Lesson 3.2: Statements, Conditionals, and Biconditionals

Wednesday, October 19, 2022 8:55 PM

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3.2
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Statements, Conditionals, and Biconditionals

Mc Graw Hill

Florida's B.E.S.T. Standards for Mathematics

MA.912.LT.4.3
Identify and accurately interpret "if…then," "if and only if," "all" and "not" statements. Find the converse, inverse and contrapositive of a statement.

MA.912.LT.4.10
Judge the validity of arguments and give counterexamples to disprove statements.

Content Objective

Students write and analyze compound statements by using logic.

Learn
Using Logic

A **statement** is any sentence that is either true T or false F, but not both. **Truth value** is the truth or falsity of a statement. Statements are often represented using a letter such as *p* or *q*.

If a statement is represented by *p*, then *not p* or $\sim p$ is the **negation** of the statement. The negation of a statement has the opposite meaning, as well as the opposite truth value, of the original statement.

Learn
Using Logic

Two or more statements joined by the word *and* or *or* form a **compound statement**. A compound statement using the word *and* is called a **conjunction**. A conjunction is true only when both statements that form it are true. A conjunction is written as *p and q* or $p \wedge q$.

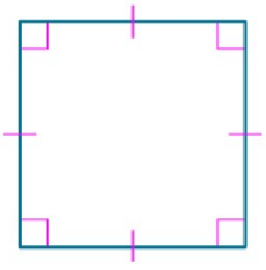
A compound statement using the word *or* is called a **disjunction**. A disjunction is true if at least one of the statements is true. A disjunction is written as *p or q* or $p \vee q$.

Example 1
Truth Values of Conjunctions

Use the statements to write each conjunction. Then find the truth values. Explain your reasoning.

trapezoid

***p*: The figure is a ~~pentagon.~~**



***q*: The figure has four congruent sides.**

***r*: The figure has four right angles.**

a. ***p* and *r*** The figure is a trapezoid and the figure has 4

b. **~*p* ∧ *q*** The figure is NOT a trapezoid and the figure ha

∧ and ∨ or

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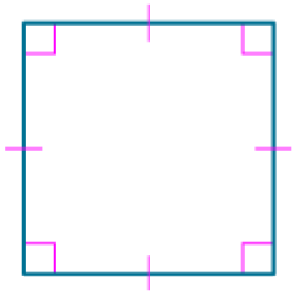
Example 1
Truth Values of Conjunctions

a. *p* and *r*

p and *r*: The figure is a pentagon, and the figure has four right angles. The statement *r* is true, and *p* is false. So, *p and r* is false.



b. ~*p* ∧ *q*

~*p* ∧ *q*: The figure is not a pentagon, and the figure has four congruent sides. Both ~*p* and *q* are true, so ~*p* ∧ *q* is true.

Example 2
Truth Values of Disjunctions

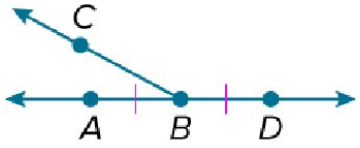
∠ABC and ∠CBD are compl. OR $\overline{AB} \not\cong \overline{BD}$

Use the statements to write the disjunction *p* ∨ ~*r*. Then find its truth value. Explain your reasoning.

***p*: ∠*ABC* and ∠*CBD* are complementary.**

***q*: ∠*ABC* and ∠*CBD* are vertical angles.**

***r*: $\overline{AB} \cong \overline{BD}$**



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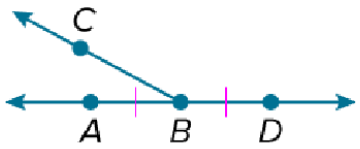
Example 2
Truth Values of Disjunctions

p ∨ ~*r*: ∠*ABC* and ∠*CBD* are complementary,

or $\overline{AB}$ and $\overline{BD}$ are not congruent.

$p \vee \sim r$ is false, because p is false and $\sim r$ is false.



