

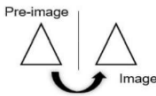
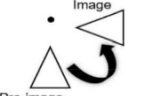
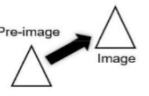
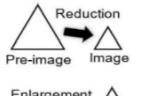

Module 4: Transformations

Sunday, December 11, 2022 4:28 PM




Module 4
Notes

Module 4: Transformations

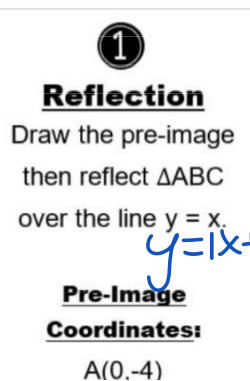
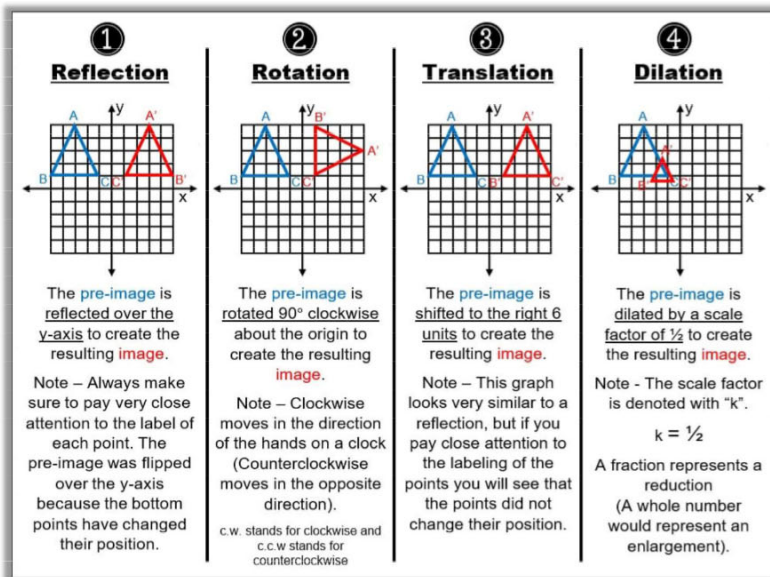
1	2	3	4
Reflections - "Flips" over a given line Example:  Think about a mirror image!	Rotations - "Turns" about a point Example:  Think about turning a doorknob!	Translations - "Slides" to a new location Example:  Think about when you slide an object such as a bookcase!	Dilations - "Enlarges" or "reduces" a figure Example:  Reduction  Enlargement

The "pre-image" is the figure before it has undergone a transformation. The "image" is the result of the transformation. We denote the difference between the "pre-image" and "image" when we label our figure. For example,

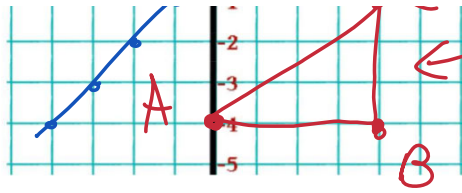
Points are denoted with capital letters



Points have a "prime" symbol now



B(4,-4)
C(4,-1)



← pre-image
original

2

Rotation

Draw the pre-image
then rotate $\triangle DEF$

90° c.c.w.

$-y, x$

Pre-Image

Coordinates:

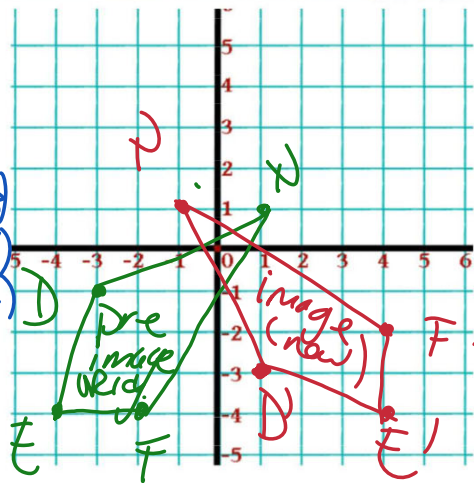
D(-3,-1)

E(-4,-4)

F(-2,-4)

N(1,1)

Counterclockwise Rotation	Clockwise Rotation	Coordinate Rule
90° counterclockwise	270° clockwise	$(x, y) \rightarrow (-y, x)$
180° counterclockwise	180° clockwise	$(x, y) \rightarrow (-x, -y)$
270° counterclockwise	90° clockwise	$(x, y) \rightarrow (y, -x)$



3

Translation

Draw the pre-image
then translate KLMN

3 units to the left
and 5 units down.

