

Module 4: Transformations

Sunday, December 11, 2022 4:28 PM



Module 4
Notes

Module 4: Transformations

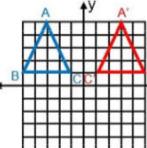
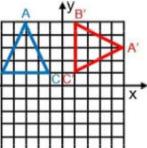
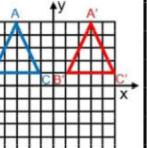
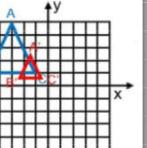
① Reflections	② Rotations	③ Translations	④ Dilations
<p>- "Flips" over a given line</p> <p>Example:</p> <p>Pre-image Image</p> <p>Think about a mirror image!</p>	<p>- "Turns" about a point</p> <p>Example:</p> <p>• Image</p> <p>Pre-image</p> <p>Think about turning a doorknob!</p>	<p>- "Slides" to a new location</p> <p>Example:</p> <p>Pre-image Image</p> <p>Think about when you slide an object such as a bookcase!</p>	<p>- "Enlarges" or "reduces" a figure</p> <p>Example:</p> <p>Reduction</p> <p>Pre-image Image</p> <p>Enlargement</p> <p>Pre-image Image</p>

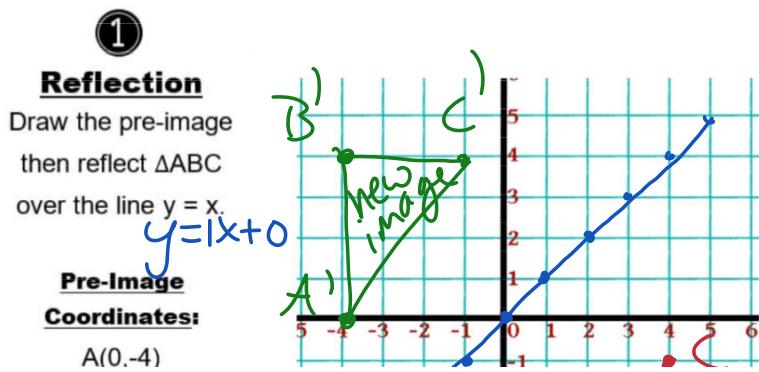
The "pre-image" is the figure before it has undergone a transformation. The "image" is the result of the transformation. We denote the difference between the "pre-image" and "image" when we label our figure. For example,

Points are denoted with capital letters

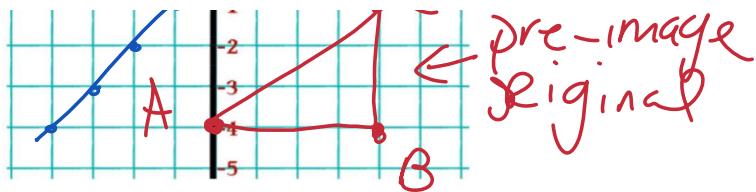
Pre-image A B C → Image A' B' C'

Points have a "prime" symbol now

1 Reflection	2 Rotation	3 Translation	4 Dilation
			
<p>The pre-image is reflected over the y-axis to create the resulting image.</p> <p>Note – Always make sure to pay very close attention to the label of each point. The pre-image was flipped over the y-axis because the bottom points have changed their position.</p>	<p>The pre-image is rotated 90° clockwise about the origin to create the resulting image.</p> <p>Note – Clockwise moves in the direction of the hands on a clock (Counterclockwise moves in the opposite direction). c.w. stands for clockwise and c.c.w stands for counterclockwise</p>	<p>The pre-image is shifted to the right 6 units to create the resulting image.</p> <p>Note – This graph looks very similar to a reflection, but if you pay close attention to the labeling of the points you will see that the points did not change their position.</p>	<p>The pre-image is dilated by a scale factor of $\frac{1}{2}$ to create the resulting image.</p> <p>Note - The scale factor is denoted with "k".</p> <p>$k = \frac{1}{2}$</p> <p>A fraction represents a reduction (A whole number would represent an enlargement).</p>



B(4,-4)
C(4,-1)



2

Rotation

Draw the pre-image

then rotate $\triangle DEF$

90° c.c.w.
 $-y, x$

Pre-Image

Coordinates:

