

Lesson 1-4: Midpoints and Bisectors

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1.4

Midpoint



Midpoints and Bisectors



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Florida's B.E.S.T. Standards for Mathematics

MA.912.GR.3.3

Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.

MA.912.GR.5.2

Construct the bisector of a segment or an angle, including the perpendicular bisector of a line segment.

Content Objective

Students will find midpoints and bisect line segments



Learn

Midpoints on a Number Line

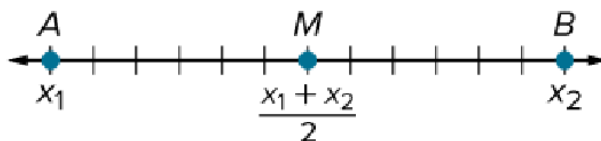
The **midpoint** of a segment is the point halfway between the endpoints of the segment. A point is **equidistant** from other points if it is the same distance from them. The midpoint separates the segment into two segments with a ratio of 1:1.

Learn

Midpoints on a Number Line

Key Concept: Midpoint on a Number Line

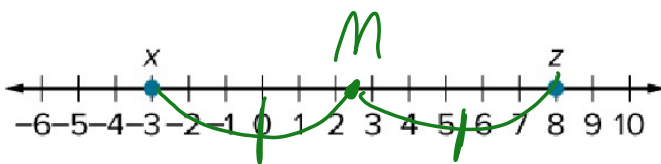
If \overline{AB} has endpoints at x_1 and x_2 on a number line, then the midpoint M of \overline{AB} has coordinate $M = \frac{x_1 + x_2}{2}$.



Example 1

Find the Midpoint on a Number Line

Find the coordinate of the midpoint of \overline{XZ} .



$$-\frac{3+8}{2} \quad \frac{5}{2} = 2.5$$



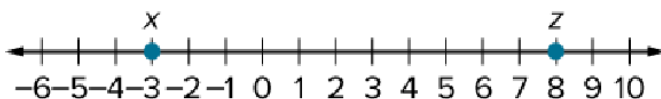
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Example 1

Find the Midpoint on a Number Line



$$\begin{aligned} M &= \frac{x_1 + x_2}{2} \\ &= \frac{8 + (-3)}{2} \\ &= \frac{5}{2} \text{ or } 2.5 \end{aligned}$$

Midpoint Formula

Substitution

Simplify.

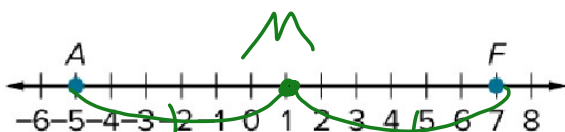
The coordinate of the midpoint of \overline{XZ} is 2.5.

Example 1

Find the Midpoint on a Number Line

Check

What is the coordinate of the midpoint of \overline{AF} ?



$$-\frac{5+7}{2} = \frac{2}{2} = 1$$



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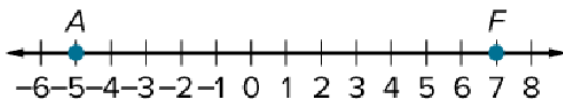


Example 1

Find the Midpoint on a Number Line

Check

What is the coordinate of the midpoint of \overline{AF} ?



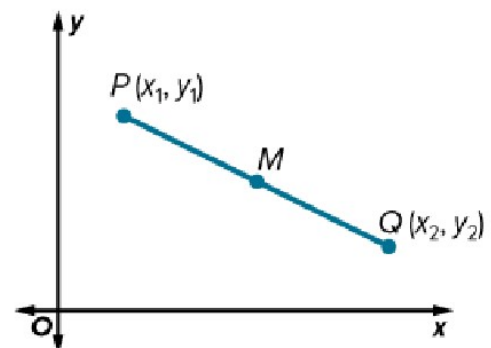
1

Learn

Midpoints on the Coordinate Plane

Key Concept: Midpoint Formula on the Coordinate Plane

If \overline{PQ} has endpoints at $P(x_1, y_1)$ and $Q(x_2, y_2)$ on the coordinate plane, then the midpoint M of \overline{PQ} has coordinates $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Example 3

Find the Midpoint on the Coordinate Plane

Find the coordinates of M , the midpoint of \overline{AB} , for

$A(-2, 1)$ and $B(8, 3)$.

Handwritten student work showing the midpoint calculation for $A(-2, 1)$ and $B(8, 3)$.

Coordinates: $A(-2, 1)$ and $B(8, 3)$

Calculation for x-coordinate:

$$\frac{-2 + 8}{2} = \frac{6}{2} = 3$$

Calculation for y-coordinate:

$$\frac{1 + 3}{2} = \frac{4}{2} = 2$$

Midpoint: $(3, 2)$



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Example 3

Find the Midpoint on the Coordinate Plane

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 8}{2}, \frac{1 + 3}{2} \right) \\ &= \left(\frac{6}{2}, \frac{4}{2} \right) \text{ or } (3, 2) \end{aligned}$$

Midpoint Formula

Substitution

Simplify.

