Lesson 3.8 Slope and Equations of Lines

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Workbook pages 191-196

Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.3.3

Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

Content Objective

Students classify lines as parallel, perpendicular, or neither by using the slope criteria.

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Learn

Slope Criteria for Parallel and Perpendicular Lines

Slope is the ratio of the change in the *y*-coordinate (rise) to the corresponding change in the *x*-coordinate (run) as you move from one point to another along a line. The **slope criteria** outlines a method for proving the relationship between lines based on a comparison of the slopes of the lines. You can use the slopes of two lines to determine whether the lines are parallel, perpendicular, or neither.

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Learn

Slope Criteria for Parallel and Perpendicular Lines

Postulate 3.13: Slope Criteria for Parallel and Perpendicular Lines

Slopes of Parallel Lines

Two distinct nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

Slopes of Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is −1. Vertical and horizontal lines are perpendicular.

negative Reciprocals

$-\frac{2}{3}$ $\frac{3}{2}$ $\frac{4}{1}$ $-\frac{1}{4}$

$\frac{1}{3} \times \frac{-3}{1} = \frac{-3}{3}$ (neg. reci

Example 1

Determine Line Relationships When Given Points

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for

Slope $\overline{AB}$

$\frac{y_2 - y_1}{}$ $\frac{6-2}{}$

A(3, 6), B(−9, 2), C(5, 4), and D(2, 3).
Graph each line to verify your answer.

$x_2 y_2$ $x_1 y_1$ Same slope parallel

$x_2 - x_1$ Slope CD $\frac{4-3}{5-2}$

Example 1
Determine Line Relationships When Given Points

Step 1 Find the slope of each line.

slope $= \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

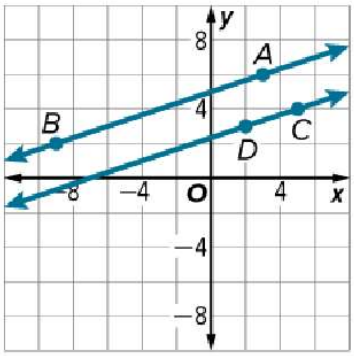
slope of $\overleftrightarrow{AB} = \frac{6-2}{3-(-9)} = \frac{4}{12}$ or $\frac{1}{3}$

slope of $\overleftrightarrow{CD} = \frac{4-3}{5-2}$ or $\frac{1}{3}$

Example 1
Determine Line Relationships When Given Points

Step 2 Determine the relationship.

The two lines have the same slope, so they are parallel.



Example 1
Determine Line Relationships When Given Points

Check

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for $A(14, 13)$, $B(-11, 0)$, $C(-3, 7)$, and $D(-4, -5)$. Graph each line to verify your answer.

x_1, y_1, x_2, y_2 x_1, y_1 x_2, y_2

neither

$\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{0 - 13}{-11 - 14} = \frac{-13}{-25}$

Slope AB

$\frac{13}{25}$

$\frac{-5-7}{-4+3}$

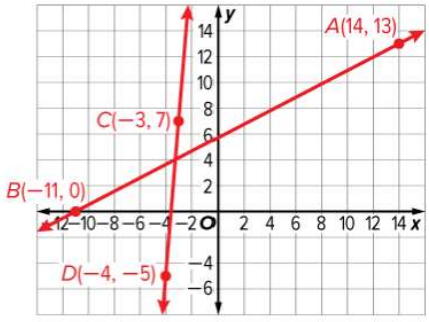
$\frac{-12}{-1}$

$\frac{12}{1}$ slope

Example 1
Determine Line Relationships When Given Points

Check

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for $A(14, 13)$, $B(-11, 0)$, $C(-3, 7)$, and $D(-4, -5)$. Graph each line to verify your answer. neither