Pre-Image

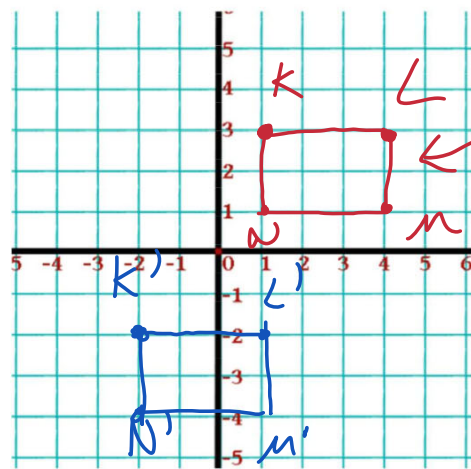
Coordinates:

K(1,3)

L(4,3)

M(4,1)

N(1,1)



4

Dilation

Draw the pre-image
then dilate $\triangle XYZ$ by
a scale factor of 2.

multiply by
Pre-Image

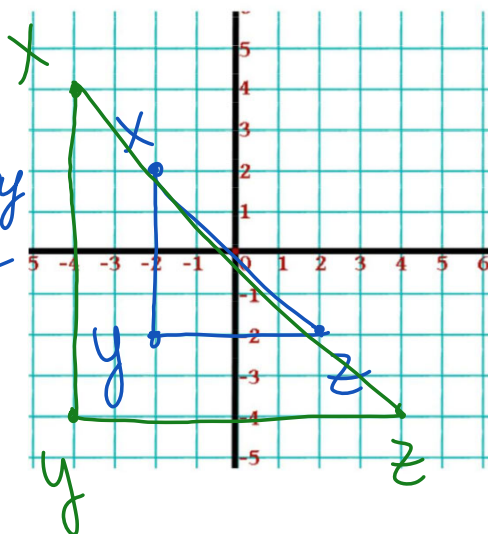
Coordinates:

$X(-2, 2)$

$Y(-2, -2)$

$Z(2, -2)$

(-4, 4)
(-4, -4)
(4, -4)



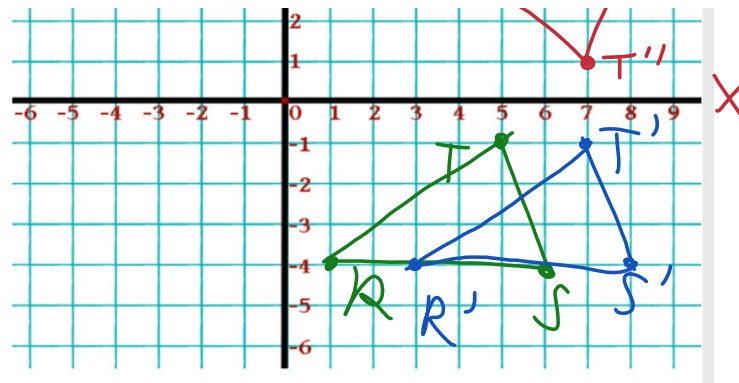
#5 Graph each figure with the given vertices and its image after the indicated glide reflection.

