

Lesson 2.2: Angle Relationships

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2-2 Angle
Relations

Lesson 2.2

Angle Relationships



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Lesson Objectives



Content Objective

Students use the properties of perpendicular lines to find the measures of angles.

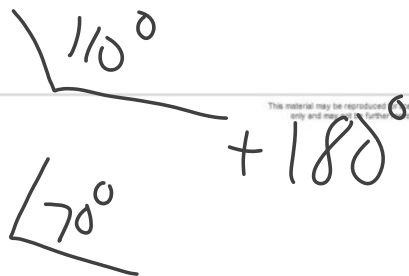
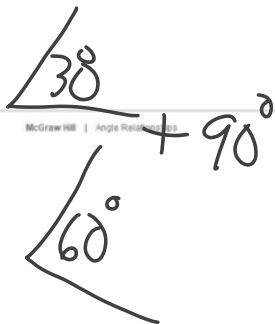
Learn

Complementary and Supplementary Angles

Complementary and Supplementary Angles	
Complementary Angles	Supplementary Angles
Definition	
two angles with measures that have a sum of 90°	two angles with measures that have a sum of 180°

90°
C

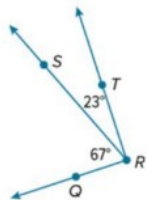
180°
S



Learn

Complementary and Supplementary Angles

Examples of Complementary Angles



$$\begin{aligned} m\angle QRS + m\angle SRT &= 90^\circ \\ 67^\circ + 23^\circ &= 90^\circ \end{aligned}$$

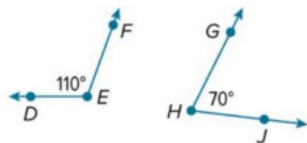


$$\begin{aligned} m\angle JKL + m\angle ABC &= 90^\circ \\ 30^\circ + 60^\circ &= 90^\circ \end{aligned}$$

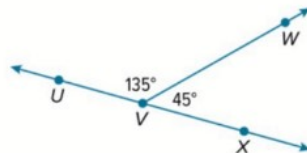
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Complementary and Supplementary Angles

Examples of Supplementary Angles



$$\begin{aligned} m\angle DEF + m\angle GHJ &= 180^\circ \\ 110^\circ + 70^\circ &= 180^\circ \end{aligned}$$



$$\begin{aligned} m\angle UVW + m\angle WVX &= 180^\circ \\ 135^\circ + 45^\circ &= 180^\circ \end{aligned}$$

Example 1

Complementary and Supplementary Angles

Find the measures of two complementary angles if the measure of the larger angle is five more than four times the measure of the smaller angle.

$$4x$$

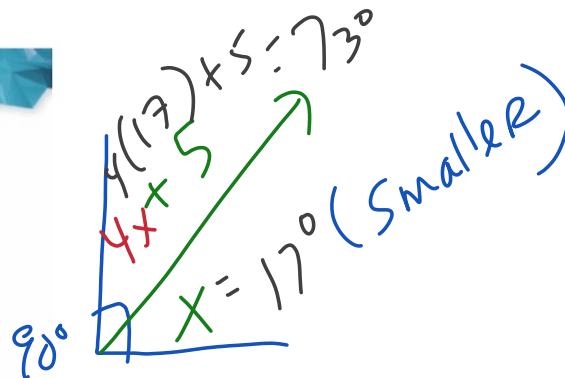
$$90^\circ$$

$$x$$

$$4x + 5 + x = 90^\circ$$

$$5x + 5 = 90$$

$$\frac{5x}{5} = \frac{85}{5}$$



$$x = 17$$

Example 1

Complementary and Supplementary Angles



If two angles are complementary, then the sum of the angle measures is 90° . To find the measures of each angle, first write an equation. Let x° be the measure of the smaller angle. Then the measure of the larger angle is $(4x + 5)^\circ$.