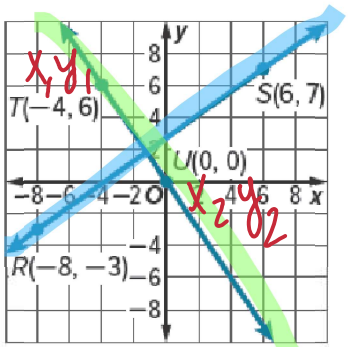


Example 2
Determine Line Relationships When Given Graphs

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

a. $\overleftrightarrow{RS}$ and $\overleftrightarrow{TU}$

b. $\overleftrightarrow{EF}$ and $\overleftrightarrow{DG}$



Rise/Run $\frac{10}{14} = \frac{5}{7}$

TU $\frac{-6}{4} = -\frac{3}{2}$

neither

EF $-\frac{7}{3}$

DG $\frac{6}{14} = \frac{3}{7}$

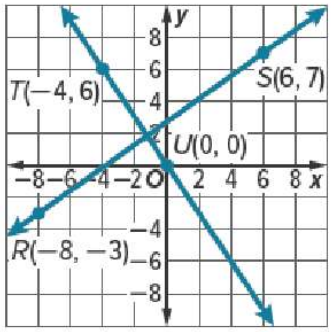
perpendicular (negative reciprocal)

$-\frac{7}{3} \cdot \frac{3}{7} = -\frac{21}{21} = -1$ ✓

Example 2

Determine Line Relationships When Given Graphs

a. $\overleftrightarrow{RS}$ and $\overleftrightarrow{TU}$



Step 1 Find the slope of each line.

slope $= \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

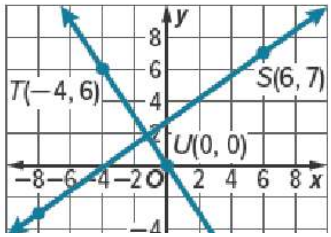
slope of $\overleftrightarrow{RS} = \frac{7-(-3)}{6-(-8)} = \frac{10}{14}$ or $\frac{5}{7}$

slope of $\overleftrightarrow{TU} = \frac{0-6}{0-(-4)} = -\frac{6}{4}$ or $-\frac{3}{2}$

Example 2

Determine Line Relationships When Given Graphs

b. $\overleftrightarrow{EF}$ and $\overleftrightarrow{DG}$

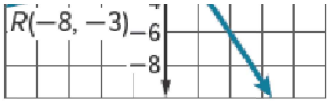


Step 1 Find the slope of each line.

slope $= \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

slope of $\overleftrightarrow{EF} = \frac{-1-6}{6-3} = -\frac{7}{3}$

slope of $\overleftrightarrow{DG} = \frac{5-(-1)}{12-(-2)} = \frac{6}{14}$ or $\frac{3}{7}$

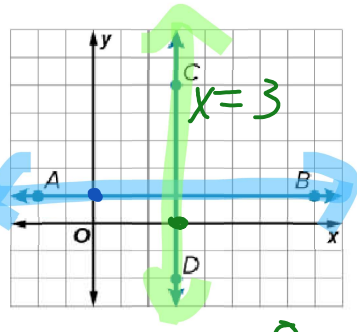


Example 2
Determine Line Relationships When Given Graphs

Check

Determine whether the pair of lines is *parallel*, ~~*perpendicular*~~, or *neither*.

vertical line
$x =$ x-int
Slope undefined $\frac{5}{0}$

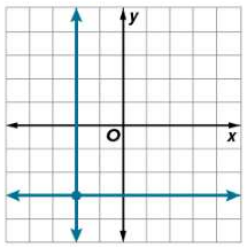


$y = 1$

$y =$ horizontal line
Slope $\frac{0}{8} = 0$

Learn
Equations of Lines

The equations of horizontal and vertical lines involve only one variable.

Key Concept: Horizontal and Vertical Line Equations	
The equation of a horizontal line is $y = b$, where b is the y-intercept of the line.	
The equation of a vertical line is $x = a$, where a is the x-intercept of the line.	

When given the equations of two lines, you can compare the equations determine the relationship between the lines.

Example 3
Determine Line Relationships When Given Equations

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

a. $y = 3x - 2$; $y - 0 = -\frac{1}{3}(x - 2)$

b. $y = 3$; $x = 1$

c. $y - 5 = -\frac{3}{4}(x + 2)$; $y = -\frac{3}{4}x + 2$

d. $y = 2x + 3$; $y - 1 = \frac{1}{2}(x + 2)$

e. $x = -2$; $x = 4$

Example 3
Determine Line Relationships When Given Equations

a. $y = 3x - 2$; $y - 0 = -\frac{1}{3}(x - 2)$

slope-intercept form: $y = 3x - 2$

point-slope form: $y - 0 = -\frac{1}{3}(x - 2)$

slope

Example 3
Determine Line Relationships When Given Equations

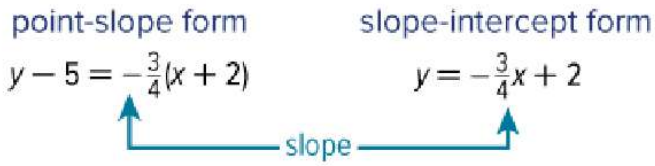
b. $y = 3$; $x = 1$

The line $y = 3$ is a horizontal line. The line $x = 1$ is a vertical line. Vertical and horizontal lines are always perpendicular.

