

Chapter One

Identify points, lines, planes, segments, rays, collinear points, non-collinear points, coplanar points, angles, and opposite rays

Use the Midpoint and Distance Formulas to find missing coordinate values

Know and apply the Segment Addition and the Angle Addition Postulates

Classify angles as Right, Acute, or Obtuse

Identify and apply properties of adjacent angles, vertical angles, straight angles, right angles and linear pairs of angles

Understand and use complementary and supplementary angles

Understand and use concepts of midpoint, segment bisectors, and angle bisectors to solve problems (radius of a circle)

Know the undefined terms in Geometry

$$MP = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

- 101.) If E is between Y and S,  $YS = 2x + 6$ ,  $YE = 4$  and  $ES = x + 10$ , is E the midpoint of YS?

NO

$$2x + 6 = 4 + x + 10$$

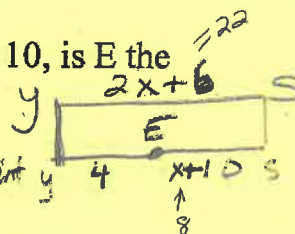
$$2x + 6 = x + 14$$

$$x = 8$$

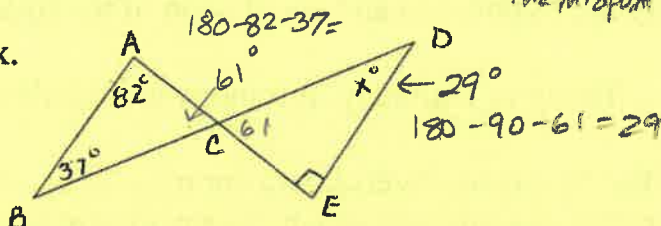
$$ES = 18$$

$$YE = 4$$

E is not the midpoint



- 12.649 2.) Find the value of x.



- 12.649 3.) Estimate the diameter of a circle that is centered at the origin and contains the point (2, 6).

$$r = \sqrt{(0-2)^2 + (0-6)^2}$$

$$= \sqrt{4 + 36}$$

$$= \sqrt{40} = 2\sqrt{10}$$

$$d = 2r$$

$$= 4\sqrt{10}$$

$$= 12.649$$

- 4.) The supplement of an angle is 3 more than twice its complement. Find the measure of the angle.

let  $x = \text{angle measurement}$

$$180 - x = 2(90 - x) + 3$$

$$180 - x = 180 - 2x + 3$$

$$x = 3$$

- 12.649 5.) If  $\overline{RT} \perp \overline{RV}$  and  $m\angle TRV = 3x + 15$ . Find x.

$$3x + 15 = 90$$

$$3x = 75$$

$$x = 25$$



- 12.649 6.) A segment has one endpoint at (3, 6) and a midpoint at (0, -10). Find the y - coordinate of the other endpoint.

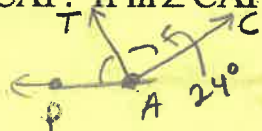
$$\frac{3+x}{2} = 0$$

$$\frac{6+y}{2} = -10$$

$$6+y = -20$$

$$y = -26$$

- 12.649 7.)  $\overrightarrow{AT}$  bisects  $\angle CAP$ . If  $m\angle CAP = 3x + 6$  and  $m\angle CAT = 24$ , find the value of x.



$$\angle CAP = 2(24)$$

$$= 48$$

$$3x + 6 = 48$$

$$3x = 42$$

$$x = 14$$

if  $p \rightarrow q$  statement  
 $q \rightarrow p$  converse  
 $\sim p \rightarrow \sim q$  inverse  
 $\sim q \rightarrow \sim p$  contrapositive

## Chapter Two

Use inductive reasoning and logic to make conclusions and conjectures  
 Provide and/or identify counterexamples  
 Write conditional statements and identify the hypothesis and conclusion  
 Identify the converse, inverse, and/or contrapositive of a conditional statement  
 Determine whether statements are logically equivalent  
 Use deductive reasoning (Law of Detachment and Law of Syllogism)  
 Write two-column proofs (algebra, segment, and angle)  
 Identify and use properties of algebra including but not limited to reflexive prop of =,  
 symmetric prop of =, and transitive prop of =  
 Know the difference between a postulate, definition, and theorem

1.) Write the statement "Vertical angles are congruent." as a conditional.

if  $2 \angle s$  are vertical angles then they are congruent

2.) Identify the hypothesis and conclusion of the following statement.