Example 1

Complementary and Supplementary Angles

Step 1 Solve for x .

$$x^\circ + (4x + 5)^\circ = 90^\circ \quad \text{Complementary angle measures add to } 90^\circ.$$

$$5x^\circ + 5^\circ = 90^\circ \quad \text{Combine like terms.}$$

$$5x^\circ = 85^\circ \quad \text{Subtract } 5^\circ \text{ from each side.}$$

$$x = 17 \quad \text{Divide each side by } 5^\circ.$$

So, the measure of the smaller angle is 17° .



Example 1

Complementary and Supplementary Angles

Step 2 Find the measure of the larger angle.

Use substitution to find the measure of the larger angle.

$$\begin{aligned}(4x + 5)^\circ &= 4(17)^\circ + 5^\circ && \text{Substitute 17 for } x. \\ &= 68^\circ + 5^\circ && \text{Multiply.} \\ &= 73^\circ && \text{Solve.}\end{aligned}$$

The measures of the angles are 17° and 73° .

Example 1

Complementary and Supplementary Angles

CHECK

Does your answer seem reasonable?

Yes; $17^\circ + 73^\circ = 90^\circ$, so the two angles are complementary.

Example 1

Complementary and Supplementary Angles

Check —

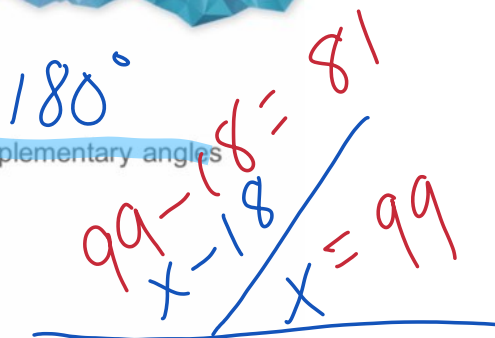
The difference between the measures of two supplementary angles is 18° . Find the measure of each angle.

$$x - 18 + x = 180$$

$$\begin{array}{r} 2x - 18 = 180 \\ + 18 \quad + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 2x = 198 \\ \hline 2 \quad 2 \end{array}$$

$$x = 99$$



$$= 180$$



Example 1

Complementary and Supplementary Angles

Check

The difference between the measures of two supplementary angles is 18° . Find the measure of each angle.

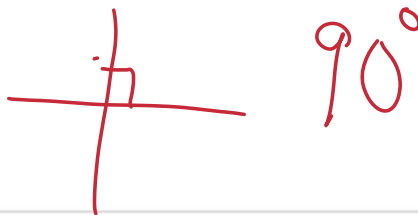
The measure of the smaller angle is 81° , and the measure of the larger angle is 99° .

Learn

Perpendicularity



Lines, segments, or rays that intersect at right angles are **perpendicular**. Segments or rays can be perpendicular to lines or other line segments and rays. The right angle symbol indicates that the lines are perpendicular.

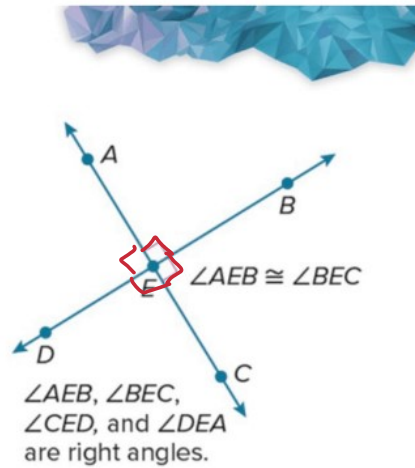


Learn

Perpendicularity

Perpendicular lines intersect to form four right angles.

Perpendicular lines intersect to form congruent adjacent angles.

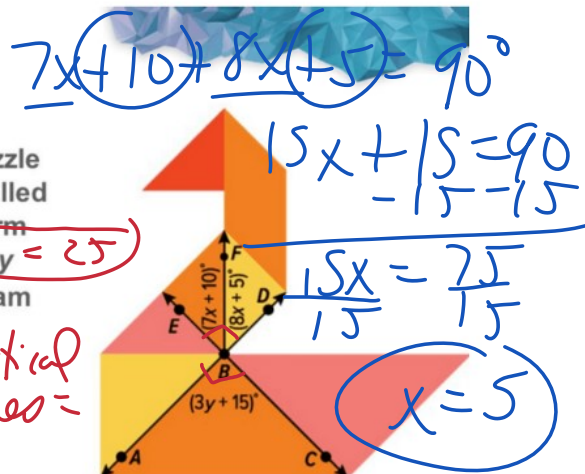


Example 2
Perpendicular Lines

TANGRAMS The tangram is a puzzle consisting of eight flat shapes called *tans* which are put together to form images. Find the values of x and y such that \overline{AD} and \overline{EC} in the tangram are perpendicular.

