle measures of an *n*-sided polygon is 180°(n-2).



4 × 180° = 720° | 5 × 180° = 900° | 6 × 180° = 1080°

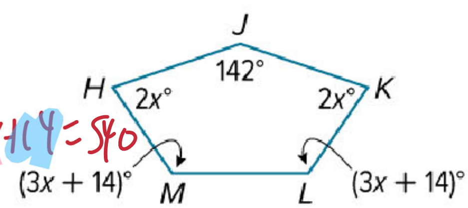
Learn
Exterior Angles of Polygons

Polygon Exterior Angles Sum Theorem

Words	The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360°.
Example	$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360°$ 

Example 1
Find the Interior Angles Sum of a Polygon

Find the measure of each interior angle of pentagon *HJKLM*.



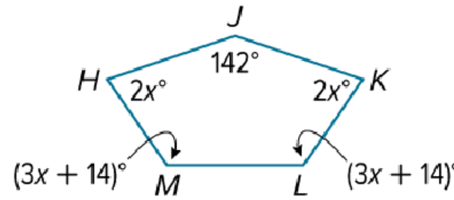
$180(n-2)$
$180(5-2)$
$180(3)$
540

$2x + 142 + 2x + 3x + 14 + 3x + 14 = 540$
$10x + 170 = 540$
$-170 \quad -170$
$10x = 370$
$\frac{10x}{10} = \frac{370}{10}$
$x = 37$

Example 1
Find the Interior Angles Sum of a Polygon

Step 1 Find the sum.

A pentagon has 5 sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.



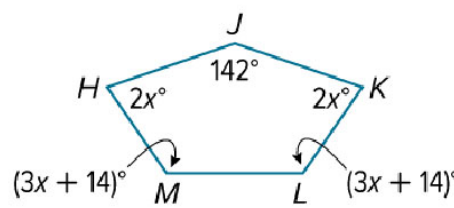
$m\angle H + m\angle J + m\angle K + m\angle L + m\angle M$

$= 180(n - 2)$	Polygon Interior Angles Sum Thm.
$= 180° (5-2)$	Substitute.
$= 180 (3)$	
$= 540°$	Solve.

Example 1
Find the Interior Angles Sum of a Polygon

Step 2 Find the value of *x*.

Use the sum of the interior angle measures to determine the value of *x*.



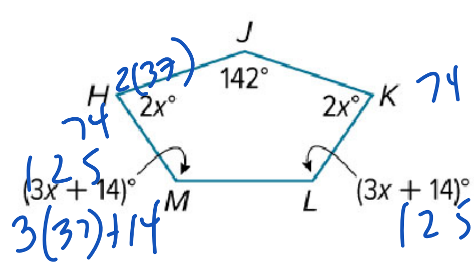
$2x° + 2x° + (3x + 14)° + (3x + 14)° + 142° = 540°$ Write an equation.

$x = 37$ Solve.

Example 1
Find the Interior Angles Sum of a Polygon

Step 3 Find the measure of each angle.

Use the value of *x* to find the measure of each angle.



$m\angle J = 142°$ $m\angle K = 2(37)°$ or $74°$ $m\angle L = [3(37) + 14]°$ or $125°$

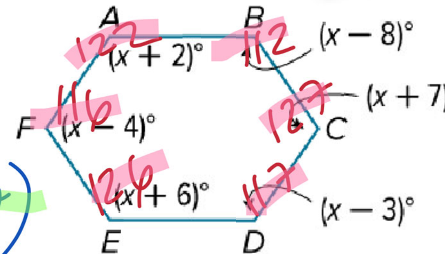
$m\angle M = [3(37) + 14]°$ or $125°$ $m\angle H = 2x° = 2(37)°$ or $74°$

Example 1

Find the Interior Angles Sum of a Polygon

Check

Find the measure of all the angles.



$180(n-2)$
$180(6-2)$
$180(4)$
720

$(x+2)+(x-8)+(x+7)+(x-3)+(x+6)+(x-4) = 720$

$\frac{6x}{6} = \frac{720}{6}$

$x = 120$

Example 2

Identify the Polygon Given the Sum of the Interior Angle Measures

Let n = the number of sides in the polygon. If the sum of the interior angle measures is 1440°, what kind of polygon is it? By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as 180°(n-2).

$1440 = 180(n - 2)$
$1440 = 180n - 360$
$+360 \qquad +360$
$1800 = 180n$

$\frac{1800}{180} = \frac{180n}{180}$

$10 = n$

$180(10-2)$
$180(8) = 1440$

The polygon has 10 sides, so it is a regular decagon.

