Lesson 5.2: Congruent Triangles

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Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.1.3 Prove relationships and theorems about triangles. Solve mathematical and real-world problems involving postulates, relationships and theorems of triangles.

MA.912.GR.1.6 Solve mathematical and real-world problems involving congruence or similarity in two-dimensional figures.

MA.912.GR.2.6 Apply rigid transformations to map one figure onto another to justify that the two figures are congruent.

Lesson Objectives

Content Objective

Students will use congruence criteria and the Third Angles Theorem to solve problems.

Language Objectives

- Students explain congruence between triangles based on their corresponding parts using *same, equal, corresponding*, and *congruent*.

Learn
Interior Angles of Triangles

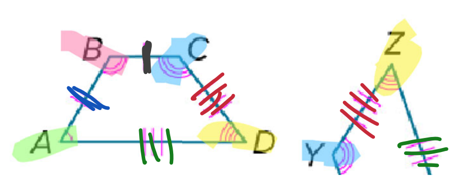
Key Concept: Congruent Triangles

Two triangles are congruent if and only if their corresponding parts are congruent.

For triangles, we say *Corresponding parts of congruent triangles are congruent*, or CPCTC.

Example 1
Identify Corresponding Congruent Parts

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a



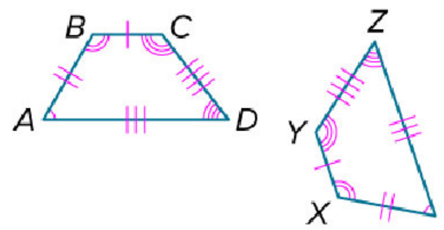
congruence statement.

$ABCD \cong WXYZ$

Example 1
Identify Corresponding Congruent Parts

Angles: $\angle A \cong \angle W$; $\angle B \cong \angle X$; $\angle C \cong \angle Y$; $\angle D \cong \angle Z$

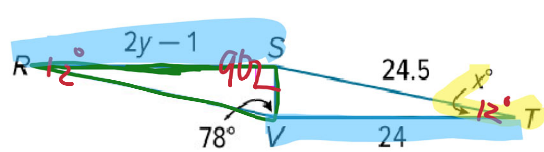
Sides: $\overline{BC} \cong \overline{XY}$; $\overline{AB} \cong \overline{WX}$; $\overline{DA} \cong \overline{ZW}$; $\overline{CD} \cong \overline{YZ}$



All corresponding parts of the two polygons are congruent. Therefore, polygon *ABCD* $\cong$ polygon *WXYZ*.

Example 2
Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y.



Part A Find the value of x. 12°

Part B Find the value of y.

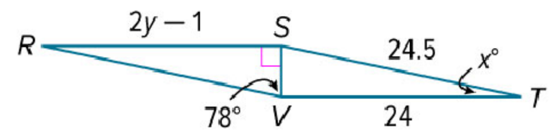
180 -

$$2y - 1 = 24$$
$$\underline{\quad +1 \quad +1}$$
$$\frac{2y}{2} = \frac{25}{2}$$
$$y = 12.5$$

Example 2

Use Corresponding Parts of Congruent Triangles

Part A Find the value of *x*.



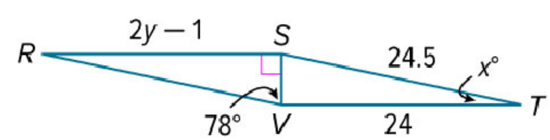
$\angle T \cong \angle R$	CPCTC
$m\angle T = m\angle R$	Definition of congruence
$= 180° - 90° - 78°$	Triangle Angle-Sum Theorem
$= 12°$	Solve.

The value of x is 12.

Example 2

Use Corresponding Parts of Congruent Triangles

Part B Find the value of *y*.



$\overline{RS} \cong \overline{TV}$	CPCTC
$RS = TV$	Definition of congruence
$2y - 1 = 24$	Substitution
$y = 12.5$	Solve.

The value of y is 12.5.