90°

Vertical angles =



$$\begin{array}{r} 3y + 15 = 90 \\ -15 \quad -15 \\ \hline 3y = 75 \\ \div 3 \quad \div 3 \\ \hline y = 25 \end{array}$$

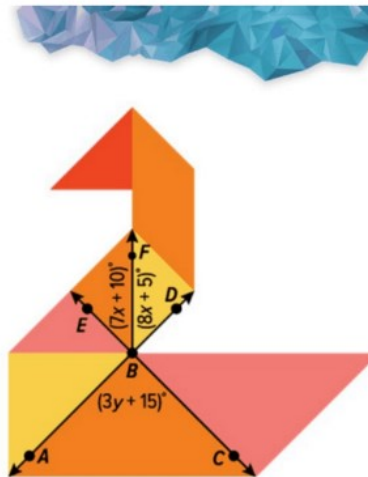
Example 2

Perpendicular Lines

If \overleftrightarrow{AD} and \overleftrightarrow{EC} are perpendicular, then
 $m\angle ABC = 90^\circ$ and $m\angle EBD = 90^\circ$.

Step 1 Solve for y .

$$\begin{aligned}(3y + 15)^\circ &= 90^\circ & m\angle ABC &= 90^\circ \\ y &= 25 & \text{Solve for } y.\end{aligned}$$



Example 2

Perpendicular Lines



Step 2 Solve for x .

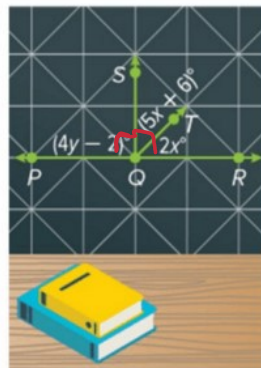
$$\begin{aligned} m\angle EBF + m\angle FBD &= m\angle EBD && \text{sum of parts} = \text{whole} \\ (7x + 10)^\circ + (8x + 5)^\circ &= 90^\circ && \text{Substitution} \\ x &= 5 && \text{Solve for } x. \end{aligned}$$

Example 2

Perpendicular Lines

Check

DESIGN Find the values of x and y such that \overrightarrow{PR} and \overrightarrow{QS} are perpendicular.



$$\begin{aligned}
 &5x + 6 + 2x = 90 \\
 &7x + 6 = 90 \\
 &\quad -6 \quad -6 \\
 \hline
 &7x = 84
 \end{aligned}$$

$$\frac{7x}{7} = \frac{84}{7} \quad x = 12$$

$$\begin{aligned}
 &4y - 2 = 90 \\
 &\quad +2 \quad +2 \\
 \hline
 &4y = 92 \\
 &\quad \div 4 \quad \div 4 \\
 &y = 23
 \end{aligned}$$

Example 2

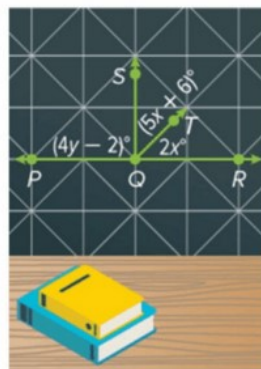
Perpendicular Lines

Check

DESIGN Find the values of x and y such that \overrightarrow{PR} and \overrightarrow{QS} are perpendicular.

$$x = 12$$

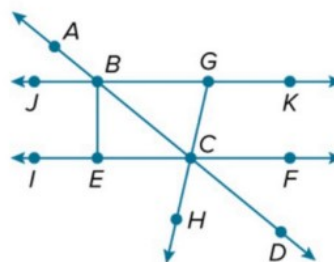
$$y = 23$$



Example 3

Interpreting Diagrams

Determine whether each statement can be assumed from the figure. Explain.



