

Lesson 3.1: Conjectures and Counterexamples

Monday, October 17, 2022 5:28 PM

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Lesson 3.1
Conjectures



Conjectures and Counterexamples



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Florida's B.E.S.T. Standards for Mathematics

MA.912.LT.4.10

Judge the validity of arguments and give counterexamples to disprove statements.

MA.K12.MTR.6.1



Assess the reasonableness of solutions.

MA.K12.MTR.7.1

Apply mathematics to real-world contexts.

Content Objective

Students analyze conjectures by using inductive reasoning and disprove conjectures by using counterexamples.

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Learn

Inductive Reasoning and Conjecture

Inductive reasoning is the process of reaching a conclusion based on a pattern of examples. When you assume that an observed pattern will continue, you are applying inductive reasoning. You can use inductive reasoning to make an educated guess based on known information and specific examples. This educated guess is also known as a **conjecture**.

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Example 1

Patterns and Conjectures

guess

inductive reasoning

Write a **conjecture** that describes the pattern in the sequence. Then use your conjecture to find the next term in the sequence.

term in the sequence.

Appointment times: 8:30 A.M., 9:15 A.M., 10:00 A.M., 10:45 A.M., ...

add
45 min each
time

$$\begin{array}{r} 11:00 \\ + 75 \text{ min} \\ \hline 11:30 \end{array}$$



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Example 1

Patterns and Conjectures

Step 1 Look for a pattern.

8:30 A.M. 9:15 A.M. 10:00 A.M. 10:45 A.M.

+45 min +45 min +45 min

Step 2 Make a conjecture.

Each appointment time is 45 minutes after the previous appointment time. The next appointment time will be 10:45 A.M. + 0:45 or 11:30 A.M.

Example 1

Patterns and Conjectures

$$\frac{1}{2} + \frac{1}{2} = 1 \quad \frac{1}{2} \cdot \frac{2}{1} = \frac{2}{2} = 1$$

Check

Write a conjecture that describes the pattern in the sequence. Then use your conjecture to find the next term in the sequence.

$$\frac{1}{2}, 1, 2, 4, \dots \times 2 = 8$$

multiply by 2
or double



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Example 1

Patterns and Conjectures

Check

Write a conjecture that describes the pattern in the sequence. Then use your conjecture to find the next term in the sequence.

$$\frac{1}{2}, 1, 2, 4, \dots$$

The next number in the sequence is 2 times the preceding number.

The next number in the sequence is 8.