$\triangle RST$: $R(1, -4)$, $S(6, -4)$, $T(5, -1)$

Translation: along $\langle 2, 0 \rangle$

Reflection: in x-axis

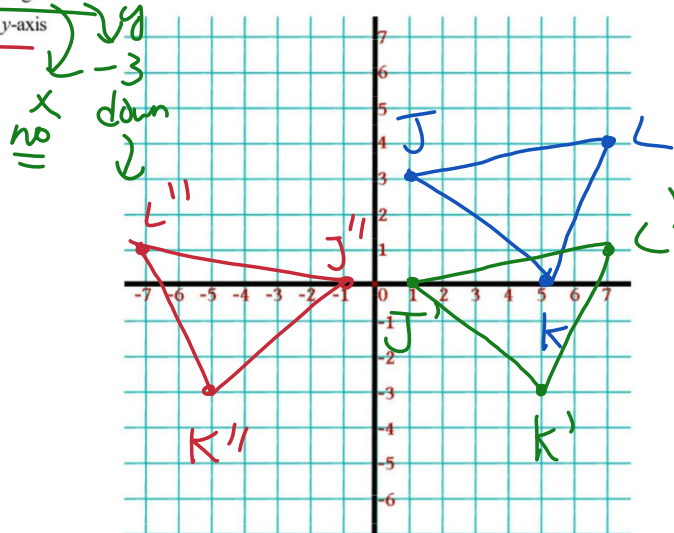




#6 $\triangle JKL: J(1, 3), K(5, 0), L(7, 4)$

Translation: along $\langle 0, -3 \rangle$

Reflection: in y-axis



#7 \overline{WX} : $W(-4, 6)$ and $X(-4, 1)$
Reflection: in x-axis
Rotation: 90° about origin

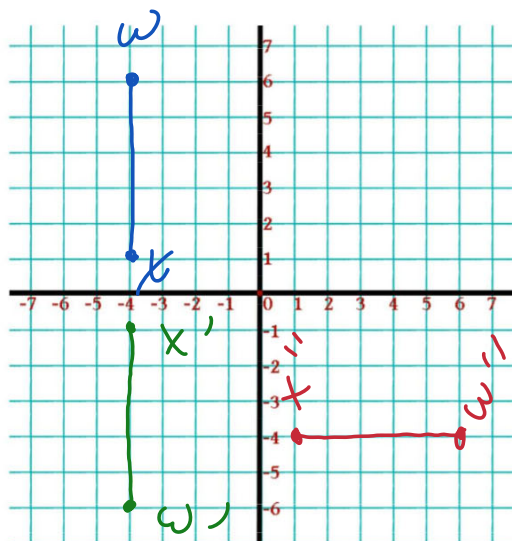
$(-y, x)$

$X'(-4, -1)$

$X''(+1, -4)$

$W'(-4, -6)$

$W''(+6, -1)$



#8 \overline{FG} : $F(1, 1)$ and $G(6, 7)$
Reflection: in x-axis
Rotation: 180° about origin

G''