Example 3

Interpreting Diagrams

a. \overrightarrow{CE} and \overrightarrow{CF} are opposite rays. *line*

b. $\angle BGC$ and $\angle KGC$ form a linear pair.

c. $\angle ABJ$ and $\angle CBG$ are vertical angles.

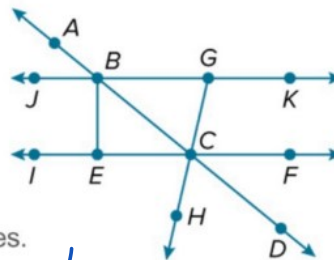
d. $\angle BCG$ and $\angle DCF$ are congruent. *equal*

e. \overline{BE} and \overline{IF} are perpendicular. *90°*

f. $\angle EBC$ and $\angle GBC$ are complementary angles.

g. $\angle ICH$ and $\angle HCD$ are adjacent angles. *next to*

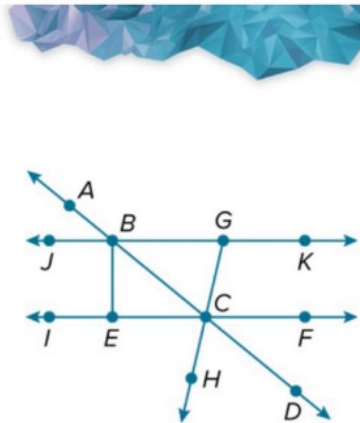
h. \overline{BC} is an angle bisector of $\angle ECG$.



Example 3

Interpreting Diagrams

- a. \overrightarrow{CE} and \overrightarrow{CF} are opposite rays.
Yes; C is a common endpoint.
- b. $\angle BGC$ and $\angle KGC$ form a linear pair.
Yes; their noncommon sides are opposite rays.
- c. $\angle ABJ$ and $\angle CBG$ are vertical angles.
Yes; these angles are nonadjacent and are formed by two intersecting lines.



Example 3

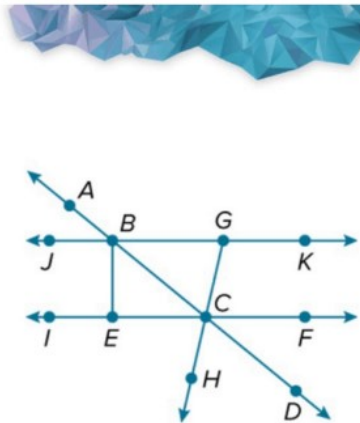
Interpreting Diagrams

- d. $\angle BCG$ and $\angle DCF$ are congruent.

No; these angles are not vertical angles. There isn't enough information given to determine this.

- e. \overline{BE} and \overleftrightarrow{IF} are perpendicular.

No; there isn't enough information given to determine this.



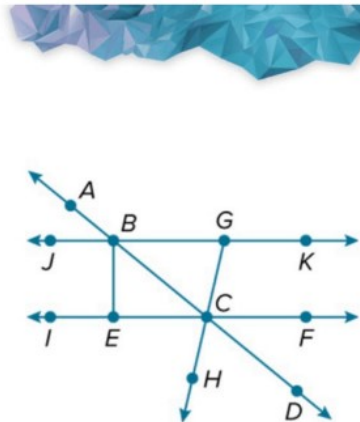
Example 3

Interpreting Diagrams

- f. $\angle EBC$ and $\angle GBC$ are complementary angles.

No; there isn't any information about perpendicularity or angle measure so this cannot be determined.

- g. $\angle ICH$ and $\angle HCD$ are adjacent angles.
Yes; these angles share a common side.

