Lesson 3.10 Perpendiculars and Distance

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Perpendiculars and Distance
Workbook pages 209-214

Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.1.1
Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.

MA.912.GR.3.3
Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

Lesson Objectives

Content Objective

Students use perpendicular lines to find distance.

Language Objectives

- Students use *have to, must,* and *need to* to explain how to find distance using perpendicular lines.

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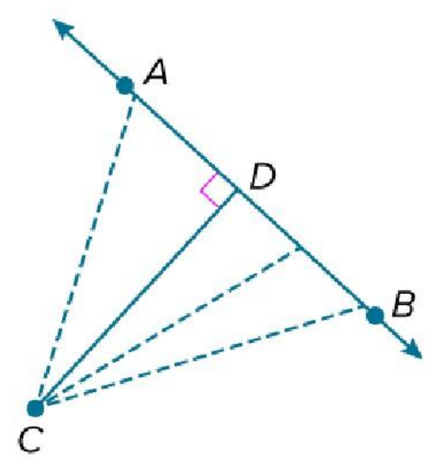
Learn

Distance Between a Point and a Line

Given $\overleftrightarrow{AB}$ and point C not on the line, there are an infinite number of lines that pass through the point and intersect the line. The shortest distance between the point and the line is the length of the segment that is perpendicular to the line through the point. So, the distance between C and $\overleftrightarrow{AB}$ is CD.



Learn

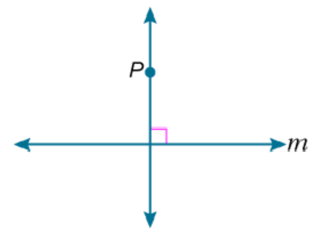
Distance Between a Point and a Line

Just as there is one shortest distance from C to $\overleftrightarrow{AB}$, there is exactly one line that passes through C and is perpendicular to $\overleftrightarrow{AB}$.

perpendicular to $\overline{AB}$.

Postulate 3.14: Perpendicular Postulate

If given a line and a point not on the line, then there exists exactly one line through the point that is perpendicular to the given line.



Example 1

Distance from a Point to a Line on the Coordinate Plane

$\frac{1}{2}$ $\frac{2}{4}$ Slope $\frac{\text{rise}}{\text{run}}$ $\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$

Line ℓ contains points (1, 2) and (5, 4). Find the distance between line ℓ and the point *P*(1, 7).

new slope (neg. reciprocal) $\frac{-2}{1}$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(3-1)^2+(3-7)^2}$

$\sqrt{2^2+-4^2}$

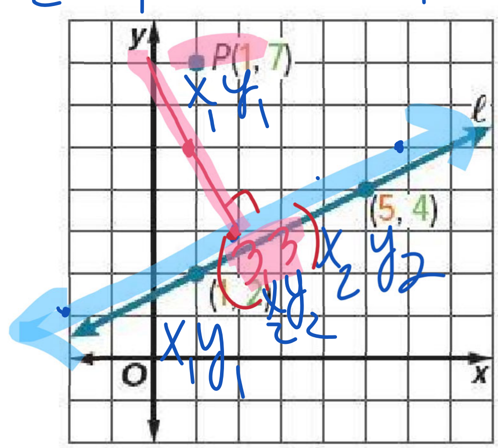
$\sqrt{4+16}$

$\sqrt{20} = 4.5$

$\sqrt{20}$

$\sqrt{4}\cdot\sqrt{5}$

$2\sqrt{5}$



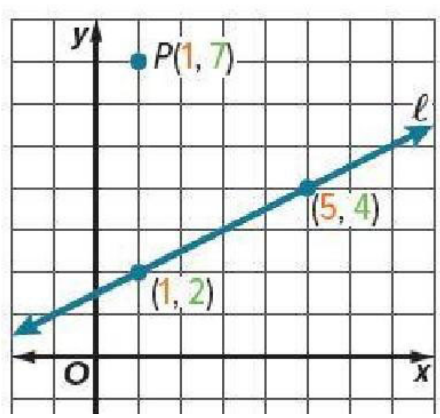
Example 1

Distance from a Point to a Line on the Coordinate Plane

Step 1 Find the equation of line ℓ.