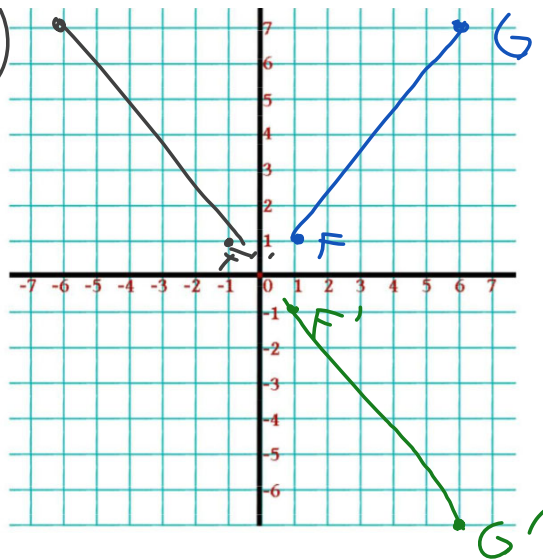
$$(-x, -y)$$

$$F'(1, -1)$$

$$F''(-1, 1) \quad \times$$

$$G'(6, 7)$$

$$G''(-6, 7)$$



#9

$\overline{RS} : R(2, -1) \text{ and } S(6, -5)$

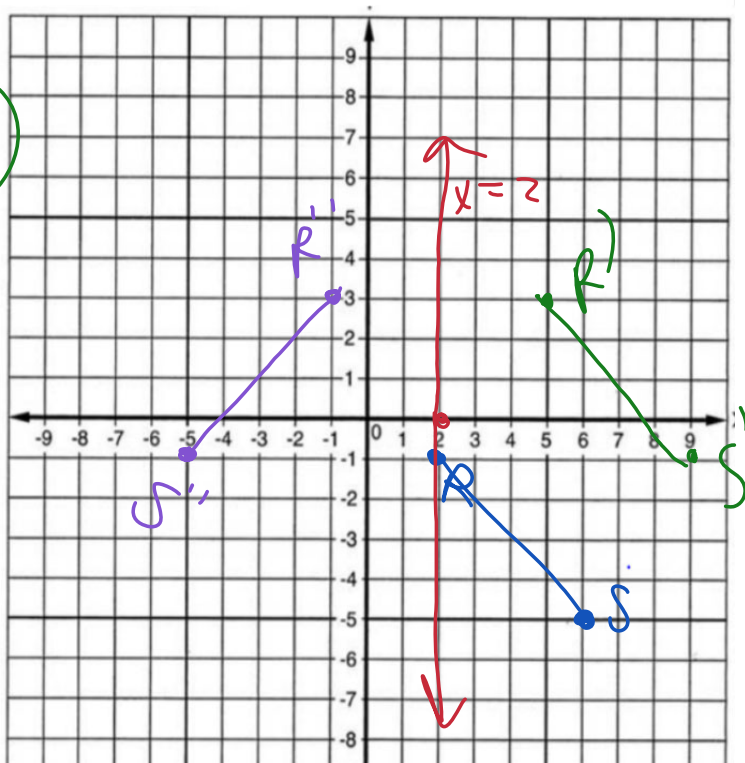
Translation: along $\langle 3, 4 \rangle$

Reflection: in $x = 2$

$$(x+3, y+4)$$

$\langle 3, 4 \rangle$

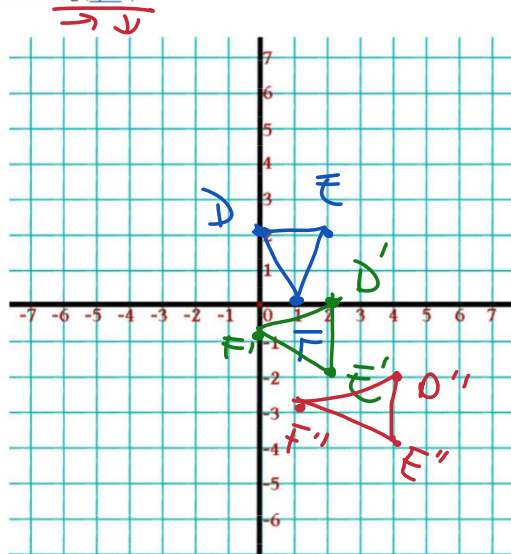
$\downarrow 3$ $\uparrow 4$



#10 270° counter clockwise rotation about the origin followed by a translation along $\langle 2, -2 \rangle$

D: (0,2), E: (2,2), F (1,0)

y, \ominus
 $D(2,0)$
 $E'(2,2)$
 $F'(0,1)$



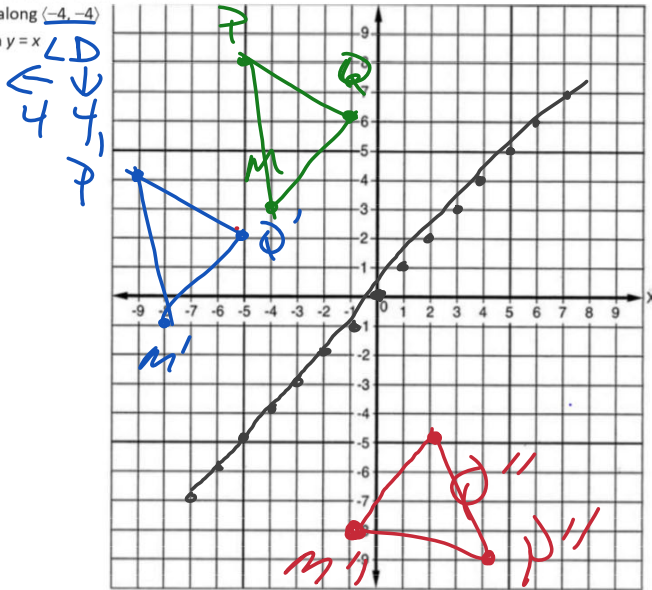
#11 $\triangle MPQ$: $M(-4, 3)$, $P(-5, 8)$, $Q(-1, 6)$