Example 3

Find the Midpoint on the Coordinate Plane

Check

Find the coordinates of B , the midpoint of \overline{AC} , for $A(-3, -2)$ and $C(5, 10)$.

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$\frac{-3 + 5}{2}, \frac{-2 + 10}{2}$$

$$\frac{-2 + 10}{2}$$

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$(1, 4)$$



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Example 3

Find the Midpoint on the Coordinate Plane

Check

Find the coordinates of B , the midpoint of \overline{AC} , for $A(-3, -2)$ and $C(5, 10)$.

$(1, 4)$

Example 4

Find Missing Coordinates

Find the coordinates of A if $P(3, \frac{1}{2})$ is the midpoint of \overline{AB} and B has coordinates $(8, 3)$.

$$\frac{x_1 + x_2}{2} = \frac{x + 8}{2} = 3$$

$$\frac{y_1 + y_2}{2} = \frac{y + 3}{2} = \frac{1}{2}$$

$$x + 8 = 6$$

$$x = -2$$

$$y + 3 = 1$$

$$y = -2$$

$$A(-2, -2)$$



$$\begin{array}{r} x + 8 = 6 \\ -8 \quad -8 \\ \hline x = -2 \end{array}$$



$$\begin{array}{r} y + 3 = 1 \\ -3 \quad -3 \\ \hline y = -2 \end{array}$$

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Example 4

Find Missing Coordinates

First, substitute the known information into the Midpoint Formula. Let A be (x_1, y_1) and B be (x_2, y_2) .

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Midpoint Formula

$$\left(3, \frac{1}{2} \right) = \left(\frac{x_1 + 8}{2}, \frac{y_1 + 3}{2} \right)$$

Substitution

Example 4

Find Missing Coordinates

Next, write two equations to solve for x_1 and y_1 .

$$3 = \frac{x_1 + 8}{2}$$

Equation for x_1

$$6 = x_1 + 8$$

Multiply each side by 2.

$$-2 = x_1$$

Solve.

$$\frac{1}{2} = \frac{y_1 + 3}{2}$$

Equation for y_1

$$1 = y_1 + 3$$

Multiply each side by 2.

$$-2 = y_1$$

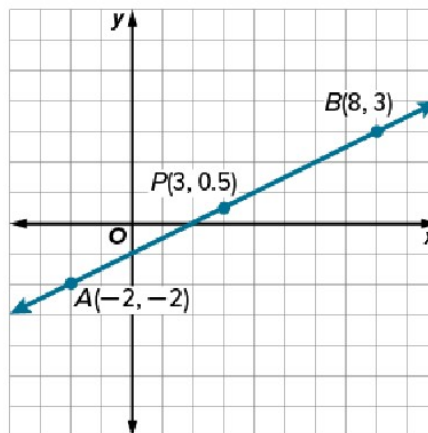
Solve.

Example 4

Find Missing Coordinates

The coordinates of A are $(-2, -2)$.

Plot the points on a coordinate plane to check your answer for reasonableness.



Example 4

Find Missing Coordinates

Check $Q(0, -6)$ $S(12, 4)$ $R(6, -1)$ \times midpoint

Find the coordinates of Q if $R(6, -1)$ is the midpoint of \overline{QS} and S has coordinates $(12, 4)$.

$x_1 + x_2 = 2 \cdot x_m$
 $0 + 12 = 2 \cdot 6$
 $12 = 12$
 $x = 0$

$y_1 + y_2 = 2 \cdot y_m$
 $-6 + 4 = 2 \cdot (-1)$
 $-2 = -2$
 $y = -6$

$Q(0, -6)$ $S(12, 4)$ $R(6, -1)$



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Example 4

Find Missing Coordinates

Check

Find the coordinates of Q if $R(6, -1)$ is the midpoint of \overline{QS} and S has coordinates $(12, 4)$.

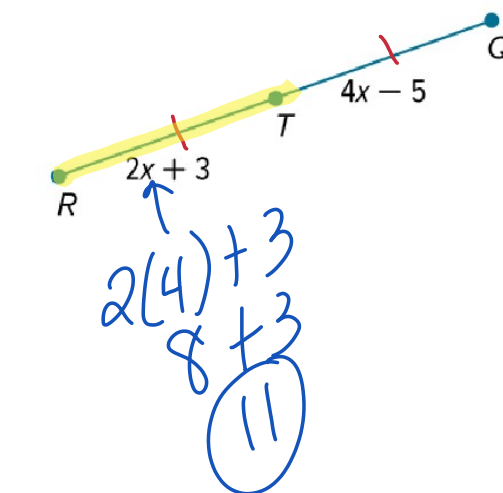
(0, -6)