Learn
Conditionals

A **conditional statement** is a compound statement that consists of a premise, or *hypothesis*, and a *conclusion*, which is false only when its hypothesis is true and its conclusion is false.

Learn
Conditionals

Conditional Statements and Related Conditionals

Words	Examples
An **if-then statement** is a compound statement of the form "if p, then q," where p and q are statements. Symbols: $p \rightarrow q$; read *if* p, *then* q, or p *implies* q	If it rains, then the parade will be canceled.
The **hypothesis** of an if-then statement is the phrase immediately following the word *if*. Symbols: $p \rightarrow q$; read *if* p, *then* q, or p *implies* q	

Learn
Conditionals

Conditional Statements and Related Conditionals

Words	Examples
The **conclusion** of an if-then statement is the phrase immediately following the word *then*. Symbols: $p \rightarrow q$; read *if p*, *then q*, or *p implies q*	If it rains, then the parade will be canceled.

Learn
Conditionals

Conditional Statements and Related Conditionals

Words	Examples
The **converse** is formed by exchanging the hypothesis and conclusion of the conditional. Symbols: $q \rightarrow p$, read *if q*, *then p*, or *q implies p*	If the parade is canceled, then it has rained.

Learn
Conditionals

Conditional Statements and Related Conditionals

Words	Examples
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional. Symbols: [illegible]	If it does not rain, then [illegible]

Symbols: $\sim p \rightarrow \sim q$, read *if not p, then not q*	then the parade will not be canceled.

Learn
Conditionals

Conditional Statements and Related Conditionals

Words	Examples
The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional. Symbols: $\sim q \rightarrow \sim p$, read *if not q, then not p*	If the parade is not canceled, then it does not rain.

Example 3
Identify the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

a. If a polygon has six sides, then it is a hexagon.

b. Another performance will be scheduled if the first one is sold out.

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Example 3
Identify the Hypothesis and Conclusion

a. If a polygon has six sides, then it is a hexagon.

Hypothesis: A polygon has six sides.

Conclusion: The polygon is a hexagon.

b. Another performance will be scheduled if the first one is sold out.

Notice that the word *if* appears in the second portion of the sentence.

Hypothesis: The first performance is sold out.

Conclusion: Another performance will be scheduled.

Example 3

Identify the Hypothesis and Conclusion

Check

Identify the hypothesis and conclusion of each conditional statement.

a. If the forecast is rain, then I will take an umbrella.

H

P *Hypothesis:* The forecast is Rain

Q *Conclusion:* I will take an umbrella

b. A number is divisible by 10 if its last digit is a 0.

H

Hypothesis: The last digit is 0

Conclusion: A # is div. by 10

Example 3

Identify the Hypothesis and Conclusion

Check

Identify the hypothesis and conclusion of each conditional statement.

a. If the forecast is rain, then I will take an umbrella.

Hypothesis: The forecast is rain.

Conclusion: I will take an umbrella.

b. A number is divisible by 10 if its last digit is a 0.

Hypothesis: The last digit of a number is 0.

Conclusion: A number is divisible by 10

Example 4
Write a Conditional in If-Then Form

Check

Identify the hypothesis and conclusion of the conditional statement *A quadrilateral with two sets of parallel sides is a parallelogram.* Then write the statement in if-then form.

Hypothesis: A quad. w/ 2 // sides

Conclusion: A parallelogram

If-then: If a quad w/ 2 set // side

Example 4
Write a Conditional in If-Then Form

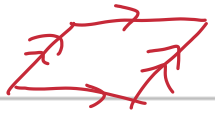
Check

Identify the hypothesis and conclusion of the conditional statement *A quadrilateral with two sets of parallel sides is a parallelogram.* Then write the statement in if-then form.

Hypothesis: A quadrilateral has two sets of parallel sides.

Conclusion: The quadrilateral is a parallelogram.

If-then: If a quadrilateral has two sets of parallel sides, then it is a parallelogram.



Learn
Biconditionals

You can use logic and *biconditional statements* to indicate exclusivity in situations. For example, a square is a parallelogram with all four sides and all four angles congruent. You can express this as two if-then