Conclusion: Today is Thursday, if tomorrow is Friday.  
 Hypothesis:

3.) Write the converse, inverse, and contrapositive of the following. Write their truth value and tell which statements are logically equivalent.

$p \rightarrow q$  Conditional: If  $x = 2$ , then  $x^2 = 4$ . True

$q \rightarrow p$  Converse: if  $x^2 = 4$ , then  $x = 2$  False,  $x$  can be  $\pm 2$

$\sim p \rightarrow \sim q$  Inverse: if  $x \neq 2$ , then  $x^2 \neq 4$  False

$\sim q \rightarrow \sim p$  Contrapositive: if  $x^2 \neq 4$ , then  $x \neq 2$  True

4.) Using laws of logic, what conclusion can you make from the following:

All fish can swim. Gertrude is a fish.

Gertrude can swim (Law of Detachment)

5.) Write a proof for the statement. If  $3(x + 10) = x + 6$ , then  $x = -12$ .

Statement	Reason
① $3(x + 10) = x + 6$	① Given
② $3x + 30 = x + 6$	② Distribution Property
③ $2x + 30 = 6$	③ Subtraction POE
④ $2x = -24$	④ "
⑤ $x = -12$	⑤ Division "

PEMDAS

## Chapter Three

Know and identify parallel, perpendicular, intersecting, and skew lines

Identify transversals and special pairs of angles

Alternate interior, alternate exterior, consecutive interior, consecutive exterior, and corresponding angles

Using parallel lines and transversals find the values of the special pairs of angles

Prove lines are parallel using special pairs of angles

Find slopes and equations of lines in both point-slope and slope-intercept form

Write equations of lines parallel and perpendicular

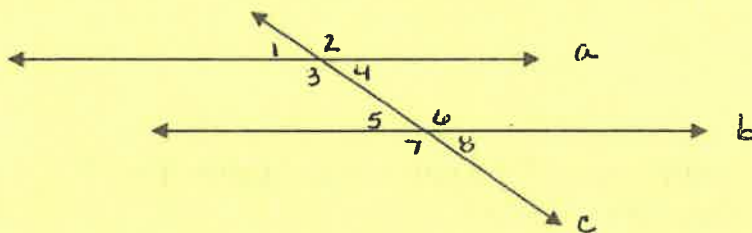
Write linear equations given real world information

Solve "Crook" problems

The midsegment of a triangle is a segment that connects the midpoints of two sides of a triangle, is parallel to the third side, and half the length of the third side

False 1.) **True or False:** Two non-coplanar lines that do not intersect are called parallel lines. *skew*

Use the diagram and answer the following questions.



2.) Name a pair of alternate interior angles.  $\angle 3, \angle 6$   $\angle 4, \angle 5$

3.) Name a pair of corresponding angles.  $\angle 1, \angle 5$   $\angle 2, \angle 6$   $\angle 3, \angle 7$   $\angle 4, \angle 8$

4.) Name a transversal. *line c*

5.) Write the equation of the line that passes through (5, 4) and (-3, -5).

$$m = \frac{-5-4}{-3-5} = \frac{-9}{-8} = \frac{9}{8} \quad Y+5 = \frac{9}{8}(X+3) \quad \text{or} \quad Y-4 = \frac{9}{8}(X-5)$$

$$Y = \frac{9}{8}X - \frac{13}{8}$$

6.) A plumber charges \$80 per visit and then charges \$60 an hour. Write the equation for the cost of this plumber's visit.  $C = 80 + 60h$

7.) Find the values of p, x, y, and z.

