Lesson 3.1: Conjectures and Counterexamples

Monday, October 17, 2022   5:28 PM

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pdf
Lesson 3.1
Conjectu

Conjectures and Counterexamples

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Florida's B.E.S.T. Standards for Mathematics

MA.912.LT.4.10
Judge the validity of arguments and give counterexamples to disprove statements.

MA.K12.MTR.6.1

Assess the reasonableness of solutions.

MA.K12.MTR.7.1
Apply mathematics to real-world contexts.

Content Objective
Students analyze conjectures by using inductive reasoning
and disprove conjectures by using counterexamples.

Learn
Inductive Reasoning and Conjecture

Inductive reasoning is the process of reaching a
conclusion based on a pattern of examples. When you
assume that an observed pattern will continue, you are
applying inductive reasoning. You can use inductive
reasoning to make an educated guess based on known
information and specific examples. This educated guess is
also known as a **conjecture**.

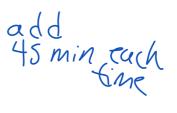## Example 1
Patterns and Conjectures

guess

inductive reasoning

Write a **conjecture** that describes the **pattern in the
sequence**. Then use your **conjecture** to **find the next**
term in the sequence.

Appointment times: 8:30 A.M., 9:15 A.M., 10:00 A.M., 10:45 A.M., . . .

[handwritten] +15 min   11:30

11:00
+30 min
11:30

[handwritten, right] add 45 min each time

Example 1
Patterns and Conjectures

Step 1   Look for a pattern.

8:30 A.M.   9:15 A.M.   10:00 A.M.   10:45 A.M.

+45 min   +45 min   +45 min

Step 2   Make a conjecture.

Each appointment time is 45 minutes after the previous appointment time. The next appointment time will be 10:45 A.M. + 0:45 or 11:30 A.M.
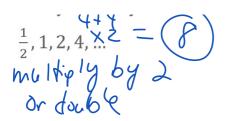
Example 1
Patterns and Conjectures

[handwritten] $\frac{1}{2} + \frac{1}{2} = 1$ $\frac{1}{2} \cdot \frac{2}{1} = \frac{2}{2} = 1$

Check

Write a conjecture that describes the pattern in the sequence. Then use your conjecture to find the next term in the sequence.

$\frac{1}{2}, 1, 2, 4, ...$ $\overset{4+4}{\times 2} = \boxed{8}$

multiply by 2
or double

Example 1
Patterns and Conjectures

Check

Write a conjecture that describes the pattern in the sequence. Then use your conjecture to find the next term in the sequence.

$\frac{1}{2}, 1, 2, 4, ...$

The next number in the sequence is 2 times the preceding number.

The next number in the sequence is 8.

Example 2
Algebraic Conjectures

$\frac{2}{\uparrow}$ *exponent*

2^2

4^{2a}

$4^3 = 4 \times 4 \times 4$
$= 64$
(NOT $4 \times 13 = 12$) *add*

9×9
81

$9^2 = 18$
9×2 NO

2 2

$2 + 3$

$4 + 9 = 13$

NO
$3 \times 2 = 6$

Make a conjecture about the sum of the squares of two consecutive natural numbers. List or draw some examples

that support your conjecture.

always ← be an odd # as answer (handwritten)

$$10^2 + 11^2$$
$$100 + 121 = 221$$

$$6^2 + 7^2$$
$$36 + 49 = 85$$

Example 2
Algebraic Conjectures

Step 1   List examples.

$1^2 + 2^2 = 5$ $6^2 + 7^2 = 85$
$2^2 + 3^2 = 13$ $10^2 + 11^2 = 221$

Step 2   Look for a pattern.
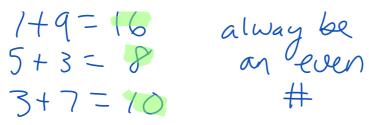Notice that all the sums are odd numbers.

Step 3   Make a conjecture.
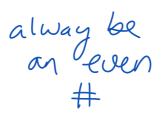The sum of the squares of two consecutive
natural numbers is an odd number.

Example 2
Algebraic Conjectures

Check

add (handwritten)

Make a conjecture about the sum of two odd numbers.