Example 2

Algebraic Conjectures

2 exponent
2 → 4²
4² → 16
16 → 25
25 → 36
36 → 49
49 → 64
64 → 81
81 → 100
100 → 121
121 → 144
144 → 169
169 → 196
196 → 225
225 → 256
256 → 289
289 → 324
324 → 361
361 → 400
400 → 441
441 → 484
484 → 529
529 → 576
576 → 625
625 → 676
676 → 729
729 → 784
784 → 841
841 → 900
900 → 961
961 → 1024
1024 → 1089
1089 → 1156
1156 → 1225
1225 → 1296
1296 → 1369
1369 → 1444
1444 → 1521
1521 → 1600
1600 → 1681
1681 → 1764
1764 → 1849
1849 → 1936
1936 → 2025
2025 → 2116
2116 → 2209
2209 → 2304
2304 → 2401
2401 → 2500
2500 → 2601
2601 → 2704
2704 → 2809
2809 → 2916
2916 → 3025
3025 → 3136
3136 → 3249
3249 → 3364
3364 → 3481
3481 → 3600
3600 → 3721
3721 → 3844
3844 → 3969
3969 → 4096
4096 → 4225
4225 → 4356
4356 → 4489
4489 → 4624
4624 → 4761
4761 → 4900
4900 → 5041
5041 → 5184
5184 → 5329
5329 → 5476
5476 → 5625
5625 → 5776
5776 → 5929
5929 → 6084
6084 → 6241
6241 → 6400
6400 → 6561
6561 → 6724
6724 → 6889
6889 → 7056
7056 → 7225
7225 → 7396
7396 → 7569
7569 → 7744
7744 → 7921
7921 → 8100
8100 → 8281
8281 → 8464
8464 → 8649
8649 → 8836
8836 → 9025
9025 → 9216
9216 → 9409
9409 → 9604
9604 → 9801
9801 → 10000
10000 → 10201
10201 → 10404
10404 → 10609
10609 → 10816
10816 → 11025
11025 → 11236
11236 → 11449
11449 → 11664
11664 → 11881
11881 → 12096
12096 → 12313
12313 → 12531
12531 → 12751
12751 → 12972
12972 → 13195
13195 → 13419
13419 → 13644
13644 → 13871
13871 → 14099
14099 → 14329
14329 → 14560
14560 → 14792
14792 → 15025
15025 → 15259
15259 → 15494
15494 → 15731
15731 → 15969
15969 → 16209
16209 → 16450
16450 → 16692
16692 → 16935
16935 → 17179
17179 → 17424
17424 → 17671
17671 → 17919
17919 → 18168
18168 → 18419
18419 → 18671
18671 → 18924
18924 → 19178
19178 → 19433
19433 → 19689
19689 → 19946
19946 → 20204
20204 → 20463
20463 → 20723
20723 → 20984
20984 → 21246
21246 → 21509
21509 → 21773
21773 → 22038
22038 → 22304
22304 → 22571
22571 → 22839
22839 → 23108
23108 → 23378
23378 → 23649
23649 → 23921
23921 → 24194
24194 → 24468
24468 → 24743
24743 → 25019
25019 → 25296
25296 → 25574
25574 → 25853
25853 → 26133
26133 → 26414
26414 → 26696
26696 → 26979
26979 → 27263
27263 → 27548
27548 → 27834
27834 → 28121
28121 → 28409
28409 → 28698
28698 → 28988
28988 → 29279
29279 → 29571
29571 → 29864
29864 → 30158
30158 → 30453
30453 → 30749
30749 → 31046
31046 → 31344
31344 → 31643
31643 → 31943
31943 → 32244
32244 → 32546
32546 → 32849
32849 → 33153
33153 → 33458
33458 → 33764
33764 → 34071
34071 → 34379
34379 → 34688
34688 → 34998
34998 → 35309
35309 → 35621
35621 → 35934
35934 → 36248
36248 → 36563
36563 → 36879
36879 → 37196
37196 → 37514
37514 → 37833
37833 → 38153
38153 → 38474
38474 → 38796
38796 → 39119
39119 → 39443
39443 → 39768
39768 → 40094
40094 → 40421
40421 → 40749
40749 → 41078
41078 → 41408
41408 → 41739
41739 → 42071
42071 → 42404
42404 → 42738
42738 → 43073
43073 → 43409
43409 → 43746
43746 → 44084
44084 → 44423
44423 → 44763
44763 → 45104
45104 → 45446
45446 → 45789
45789 → 46133
46133 → 46478
46478 → 46824
46824 → 47171
47171 → 47519
47519 → 47868
47868 → 48218
48218 → 48569
48569 → 48921
48921 → 49274
49274 → 49628
49628 → 49983
50000

Make a conjecture about the sum of the squares of two consecutive natural numbers. List or draw some examples

$$2^2 + 3^2 = 4 + 9 = 13$$

2 2 → 3 × 2 = 6

that support your conjecture.

always
be an odd #
as answer

$$10^2 + 11^2 = 221$$

$$6^2 + 7^2 = 85$$



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Example 2

Algebraic Conjectures

Step 1 List examples.

$$1^2 + 2^2 = 5$$

$$6^2 + 7^2 = 85$$

$$2^2 + 3^2 = 13$$

$$10^2 + 11^2 = 221$$

Step 2 Look for a pattern.

Notice that all the sums are odd numbers.

Step 3 Make a conjecture.

The sum of the squares of two consecutive natural numbers is an odd number.

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Example 2

Algebraic Conjectures

Check

+
add

Make a conjecture about the sum of two odd numbers.

$$1 + 9 = 10$$

$$5 + 3 = 8$$

$$3 + 7 = 10$$

always be
an even
#



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Example 2

Algebraic Conjectures

Check

Make a conjecture about the sum of two odd numbers.

The sum of two odd numbers is always an even number.

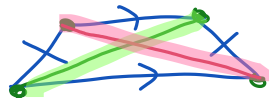
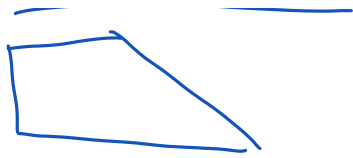
Example 3

Geometric Conjectures

2 sides
↖ ↗

Make a conjecture about the relationship between the segments joining opposite vertices of isosceles trapezoids.

→ cornes



vertex angle

diagonals
always congruent (equal)



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Example 3

Geometric Conjectures

Step 1 Draw several examples.

An isosceles trapezoid is a trapezoid with two opposite congruent legs.



Example 3

Geometric Conjectures

Step 2 Look for a pattern.