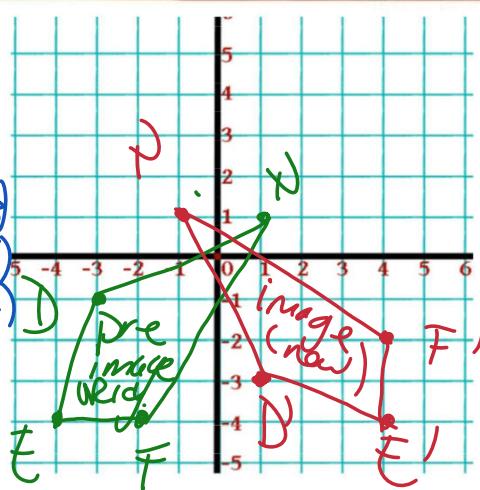
D(-3, -1)

E(-4, -4)

F(-2, -4)

N(1, 1)

Counterclockwise Rotation	Clockwise Rotation	Coordinate Rule
90° counterclockwise	270° clockwise	$(x, y) \rightarrow (-y, x)$
180° counterclockwise	180° clockwise	$(x, y) \rightarrow (-x, -y)$
270° counterclockwise	90° clockwise	$(x, y) \rightarrow (y, -x)$



③

Translation

Draw the pre-image
then translate KLMN

3 units to the left
and 5 units down.

Pre-Image

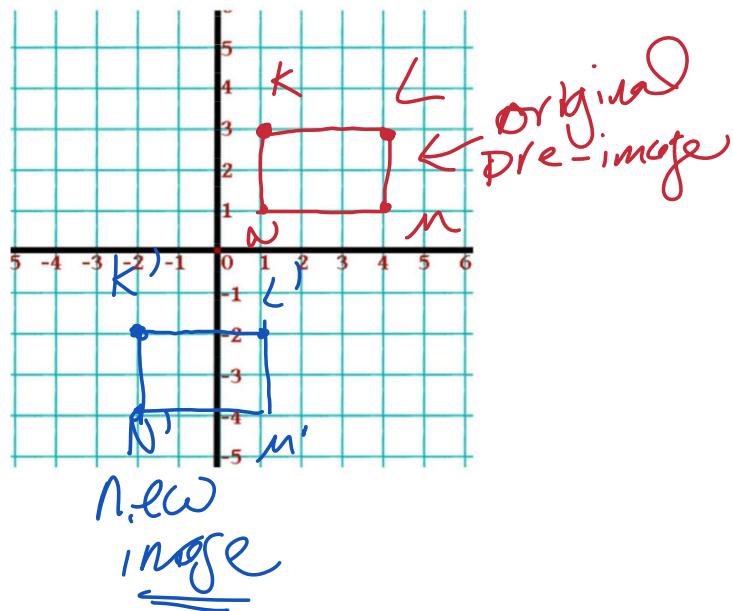
Coordinates:

K(1,3)

L(4,3)

M(4,1)

N(1,1)



4

Dilation

Draw the pre-image
then dilate $\triangle XYZ$ by
a scale factor of 2.

$$\begin{pmatrix} -4 & 4 \\ -4 & -4 \\ 4 & -4 \end{pmatrix}$$

Multiply by

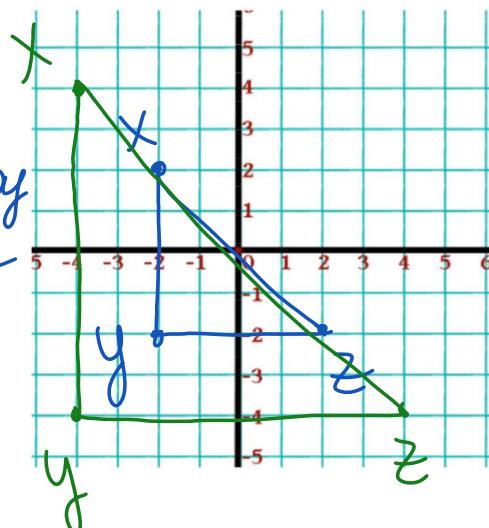
Pre-Image

Coordinates:

X(-2, 2)

Y(-2, -2)

Z(2, -2)

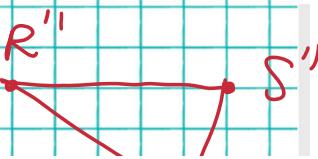


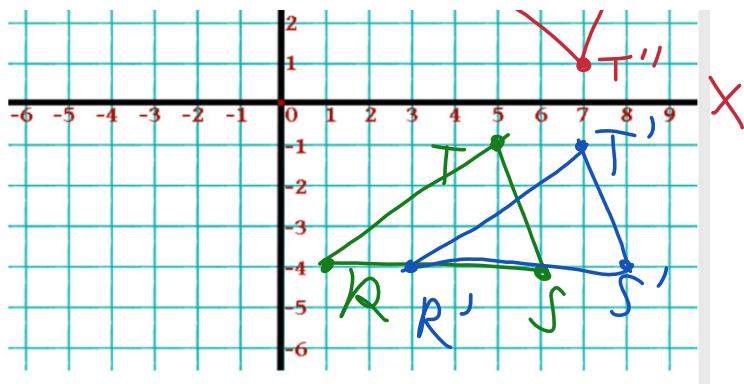
#5 Graph each figure with the given vertices and its image after the indicated glide reflection.