Example 5

Find Missing Measures

Find the measure of \overline{RT} if T is the midpoint of \overline{RQ} .

$$\begin{array}{r} 2x + 3 = 4x - 5 \\ -2x \quad -2x \\ \hline 3 = 2x - 5 \\ +5 \quad +5 \\ \hline 8 = 2x \\ \frac{8}{2} = \frac{2x}{2} \\ 4 = x \end{array} \quad (x=4)$$



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Example 5

Find Missing Measures

Because T is the midpoint, $RT = TQ$. Use this equation to solve for x .

$$\begin{aligned} RT &= TQ \\ 2x + 3 &= 4x - 5 \\ 3 &= 2x - 5 \\ 8 &= 2x \\ 4 &= x \end{aligned}$$

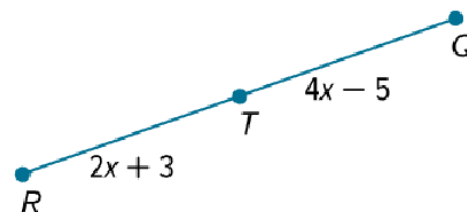
Definition of midpoint

Substitution

Subtract $2x$ from each side.

Add 5 to each side.

Divide each side by 2.



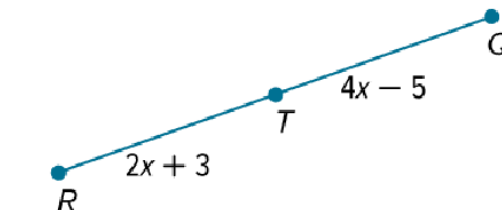
Example 5

Find Missing Measures

Substitute 4 for x in the equation for RT .

$$\begin{aligned} RT &= 2x + 3 \\ &= 2(4) + 3 \\ &= 11 \end{aligned}$$

Equation for RT
Substitution
Simplify.



Example 5

Find Missing Measures

Check

Find the measure of \overline{RS} if S is the midpoint of \overline{RT} .



- A. 56
- B. 58
- C. 112
- D. 116

$$\begin{aligned} 7x - 5 &= 6x + 4 \\ -6x &\quad -6x \\ \hline x - 5 &= 4 \\ +5 &\quad +5 \\ \hline x &= 9 \end{aligned}$$

Handwritten calculations show the substitution of $x = 9$ into the expressions for RS and ST to find the final measure.



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Example 5

Find Missing Measures

Check

Find the measure of \overline{RS} if S is the midpoint of \overline{RT} .



- A. 56
- B. 58
- C. 112
- D. 116



Students, select an option!

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Example 5

Find Missing Measures

Check

Find the measure of \overline{RS} if S is the midpoint of \overline{RT} .



- A. 56
- B. 58
- C. 112
- D. 116

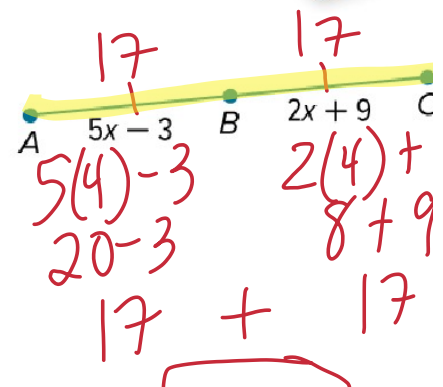
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Example 6

Find the Total Length

Find the measure of \overline{AC} if B is the midpoint of \overline{AC} .

$$\begin{array}{r} 5x - 3 = 2x + 9 \\ -2x \quad -2x \\ \hline 3x - 3 = 9 \end{array}$$



$$\begin{array}{r|l} +3 & +3 \\ \hline 3x = 12 & \\ \hline x = 4 & \end{array}$$

$$34$$



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Example 6

Find the Total Length

Because B is the midpoint, $AB = BC$.
Use this equation to solve for x .



$$AB = BC$$

$$5x - 3 = 2x + 9$$

$$3x - 3 = 9$$

$$3x = 12$$

$$x = 4$$

Definition of midpoint

Substitution

Subtract $2x$ from each side.

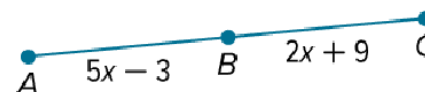
Add 3 to each side.

Divide each side by 3.

Example 6

Find the Total Length

The length of \overline{AC} is equal to the sum of \overline{AB} and \overline{BC} . So, to find the length of \overline{AC} , substitute 4 for x in the expression $5x - 3 + 2x + 9$.



$$AC = 5x - 3 + 2x + 9$$

$$= 5(4) - 3 + 2(4) + 9$$

$$= 20 - 3 + 8 + 9$$

$$= 34$$

Length of \overline{AC}

$$x = 4$$

Multiply.

Simplify.

The measure of \overline{AC} is 34.