Notice that the segments joining opposite vertices of each isosceles trapezoid appear to have the same measure. Use a ruler

isosceles trapezoid appear to have the same measure. Use a ruler or compass to confirm this.

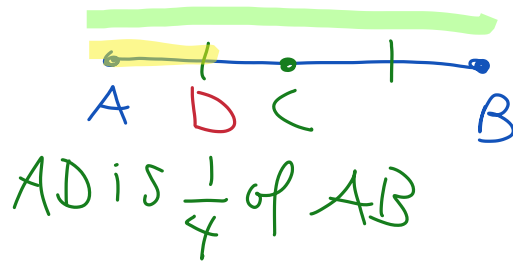
Step 3 Make a conjecture.

The segments joining opposite vertices of an isosceles trapezoid are congruent.

Example 3 Geometric Conjectures

Check

Make a conjecture about the relationship between AD and AB , if C is the midpoint of AB and D is the midpoint of AC .



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Example 3 Geometric Conjectures

Check

Make a conjecture about the relationship between AD and AB , if C is the midpoint of AB and D is the midpoint of AC .

is the midpoint of AB and D is the midpoint of AC .

AD is one fourth of AB .

Example 4

Make Conjectures from Data

GAS PRICES The table shows the average price of gasoline in the United States for the years 2010 through 2018. Make a conjecture about the price of gas in 2019. Explain how this conjecture is supported by the data given.

guess

Year	Price (dollars per gallon)
2010	2.84
2011	3.58
2012	3.68
2013	3.58
2014	3.44
2015	2.43
2016	2.14
2017	2.42
2018	2.84

increasing
decreasing
increase



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Example 4

Make Conjectures from Data

Look for patterns in the data.

The price of gasoline increased from 2010 to 2012. From 2012 to 2016

Year	Price (dollars per gallon)
2010	2.84
2011	3.58

2010 to 2012. From 2012 to 2010, the price of gas decreased, at first at a steady rate, and then more dramatically. Beginning in 2017, the price of gas began to increase at a steady rate.

2012	3.68
2013	3.58
2014	3.44
2015	2.43
2016	2.14
2017	2.42
2018	2.84

Example 4

Make Conjectures from Data

The data shows that the price of gas follows an oscillating pattern, increasing in price for several years before decreasing in price for several years.

Conjecture: In 2019, the price of gas will continue to increase.

Year	Price (dollars per gallon)
2010	2.84
2011	3.58
2012	3.68
2013	3.58
2014	3.44
2015	2.43
2016	2.14
2017	2.42
2018	2.84

Example 4

Make Conjectures from Data

Check $\frac{1}{4} \div \frac{2}{7} = \frac{1}{4} \times \frac{7}{2} = \frac{7}{8}$
HEARING LOSS The intensity of a $\frac{1}{8} \times \frac{1}{2}$

Decibel Level (dBA)	Exposure Time (hours)
----------------------------	------------------------------

$$8 \times \frac{1}{2} = \frac{8}{2} = 4$$

8 x 1

sound to the human ear is measured in A-weighted decibels, or dBA. For every 3 decibels over 85 decibels, the exposure time it takes to cause hearing damage is cut in half. How long does it take to cause hearing damage at 106 decibels? Write your answer as a decimal. 3.75 H.

85	8
88	4
91	2
94	1
97	$\frac{1}{2}$
100	$\frac{1}{4}$

$\frac{1}{16} \times \frac{60}{1} = \frac{60}{16}$

$\frac{60}{16}$

$85 \rightarrow 88 \rightarrow 91 \rightarrow 94 \rightarrow 97 \rightarrow 100$ (+3 each)

$8 \div 2 = 4$

$4 \div 2 = 2$

$2 \div 2 = 1$

$1 \div 2 = \frac{1}{2}$

$\frac{1}{2} \div 2 = \frac{1}{4}$

$\frac{1}{4} \div 2 = \frac{1}{8}$

$\frac{1}{8} \div 2 = \frac{1}{16}$



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Example 4

Make Conjectures from Data

Check

HEARING LOSS The intensity of a sound to the human ear is measured in A-weighted decibels, or dBA. For every 3 decibels over 85 decibels, the exposure time it takes to cause hearing damage is cut in half. How long does it take to cause hearing damage at 106 decibels? Write your answer as a decimal. **3.75 minutes**

Decibel Level (dBA)	Exposure Time (hours)
85	8
88	4
91	2
94	1
97	$\frac{1}{2}$
100	$\frac{1}{4}$

Learn

Counterexamples

To show that a conjecture is true for all cases, you must prove it. It only takes one example that contradicts the conjecture, however, to show that a conjecture is not