$$2x + 4 + 50 = 180$$

$$2x + 4 = 130$$

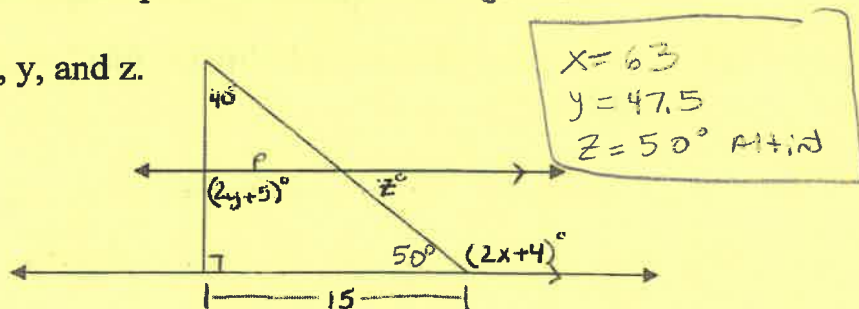
$$2x = 126$$

$$x = 63$$

$$2y + 5 = 90$$

$$2y = 85$$

$$y = 42.5$$



## Chapter Four

### Transformations:

- 1.) Reflection - flip
- 2.) Rotation - turn
- 3.) Translation - slide
- 4.) Dilation - enlarge/reduction

Name the transformation and then tell if it is an isometric transformation.

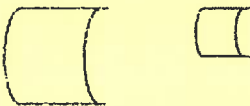
reflect 1.)



rotate 2.)



dilation  
'reduction' 3.)



4.) What are the coordinates of the point  $(-2, 3)$  after it is

- |  |                              |                  |
|--|------------------------------|------------------|
| $(2, 3)$ a.) reflected about the y-axis?         | $(x, y) \rightarrow (-x, y)$ | y-axis           |
| $(-2, -3)$ b.) reflected about the x-axis?       | $(x, y) \rightarrow (x, -y)$ | x-axis           |
| $(3, -2)$ c.) reflected about the line $y = x$ ? | $(x, y) \rightarrow (y, x)$  | switch $x$ & $y$ |

rule

5.) Rotate the capital letter F 90 degrees clockwise about the point given.



6.) Using the mapping:  $\langle x, y \rangle \rightarrow \langle x + 3, y - 2 \rangle$ , what are the coordinates of the preimage of  $(5, 4)$ ?

$\uparrow$   
 image (new)  $(2, 6)$  ← pre-image

EOC tick  
 $(8, 2)$

7.) Find the scale factor of the following dilation. Is the dilation a reduction or an enlargement?



$$k = \frac{4}{5}$$



new  
orig.

N  
O



## Chapter Five

Identify and classify triangles by their angles and sides (Right, Acute, Obtuse, Equiangular, Equilateral, Isosceles, Scalene)

Triangle Sum Theorem (all of the measures of a triangle sum to  $180^\circ$ )

Apply the Exterior Angle Theorem

Prove Triangles congruent by AAS, SAS, ASA, SSS, HL\*\*

Use CPCTC

Understand and use properties of isosceles and equilateral triangles

**You must be able to write and fill-in two-column and flow proofs**

CPCTC

Corresponding  
Parts of  
Congruent  
Triangles are  
Congruent

obtuse  
acute  
right

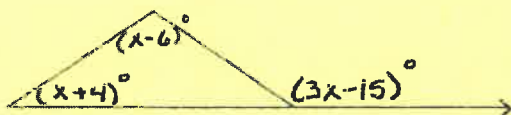
- 1.) Classify the following triangle.

right



- 2.) What is the measure of angle 1 in the above triangle?  $60^\circ$

- 3.) Find x.



2 sides  $\cong$

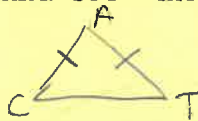
$$(x-6) + (x+4) = 3x-15$$

$$2x-2 = 3x-15$$

$$13 = x$$

- 4.) If triangle CAT is an isosceles triangle where angle A is the vertex angle and  $CA = 2x + 4$ ,  $AT = 3x - 6$ , and  $CT = 5x + 10$ , find CT.

check  
 $CA = 2(10) + 4$   
 $AT = 3(10) - 6$   
 $= 24$  ✓



$$2x+4 = 3x-6$$

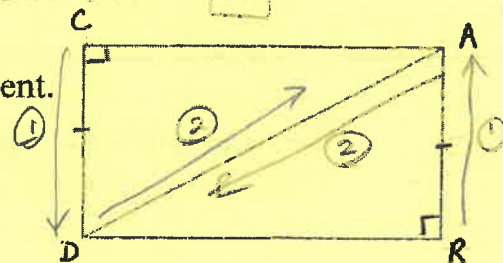
$$10 = x$$

$$CT = 5(10) + 10 = 60$$

- 5.) Using the diagram, write a congruence statement. Then, justify your answer.