 $\triangle RST: R(1, -4), S(6, -4), T(5, -1)$ Translation: along $\langle 2, 0 \rangle$

Reflection: in x-axis

Right

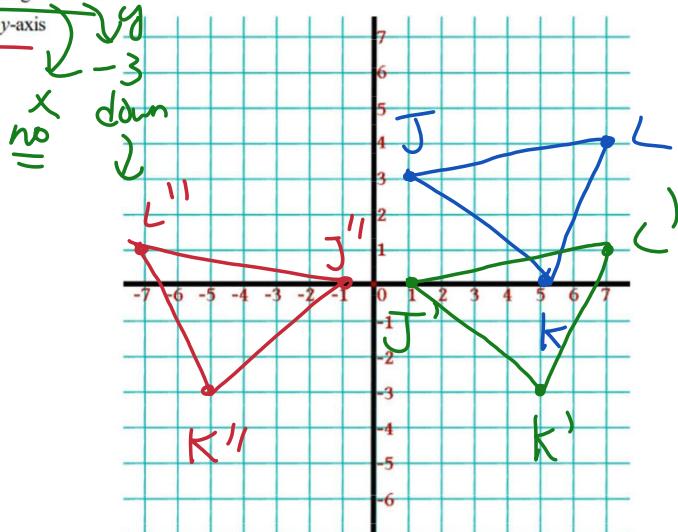




#6 $\triangle JKL: J(1, 3), K(5, 0), L(7, 4)$

Translation: along $\langle 0, -3 \rangle$

Reflection: in y -axis



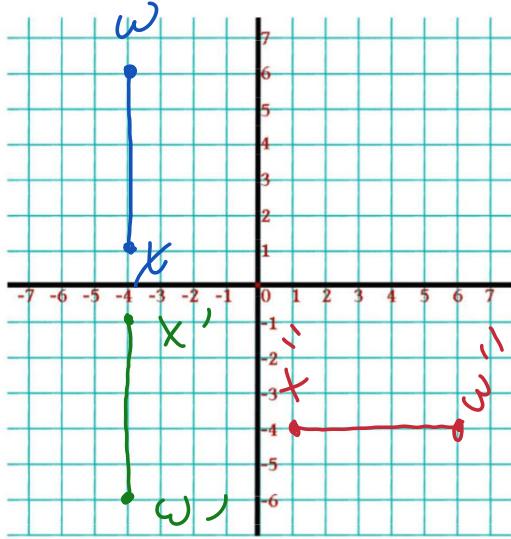
#7 \overline{WX} : $W(-4, 6)$ and $X(-4, 1)$

Reflection: in x-axis

Rotation: 90° about origin

$$(-y, x)$$

$$\begin{aligned}x' & (-4, -1) \\x'' & (+1, -4) \\w' & (-4, -6) \\w'' & (+6, -4)\end{aligned}$$