$1 + 9 = 16$

$5 + 3 = 8$

$3 + 7 = 10$

alway be an even #

Example 2
Algebraic Conjectures

Check

Make a conjecture about the sum of two odd numbers.

The sum of two odd numbers is always an even number.

Example 3
Geometric Conjectures

2 sides

Make a conjecture about the relationship between the segments joining opposite vertices of isosceles trapezoids.

→ corner

vertex angle

diagonals always congruent (equal)

Example 3
Geometric Conjectures

Step 1   Draw several examples.

An isosceles trapezoid is a trapezoid with two opposite congruent legs.

Example 3
Geometric Conjectures

Step 2   Look for a pattern.

Notice that the segments joining opposite vertices of each isosceles trapezoid appear to have the same measure. Use a ruler

or compass to confirm this.

Step 3   Make a conjecture.

The segments joining opposite vertices of an isosceles trapezoid are congruent.

Example 3
Geometric Conjectures

Check

Make a conjecture about the relationship between AD and AB, if C is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{AC}$.

A   D   C   B

AD is $\frac{1}{4}$ of AB

Example 3
Geometric Conjectures

Check

Make a conjecture about the relationship between AD and AB, if C is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{AC}$.

AD is one fourth of *AB*.

Example 4
Make Conjectures from Data

GAS PRICES The table shows the average price of gasoline in the United States for the years 2010 through 2018. Make a conjecture about the price of gas in 2019. Explain how this conjecture is supported by the data given.

Year	Price (dollars per gallon)
2010	2.84
2011	3.58
2012	3.68
2013	3.58
2014	3.44
2015	2.43
2016	2.14
2017	2.42
2018	2.84

(handwritten annotations: "guess", "increasing", "decreasing", "increase", "2019")

Example 4
Make Conjectures from Data

Look for patterns in the data.

The price of gasoline increased from 2010 to 2012. From 2012 to 2016

Year	Price (dollars per gallon)
2010	2.84
2011	3.58

the price of gas decreased, at first at a steady rate, and then more dramatically. Beginning in 2017, the price of gas began to increase at a steady rate.

2012	3.68
2013	3.58
2014	3.44
2015	2.43
2016	2.14
2017	2.42
2018	2.84

Example 4
Make Conjectures from Data

The data shows that the price of gas follows an oscillating pattern, increasing in price for several years before decreasing in price for several years.

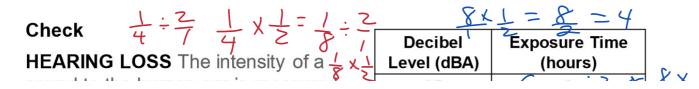Conjecture: In 2019, the price of gas will continue to increase.

Year	Price (dollars per gallon)
2010	2.84
2011	3.58
2012	3.68
2013	3.58
2014	3.44
2015	2.43
2016	2.14
2017	2.42
2018	2.84

Example 4
Make Conjectures from Data

Check

$\frac{1}{4} \div \frac{2}{7}$ $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \div \frac{2}{1}$ $\frac{8 \times 1 = \frac{8}{2} = 4}{}$

HEARING LOSS The intensity of a $\frac{1}{8} \times \frac{1}{2}$

Decibel Level (dBA)	Exposure Time (hours)

sound to the human ear is measured in A-weighted decibels, or dBA. For every 3 decibels over 85 decibels, the exposure time it takes to cause hearing damage is cut in half. How long does it take to cause hearing damage at 106 decibels? Write your answer as a decimal.

Handwritten annotations:
$\frac{1}{16} \times \frac{60}{1}$
$\left(\frac{60}{16}\right)$
3.75 H.

85	8
88	4
91	2
94	1
97	$\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
100	$\frac{1}{4} \times \frac{1}{2}$

Example 4
Make Conjectures from Data

Check

HEARING LOSS The intensity of a sound to the human ear is measured in A-weighted decibels, or dBA. For every 3 decibels over 85 decibels, the exposure time it takes to cause hearing damage is cut in half. How long does it take to cause hearing damage at 106 decibels? Write your answer as a decimal. **3.75 minutes**

Decibel Level (dBA)	Exposure Time (hours)
85	8
88	4
91	2
94	1
97	$\frac{1}{2}$
100	$\frac{1}{4}$

Learn
Counterexamples

To show that a conjecture is true for all cases, you must prove it. It only takes one example that contradicts the