Example 3

Determine Line Relationships When Given Equations

c. $y - 5 = -\frac{3}{4}(x + 2)$; $y = -\frac{3}{4}x + 2$

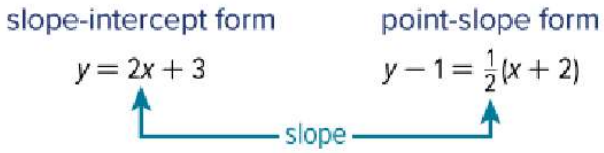


The equation $y - 5 = -\frac{3}{4}(x + 2)$ written in slope-intercept form is $y = -\frac{3}{4} + \frac{7}{2}$. Because the slopes of both lines are $-\frac{3}{4}$ and the *y*-intercepts are different, the lines are parallel.

Example 3

Determine Line Relationships When Given Equations

d. $y = 2x + 3$; $y - 1 = \frac{1}{2}(x + 2)$



Example 3
Determine Line Relationships When Given Equations

e. $x = -2$; $x = 4$

Both lines are vertical with undefined slope. Vertical lines are always parallel.

Example 3
Determine Line Relationships When Given Equations

Check

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

$y = mx + b$ $y = mx + b$ Slopes are neg. reciprocals perpendicular

$\frac{3}{1}$ $-\frac{1}{3}$

a. $y = 3x - 9$; $y = -\frac{1}{3}x + 2$

$y = mx + b$

b. $y = \frac{9}{7}x - \frac{19}{7}$; $y - 1 = \frac{9}{7}(x + 3)$ Same slope

$y = \frac{9}{7}x + \frac{}{}$

parallel

c. $x = -3$; $x = 4$ parallel

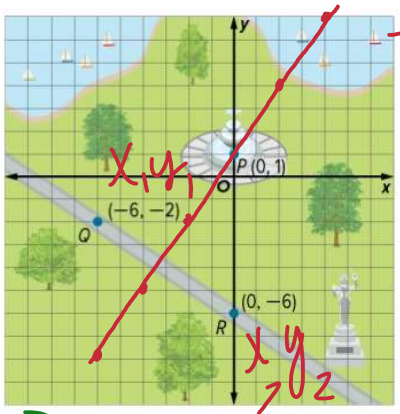
Example 4

Example 4
Use Slope to Graph a Line

Slope of $\overleftrightarrow{QR}$

DESIGN Valentina is designing a park using grid paper. She wants to build a sidewalk that connects with the fountain at $P(0, 1)$ and is perpendicular to the existing sidewalk that passes through points $Q(-6, -2)$ and $R(0, -6)$. Graph the line that represents the new sidewalk.

new $\frac{3}{2}$



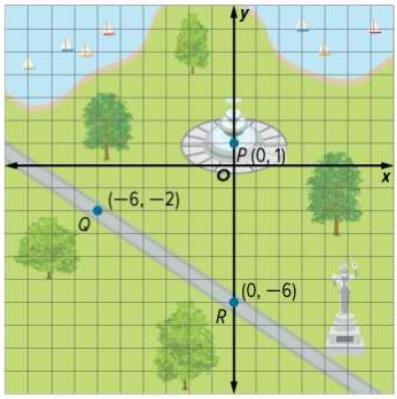
$\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{-6+2}{0+6}$ $\frac{-4}{6} = -\frac{2}{3}$

Example 4
Use Slope to Graph a Line

The slope of the existing sidewalk, $\overleftrightarrow{QR}$, is $\frac{-6-(-2)}{0-(-6)} = -\frac{4}{6}$ or $-\frac{2}{3}$.

Because $-\frac{2}{3}\left(\frac{3}{2}\right) = -1$, the slope of the line perpendicular to $\overleftrightarrow{QR}$ through P is $\frac{3}{2}$.