Begin by finding the slope of the line through points (1, 2) and (5, 4).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{5-1} = \frac{2}{4} \text{ or } \frac{1}{2}$$



Example 1

Distance from a Point to a Line on the Coordinate Plane

Then write the equation of the line using the point (1, 2).

$y = mx + b$	Slope-intercept form
$2 = \frac{1}{2}(1) + b$	$m = \frac{1}{2}$ and $(x, y) = (1, 2)$
$2 = \frac{1}{2} + b$	Simplify.
$\frac{3}{2} = b$	Subtract $\frac{1}{2}$ from each side.

The equation of line ℓ is $y = \frac{1}{2}x + \frac{3}{2}$.

Example 1

Distance from a Point to a Line on the Coordinate Plane

Step 2 Find the equation of the line perpendicular to line ℓ.

Write the equation of line w that is perpendicular to line ℓ and contains $P(1, 7)$. Because the slope of line ℓ is $\frac{1}{2}$, the slope of line w is −2. Write the equation of line w through $P(1, 7)$ with slope −2.

Example 1

Distance from a Point to a Line on the Coordinate Plane

Coordinate Plane

$y = mx + b$	Slope-intercept form
$7 = -2(1) + b$	$m = -2$, $(x, y) = (1, 7)$
$7 = -2 + b$	Simplify.
$9 = b$	Add 2 to each side.

The equation of the line is $y = -2x + 9$.

Example 1
Distance from a Point to a Line on the Coordinate Plane

Step 3 Solve the system of equations.

Find the point of intersection of lines ℓ and w.

Solve the system of equations to determine the point of intersection.

Example 1
Distance from a Point to a Line on the Coordinate Plane

$y = \frac{1}{2}x + \frac{3}{2}$	Equation of line ℓ
$y = -2x + 9$	Equation of line w

$4y = 2x + 6$	Multiply each term in the first equation by 4.
$(+)\ y = -2x + 9$	Add the second equation.
$5y = 15$	Eliminate x.
$y = 3$	Divide each side by 5.

Example 1
Distance from a Point to a Line on the Coordinate Plane

Solve for x.

$y = -2x + 9$	Equation of line w
$3 = -2x + 9$	Substitute 3 for y.
$-6 = -2x$	Subtract 9 from each side.
$3 = x$	Divide each side by −2.

The point of intersection is (3, 3). Let this be point Q.

Example 1
Distance from a Point to a Line on the Coordinate Plane

Step 4 Calculate the distance between *P* and *Q*.

Use the Distance Formula to determine the distance between $P(1, 7)$ and $Q(3, 3)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(3 - 1)^2 + (3 - 7)^2}$	$x_2 = 3$, $x_1 = 1$, $y_2 = 3$, and $y_1 = 7$
$= \sqrt{20}$	Simplify.

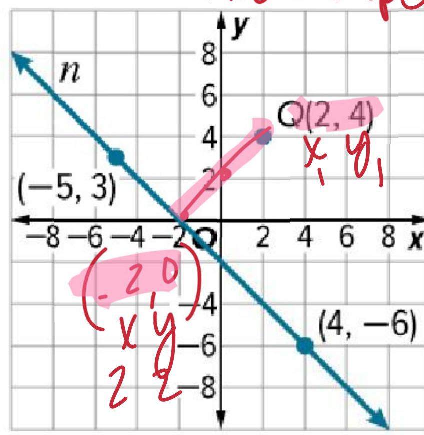
The distance between point P and line ℓ is $\sqrt{20}$ or about 4.47 units.

Example 1

Distance from a Point to a Line on the Coordinate Plane

Check

Line n contains points (−5, 3) and (4, −6). Find the distance between line n and point $Q(2, 4)$. Round to the nearest tenth, if necessary.



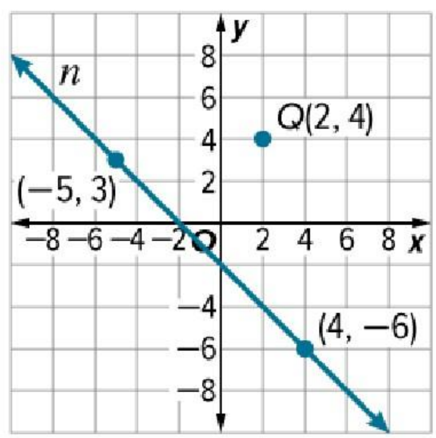
Example 1

Distance from a Point to a Line on the Coordinate Plane

Check

Line n contains points (−5, 3) and (4, −6). Find the distance between line n and point $Q(2, 4)$. Round to the nearest tenth, if necessary.