Example 2

Use Corresponding Parts of Congruent

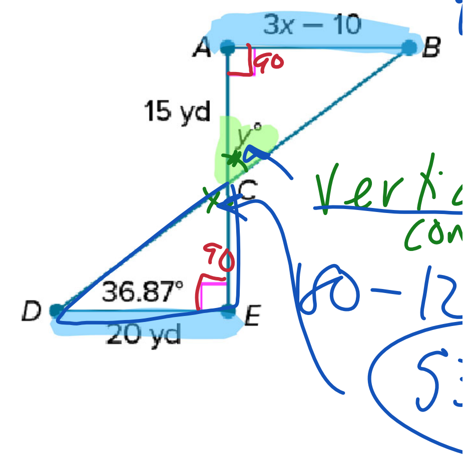
Triangles

∡ Triangle = 180

Check

In the diagram, $\triangle ABC \cong \triangle EDC$. Find the values of x and y.

$$3x - 10 = 20$$
$$+10 \quad +10$$
$$\frac{3x}{3} = \frac{30}{3}$$
$$x = 10$$

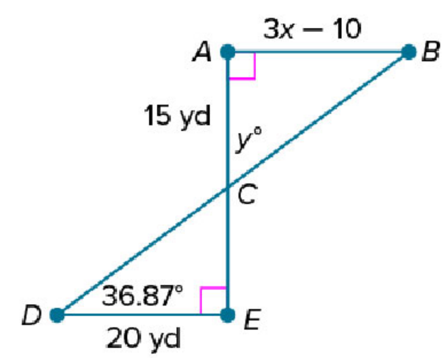


Example 2
Use Corresponding Parts of Congruent Triangles

Check

In the diagram, $\triangle ABC \cong \triangle EDC$. Find the values of x and y.

$x = 10$; $y = 53.13$

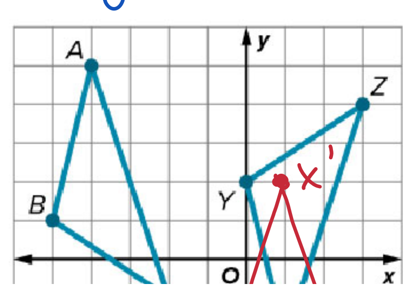


Example 3
Justify Congruence Using Rigid Transformations

yes congruent

Rigid motion

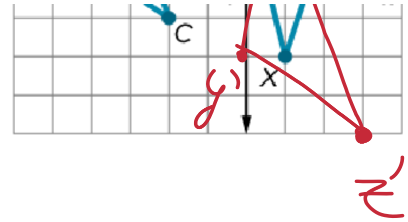
Is $\triangle ABC \cong \triangle XYZ$? Justify your answer using rigid transformations.



Reflection xyz over the x-axis

T

Translation
$(x-5, y+3)$

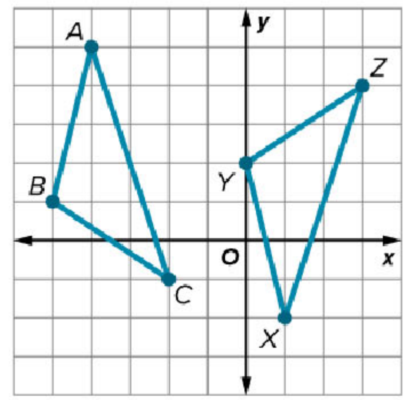


Example 3
Justify Congruence Using Rigid Transformations

Identify the rigid transformations that could map $\triangle XYZ$ onto $\triangle ABC$.

If X and A are corresponding vertices, we need to reorient $\triangle XYZ$ so that X is the topmost vertex of the triangle. Reflect $\triangle XYZ$ in the x-axis.

$$X(1, -2) \rightarrow X'(1, 2)$$
$$Y(0, 2) \rightarrow Y'(0, -2)$$
$$Z(3, 4) \rightarrow Z'(3, -4)$$



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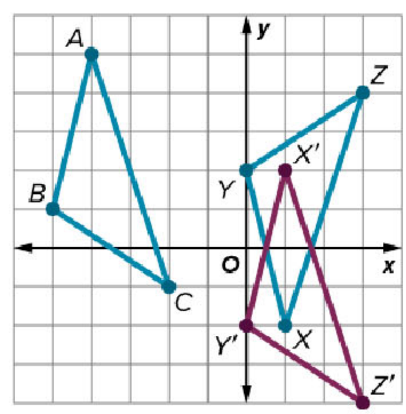
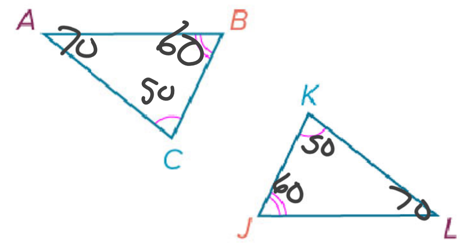
Example 3
Justify Congruence Using Rigid Transformations