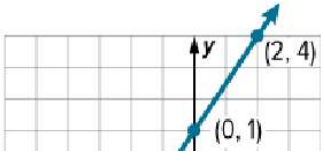
Graph the line that represents the new sidewalk.



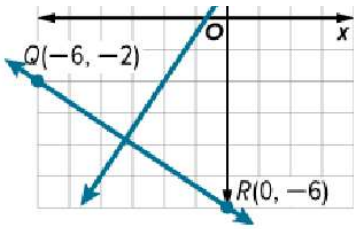
Example 4
Use Slope to Graph a Line

Step 1 Use the slope.

Use the slope of the line perpendicular to $\overleftrightarrow{QR}$ to find another point on the line that passes through



point $P(0, 1)$. From $P(0, 1)$, move up 3 units and then right 2 units. Plot a point at this location.



Step 2 **Graph the line connecting these two points.**

The new sidewalk will pass through $P(0, 1)$ and the new point that you plotted.

Example 5
Write Equations of Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line parallel to $y = -\frac{3}{4}x + 3$ containing (−3, 6).

Slope $-\frac{3}{4}$

$-\frac{3}{4} \cdot \frac{3}{1} = -\frac{9}{4}$

$y - y_1 = m(x - x_1)$

$y - 6 = -\frac{3}{4}(x + 3)$

$y - 6 = -\frac{3}{4}x - \frac{9}{4}$

$+6 \quad +\frac{6}{1} = \frac{24}{4}$

$y = \frac{-3}{4}x + \frac{15}{4}$

$y = mx + b$

$6 = -\frac{3}{4}(-3) + b$

$6 = \frac{9}{4} + b$

$-\frac{9}{4} \quad -\frac{9}{4}$

Example 5
Write Equations of Parallel and Perpendicular Lines

The slope of $y = -\frac{3}{4}x + 3$ is $-\frac{3}{4}$, so the slope of a line parallel to it is $-\frac{3}{4}$.

$y = mx + b$	Slope-intercept form
$6 = -\frac{3}{4}(-3) + b$	$m = -\frac{3}{4}$ and $(x, y) = (-3, 6)$
$6 = \frac{9}{4} + b$	Simplify.
$\frac{15}{4} = b$	Subtract $\frac{9}{4}$ from each side.

So, the equation is $y = -\frac{3}{4}x + \frac{15}{4}$.

Example 5
Write Equations of Parallel and Perpendicular Lines

Check

Write an equation in slope-intercept form for the line parallel to $y = \frac{1}{2}x + \frac{5}{2}$ containing $\left(\frac{3}{2}, 1\right)$.

Same slope

$y = mx + b$

$\frac{1}{2} \cdot -\frac{3}{2} = -\frac{3}{4}$

$y = \frac{1}{2}x + \frac{1}{4}$

$y - y_1 = m(x - x_1)$

x, y_1

$y - 1 = \frac{1}{2}\left(x - \frac{3}{2}\right)$

$y - 1 = \frac{1}{2}x - \frac{3}{4}$

$+1$ $+1$

$-\frac{3}{4}$ $\frac{4}{4}$ $\frac{1}{4}$

Example 5
Write Equations of Parallel and Perpendicular Lines

Check

Write an equation in slope-intercept form for the line parallel to $y = \frac{1}{2}x + \frac{5}{2}$ containing $\left(\frac{3}{2}, 1\right)$.

$y = \frac{1}{2}x + \frac{1}{4}$

Exit Ticket

Write the equation of the line parallel to the given line and

Write the equation of the line parallel to the given line and containing the given point.

1. $y = 3x + 2, (1, 2)$

2. $x + 2y = 6, (4, 3)$

Write the equation of the line perpendicular to the given line and containing the given point.

3. $-3x + 4y = 16, (3, 2)$

4. $4x - y = 9, (8, -6)$

Exit Ticket

Write the equation of the line parallel to the given line and containing the given point.

1. $y = 3x + 2, (1, 2)$ $y = 3x - 1$

2. $x + 2y = 6, (4, 3)$ $y = -\frac{1}{2}x + 5$

Write the equation of the line perpendicular to the given line and containing the given point.

3. $-3x + 4y = 16, (3, 2)$ $y = -\frac{4}{3}x + 6$

4. $4x - y = 9, (8, -6)$ $y = -\frac{1}{4}x - 4$