5.7 units

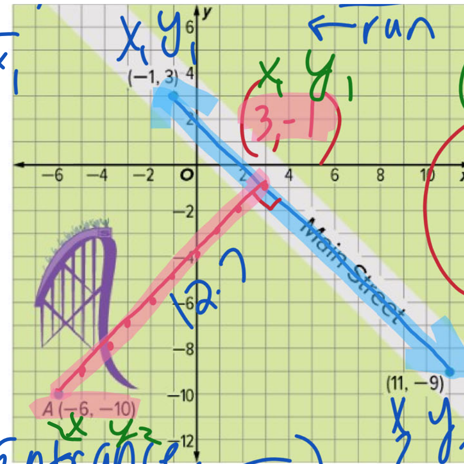


Apply Example 2

Solve a Design Problem by Using Distance

AMUSEMENT PARK The developers of an

AMUSEMENT PARK The developers of an amusement park want to build a new attraction. According to park regulations, the entrance to each attraction must be at least 10 yards from the center of Main Street. In the design plans, the entrance to the new attraction is located at *A*(−6, −10), and Main Street contains the points (−1, 3) and (11, −9). If each unit represents 1 yard, will the new attraction comply with park regulations? Explain.

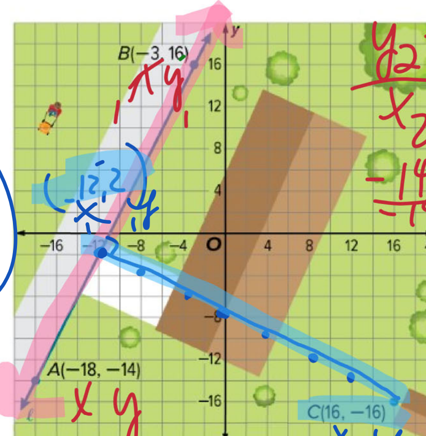


Apply Example 2

Solve a Design Problem by Using Distance

Check

ZONING Javier wants to build a shed on his property. According to zoning laws, the shed must be at least 20 feet from his property line. Javier knows that points *A* and *B* fall on his property line. If Javier plans to build the shed behind his house at point *C*, will he satisfy the zoning laws? If yes, how far away will the shed be from Javier's property line? Each unit on the coordinate plane represents 1 foot.



A. no B. yes; 31.3 ft C. yes; 34.1 ft D. yes; 37.2 ft

Apply Example 2

Solve a Design Problem by Using Distance

Check

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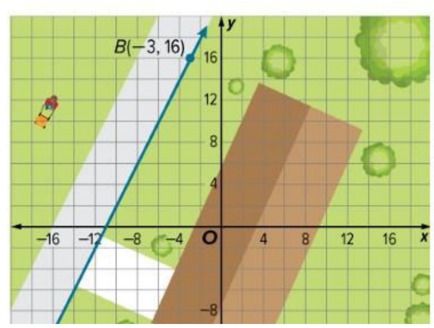
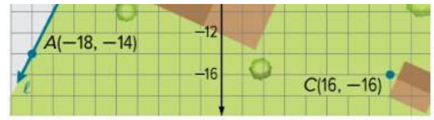


be from Javier's property line? Each unit on the coordinate plane represents 1 foot. B



A. no B. yes; 31.3 ft C. yes; 34.1 ft D. yes; 37.2 ft

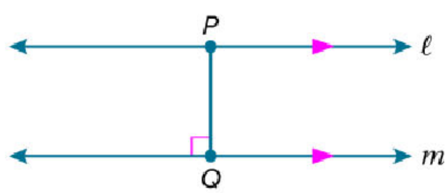
Learn
Distance Between Parallel Lines

By definition, parallel lines do not intersect. An alternate definition states that two lines in a plane are parallel if they are always equidistant. Two lines are **equidistant** from each other if the distance between the two lines, measured along a perp.

Learn
Distance Between Parallel Lines

Key Concept: Distance Between Parallel Lines

The distance between two parallel lines is the perpendicular distance between one of the lines and any point on the other line.



Example 3