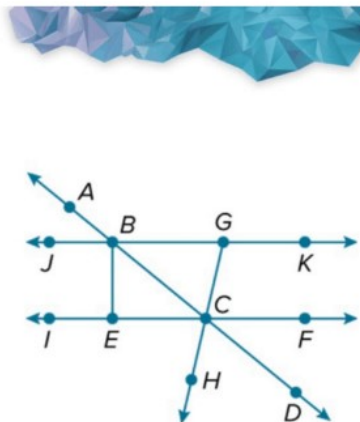


Example 3

Interpreting Diagrams

h. \overline{BC} is an angle bisector of $\angle ECG$.

No; there isn't any information about congruent angles so this cannot be determined.



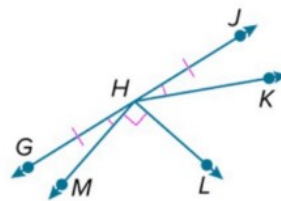
Example 3

Interpreting Diagrams

Check

Which statement(s) cannot be assumed from the figure?

- A. $\angle KHJ$ and $\angle GHM$ are complementary.
- B. $\angle GHK$ and $\angle JHK$ are a linear pair.
- C. \overrightarrow{HL} is perpendicular to \overrightarrow{HJ} .
- D. $\angle GHM$ and $\angle MHK$ are adjacent angles.
- E. \overrightarrow{HL} is perpendicular to \overrightarrow{HM} .



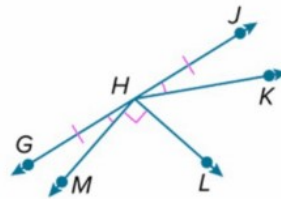
Example 3

Interpreting Diagrams

Check

Which statement(s) cannot be assumed from the figure?

- A. $\angle KHJ$ and $\angle GHM$ are complementary.
- B. $\angle GHK$ and $\angle JHK$ are a linear pair.
- C. \overrightarrow{HL} is perpendicular to \overrightarrow{HJ} .
- D. $\angle GHM$ and $\angle MHK$ are adjacent angles.
- E. \overrightarrow{HL} is perpendicular to \overrightarrow{HM} .

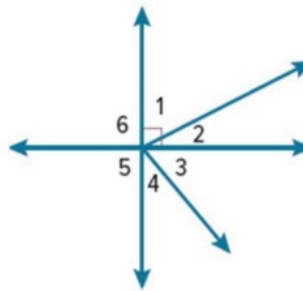


A, C

Exit Ticket

Use the figure.

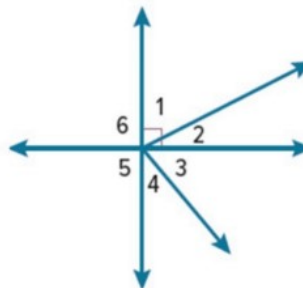
1. $\angle 1$ and what other angle are complementary angles?
2. $\angle 5$ and what other angle are supplementary angles?
3. Are $\angle 1$ and $\angle 2$ a linear pair of angles?



Exit Ticket

Use the figure.

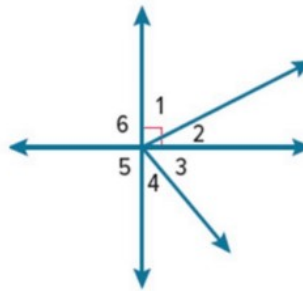
4. How are the vertical and horizontal lines in the figure related?
5. If $m\angle 2 = 27^\circ$ and $m\angle 3 = 59^\circ$, what is $m\angle 1$, $m\angle 4$, and $m\angle 5$?



Exit Ticket

Use the figure.

1. $\angle 1$ and what other angle are complementary angles? $\angle 2$
2. $\angle 5$ and what other angle are supplementary angles? $\angle 6$
3. Are $\angle 1$ and $\angle 2$ a linear pair of angles? no



Exit Ticket

Use the figure.

4. How are the vertical and horizontal lines in the figure related? **They are perpendicular.**
5. If $m\angle 2 = 27^\circ$ and $m\angle 3 = 59^\circ$, what is $m\angle 1$, $m\angle 4$, and $m\angle 5$?
 $m\angle 1 = 63^\circ$, $m\angle 4 = 31^\circ$, $m\angle 5 = 90^\circ$