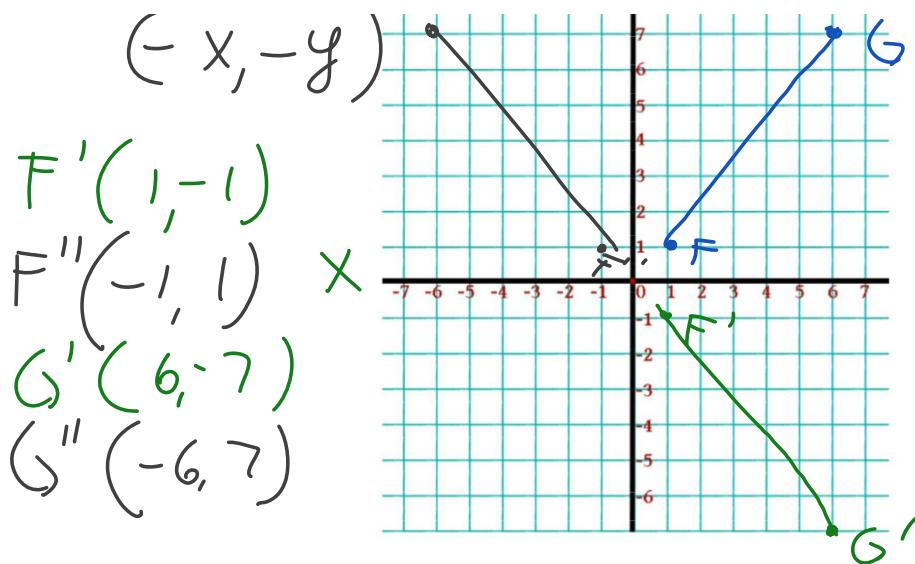


#8 \overline{FG} : $F(1, 1)$ and $G(6, 7)$

Reflection: in x-axis

Rotation: 180° about origin

$$G''$$



#9

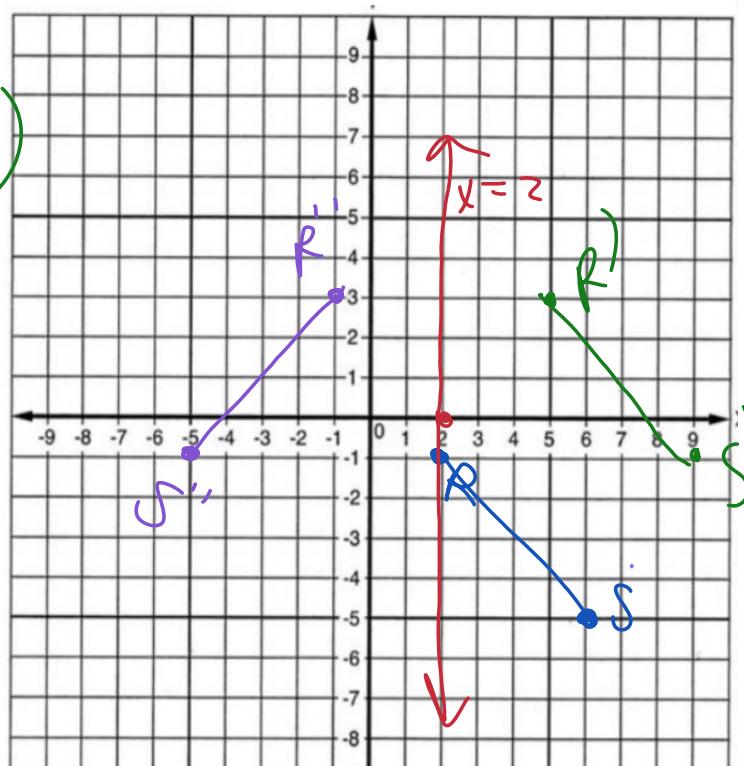
\overline{RS} : $R(2, -1)$ and $S(6, -5)$

Translation: along $\langle 3, 4 \rangle$

Reflection: in $x = 2$

$$(x+3, y+4)$$

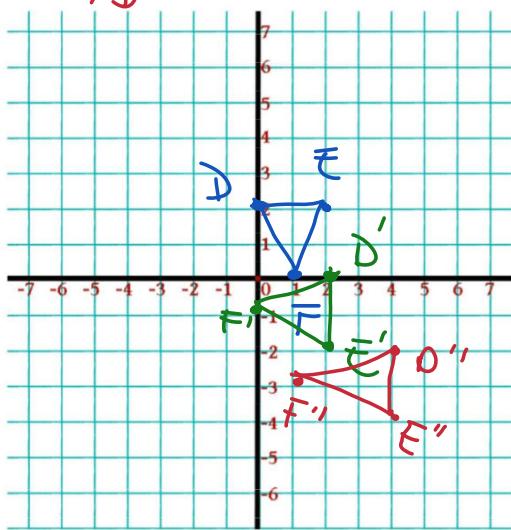
$\swarrow 3$ $\nearrow 4$



#10 **270° counter clockwise rotation about the origin followed by a translation along $\langle 2, -2 \rangle$**