Example 2: Check

Identify the Polygon Given the Sum of the Interior Angle Measures

Let n = the number of sides in the polygon. If the sum of the interior angle measures is 1080°, what kind of polygon is it?

$180(8-2) = 1080$

$180(6)$

$1080 = 180(n-2)$

$1080 = 180n - 360$

$+360 \qquad +360$

$1440 = 180n$

$\frac{1440}{180} = \frac{180n}{180}$

$8 = n$

octagon

Example 3

Identify the Polygon Given the Interior Angle Measure

The measure of an interior angle of a regular polygon is 150°. Find the number of sides in the polygon.

all sides & ∠'s are =

Let n = the number of sides in the polygon. Because all angles of a regular polygon are congruent, the sum of the interior angle measures is 150n°. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as 180°(n-2).

$150n = 180(n-2)$

$150n = 180n - 360$

$-180n \quad -180n$

$-30n = -360$

$\frac{-30n}{-30} = \frac{-360}{-30}$

$n = 12$

$180(12-2)$

$180(10) = 1800$

The polygon has 12 sides, so it is a regular dodecago

Example 3: Check

Identify the Polygon Given the Interior Angle Measure

The measure of an interior angle of a regular polygon is 120°. Find the number of sides in the polygon.

$120n = 180(n-2)$

$180(6-2)$

$120n = 180(n-2)$

$120n = 180n - 360$

$-180n \quad -180n$

$-60n = -360$

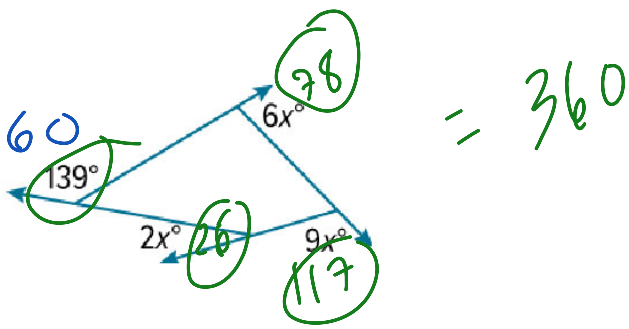
$\frac{-60n}{-60} = \frac{-360}{-60}$

$n = 6$ → $180(4)$

$\frac{720}{6} = 120°$

Example 4
Find Missing Values

Find the value of *x*.



$139 + 6x + 9x + 2x = 360$

$17x + 139 = 360$

$-139 \quad -139$

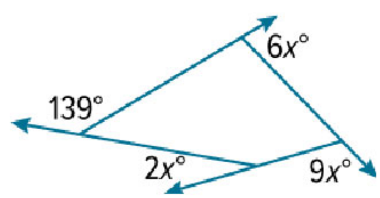
$17x = 221$

$\frac{17x}{17} = \frac{221}{17}$

$x = 13$

Example 4
Find Missing Values

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for *x*.



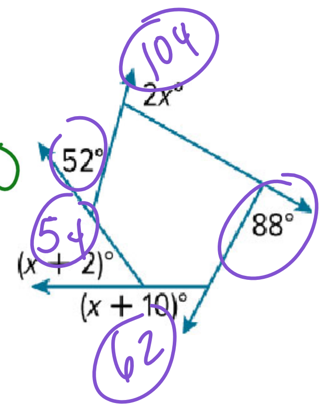
$6x° + 9x° + 2x° + 139° = 360°$ Write an equation.

$x = 13$ Solve.

Example 4
Find Missing Values

Check

Find the value of *x*.



$52 + 2x + 88 + x + 10 + x + 2 = 360$

$4x + 152 = 360$

$-152 \quad -152$

$\frac{4x}{4} = \frac{208}{4}$

$x = 52$

Example 5
Find Exterior Angle Measures of a Polygon

→ 360

Find the measure of each exterior angle of a regular dodecagon.

12

all sides
angles =

$\frac{360}{12} = 30$

$30 \times 12 = 360$

Example 5
Find Exterior Angle Measures of a Polygon

Find Exterior Angle Measures of a Polygon

A regular dodecagon has 12 congruent sides and 12 congruent interior angles. The exterior angles are also congruent, because angles supplementary to congruent angles are congruent.

Let n = the measure of each exterior angle and write and solve an equation.

$12n = 360°$	Polygon Exterior Angles Sum Theorem
$n = 30°$	Solve.

The measure of each exterior angle of a regular dodecagon is 30°.

Example 5
Find Exterior Angle Measures of a Polygon

Check

What is the measure of each exterior angle of a regular octagon?

8

360

all sides & ∠'s =

45
x 8
360

$\frac{360}{8} = 45°$