Each vertex of $\triangle X'Y'Z'$ can be mapped onto $\triangle ABC$ with a translation along the vector $\langle -5, 3\rangle$. We know that rigid transformations preserve side lengths and angle measures. Therefore, after the reflection and translation the corresponding sides and corresponding angles of $\triangle ABC$ and the image of $\triangle XYZ$ coincide. Therefore, $\triangle ABC \cong \triangle XYZ$.



Learn
Third Angles Theorem and Triangle Congruence

Theorem 5.3: Third Angles Theorem

Words	If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.
Example	If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$. 180 60+50 = −110 70 

Learn
Third Angles Theorem and Triangle Congruence

Theorem 5.4: Properties of Triangle Congruence

Reflexive Property of Triangle Congruence
$\triangle ABC \cong \triangle ABC$ (Reflection)
Same

Symmetric Property of Triangle Congruence
If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.
Symmetry

Transitive Property of Triangle Congruence
If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

~~ABC = EF~~
~~EFG = JK~~
ABC = JK

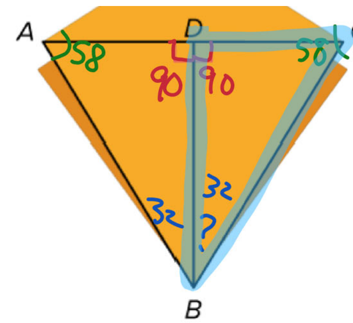
Example 4
Use the Third Angles Theorem

ORIGAMI Aika is folding origami dragons for

a party she is hosting. If $\angle ABD \cong \angle CBD$ and $\boxed{m\angle BAD = 58°}$, find $\boxed{m\angle CBD}$.

$90 + 58 = 148$

$180 - 148 = 32$

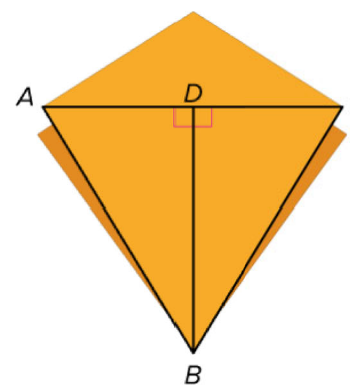


Example 4

Use the Third Angles Theorem

$m\angle BCD = 58°$	Substitute.
$m\angle CBD + 58° = 90°$	Substitute.
$m\angle CBD = 32°$	Solve.

The measure of $\angle CBD$ is 32°.



Example 4

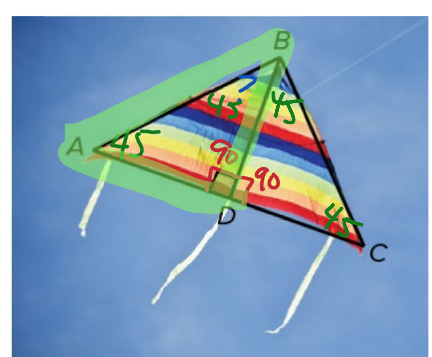
Use the Third Angles Theorem

Check

KITES In the kite shown, $\angle BAD \cong \angle BCD$ and $\boxed{m\angle BCD = 45°}$ Find $m\angle ABD$. 45°

$45 + 90 = 135$

$180 - 135 = 45$



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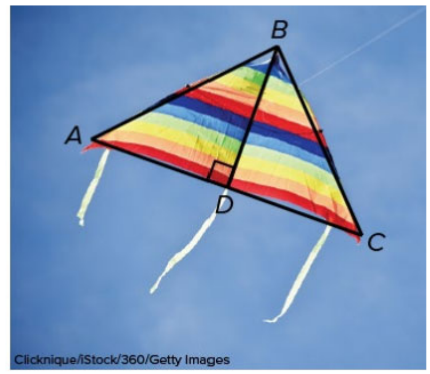
Example 4

Use the Third Angles Theorem

Check

KITES In the kite shown, $\angle BAD \cong \angle BCD$ and $m\angle BCD = 45°$. Find $m\angle ABD$.

$m\angle ABD = 45°$



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