$D(0, 2)$, $E(2, 2)$, $F(1, 0)$

$D'(2, 0)$
 $E'(2, -2)$
 $F'(-1, 1)$



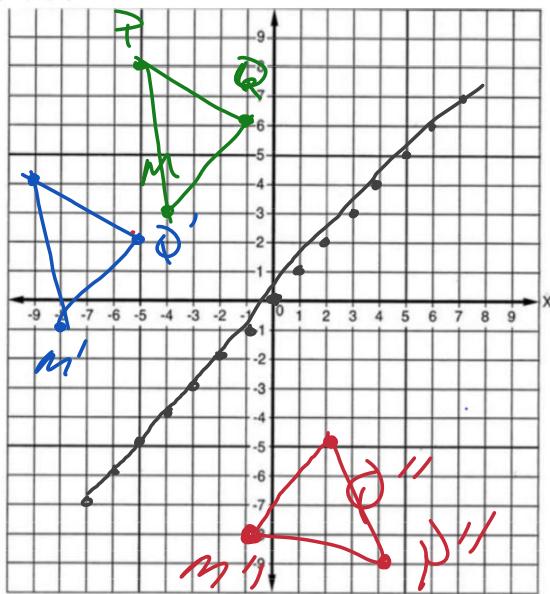
#11 $\triangle MPQ$: M (-4, 3), P (-5, 8), Q (-1, 6)

Translation: along $\langle -4, -4 \rangle$

Reflection: in $y = x$

$$y = mx + b$$

$$y = |x| + 0$$



① Reflections

$$R_{x\text{-axis}}(x,y) = (x,-y)$$

$$R_{y\text{-axis}}(x,y) = (-x,y)$$

$$R_{y=x}(x,y) = (y,x)$$

② Rotations

$$R_{90\text{cw}}(x,y) = (y,-x)$$

$$R_{90\text{ccw}}(x,y) = (-y,x)$$

$$R_{180}(x,y) = (-x,-y)$$

③ Translations

$$T_{a,b}(x,y) = (x+a,y+b)$$

"a" moves left or right
and "b" moves up or down

④ Dilations

$$D_k(x,y) = (kx,ky)$$

Scale factor = k

Example:
Determine the

$$y = -x \wedge y = -y \wedge$$

Example:

Determine the coordinates of $\Delta A'B'C'$ given that ΔABC is reflected over the y-axis: $A=(2,3)$, $B=(-1,4)$, $C=(3,-5)$

Solution:

The rule needed is:
 $y\text{-axis}(x,y) = (-x,y)$
 I will take the "opposite" of the x-coordinate and keep the y-coordinate the same. Hence,
 $A=(2,3) \rightarrow A'=(-2,3)$
 $B=(-1,4) \rightarrow B'=(1,4)$
 $C=(3,-5) \rightarrow C'=(-3,-5)$

Example:

Determine the coordinates of $\Delta F'G'H'$ given that ΔFGH is rotated 90° c.c.w: $F=(4,-2)$, $G=(-7,1)$, $H=(1,0)$

Solution:

The rule needed is:
 $R_{90\text{ccw}}(x,y) = (-y,x)$
 I will take the "opposite" of the y-coordinate and keep the x-coordinate the same, then SWITCH the position of the coordinates. Hence,
 $F=(4,-2) \rightarrow F'=(2,4)$
 $G=(-7,1) \rightarrow G'=(-1,-7)$
 $H=(1,0) \rightarrow H'=(0,1)$

Example:

Determine the coordinates of $\Delta X'Y'Z'$ given that ΔXYZ is shifted left 5 units and up 3 units: $X=(-4,-6)$, $Y=(1,8)$, $Z=(0,1)$

Solution:

The rule is:
 $T_{a,b}(x,y) = (x+a,y+b)$
 $T_{-5,3}(x,y) = (x-5,y+3)$
 I will subtract 5 from the x-coordinate and add 3 to the y-coordinate. Hence,
 $X=(-4,-6) \rightarrow X'=(-9,-3)$
 $Y=(1,8) \rightarrow Y'=(-4,11)$
 $Z=(0,1) \rightarrow Z'=(-5,4)$

coordinates of $\Delta M'N'O'$

given that ΔMNO is dilated by a scale factor of $k = \frac{1}{3}$: $M=(-3,0)$, $N=(6,-6)$, $O=(3,1)$

Solution:

The rule is:
 $D_k(x,y) = (kx,ky)$
 $D_{1/3}(x,y) = (\frac{1}{3}x,\frac{1}{3}y)$
 I will multiply both the x- and y-coordinate by a factor of $\frac{1}{3}$ (Divide each coordinate by 3). Hence,
 $M=(-3,0) \rightarrow M'=(-1,0)$
 $N=(6,-6) \rightarrow N'=(2,-2)$
 $O=(3,1) \rightarrow O'=(1,\frac{1}{3})$