Translation: along $\langle -4, -4 \rangle$

Reflection: in $y = x$

$$y = mx + b$$

$$\tilde{y} = |x| + 0$$



①

Reflections

$$r_{x\text{-axis}}(x, y) = (x, -y)$$

$$r_{y\text{-axis}}(x, y) = (-x, y)$$

$$r_{y=x}(x, y) = (y, x)$$

$$r_{y=-x}(x, y) = (-y, -x)$$

②

Rotations

$$R_{90\text{cw}}(x, y) = (y, -x)$$

$$R_{90\text{ccw}}(x, y) = (-y, x)$$

$$R_{180}(x, y) = (-x, -y)$$

$$R_{270}(x, y) = (x, -y)$$

③

Translations

$$T_{a,b}(x, y) = (x+a, y+b)$$

"a" moves left or right
and "b" moves up or
down

④

Dilations

$$D_k(x, y) = (kx, ky)$$

Scale factor = k

Example:

Determine the

$$r_y = (x(x,y), y) \rightarrow (-y, x)$$

Example:

Determine the coordinates of $\Delta A'B'C'$ given that ΔABC is reflected over the y-axis: $A = (2,3)$, $B = (-1,4)$, $C = (3,-5)$

Solution:

The rule needed is:
 $r_{y\text{-axis}}(x,y) = (-x,y)$
 I will take the "opposite" of the x-coordinate and keep the y-coordinate the same. Hence,
 $A = (2,3) \rightarrow A' = (-2,3)$
 $B = (-1,4) \rightarrow B' = (1,4)$
 $C = (3,-5) \rightarrow C' = (-3,-5)$

Example:

Determine the coordinates of $\Delta F'G'H'$ given that ΔFGH is rotated 90° c.c.w: $F = (4,-2)$, $G = (-7,1)$, $H = (1,0)$

Solution:

The rule needed is:
 $R_{90\text{ccw}}(x,y) = (-y,x)$
 I will take the "opposite" of the y-coordinate and keep the x-coordinate the same, then SWITCH the position of the coordinates. Hence,
 $F = (4,-2) \rightarrow F' = (2,4)$
 $G = (-7,1) \rightarrow G' = (-1,-7)$
 $H = (1,0) \rightarrow H' = (0,1)$

Example:

Determine the coordinates of $\Delta X'Y'Z'$ given that ΔXYZ is shifted left 5 units and up 3 units: $X = (-4,-6)$, $Y = (1,8)$, $Z = (0,1)$

Solution:

The rule is:
 $T_{a,b}(x,y) = (x+a,y+b)$
 $T_{-5,3}(x,y) = (x-5,y+3)$
 I will subtract 5 from the x-coordinate and add 3 to the y-coordinate. Hence,
 $X = (-4,-6) \rightarrow X' = (-9,-3)$
 $Y = (1,8) \rightarrow Y' = (-4,11)$
 $Z = (0,1) \rightarrow Z' = (-5,4)$

coordinates of $\Delta M'N'O'$ given that ΔMNO is dilated by a scale factor of $k = \frac{1}{3}$: $M = (-3,0)$, $N = (6,-6)$, $O = (3,1)$

Solution:

The rule is:
 $D_k(x,y) = (kx,ky)$
 $D_{1/3}(x,y) = (\frac{1}{3}x, \frac{1}{3}y)$
 I will multiply both the x- and y-coordinate by a factor of $\frac{1}{3}$ (Divide each coordinate by 3). Hence,
 $M = (-3,0) \rightarrow M' = (-1,0)$
 $N = (6,-6) \rightarrow N' = (2,-2)$
 $O = (3,1) \rightarrow O' = (1, \frac{1}{3})$