which  
theorem

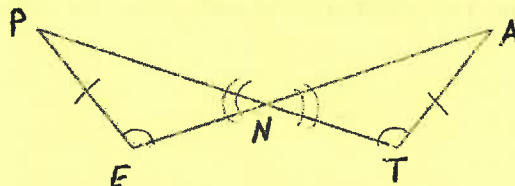
$$\triangle CDA \sim \triangle RAD \text{ by HL}$$



- 6.) Given:  $\overline{PE} \cong \overline{TA}$

$$\angle E \cong \angle T$$

Prove:  $\angle P \cong \angle A$



Statement	Reason
S ① $\overline{PE} \cong \overline{TA}$	Given
A ② $\angle E \cong \angle T$	Given
A ③ $\angle PNE \cong \angle ANT$	Vertical $\angle$ s are congruent
④ $\triangle PNE \cong \triangle ANT$	AAS
⑤ $\angle P \cong \angle A$	CPCTC

## Chapter Six

Identify and use angle bisectors, perpendicular bisectors, medians, and altitudes of triangles

Know the points of concurrency of angle bisectors (incenter), perpendicular bisectors (circumcenter), medians (centroid), and altitudes (orthocenter)

Know the properties of incenters, circumcenters, centroid, and orthocenters

Remember the smallest angle is across from the shortest side of a triangle

★ Remember that the sum of the lengths of two sides of a triangle must be greater than the length of the third side.  $S_1 + S_2 > S_3$      $S_2 + S_3 > S_1$      $S_1 + S_3 > S_2$

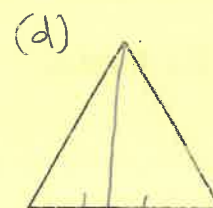
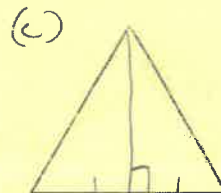
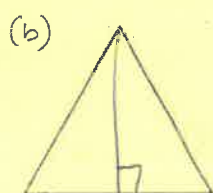
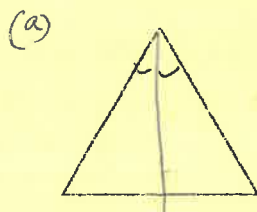
Exterior angle inequality: Measure of the exterior angle  $>$  each of the measures of the remote interior angles

The Hinge Theorem and the Converse of the Hinge Theorem (Inequalities in two triangles)

Know and be able to apply the theorems involving segments divided proportionally (Midsegment Theorem)

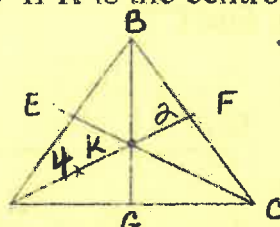


- 1.) Draw an <sup>(a)</sup> angle bisector, <sup>(b)</sup> altitude, <sup>(c)</sup> perpendicular bisector, and <sup>(d)</sup> median and list its properties.

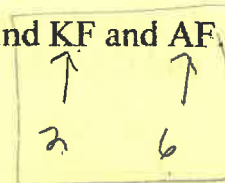


- 2.) If K is the centroid of triangle ABC and  $AK = 4$ . Find KF and AF.

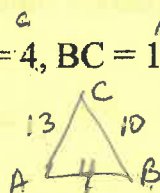
Medians



2 to 1  
longest part by vertex

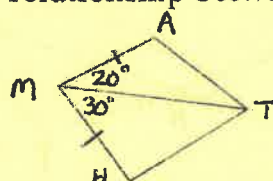


- 3.) In triangle ABC,  $AB = 4$ ,  $BC = 10$  and  $AC = 13$ . List the angles from smallest to largest.



Smallest  $\rightarrow$  largest  
 $\angle C, \angle A, \angle B$

- 4.) What is the relationship between AT and TH in the following diagram?



TH is longer than AT

- ★ 5.) If two sides of a triangle have lengths 3 and 6, what are the possible values of the third side?

3 or 9

$$6 - 3 = 3$$

$$6 + 3 = 9$$

## Chapter Seven

### Properties of Parallelograms

- 1.) Opposite sides  $\cong$
- 2.) Opposite  $\angle$ s  $\cong$
- 3.) Consecutive  $\angle$ s supplementary
- 4.) diagonals bisect each other
- 5.) opposite sides parallel

### Special Properties of Rectangles

- 1.) diagonals  $\cong$   
    angles
- 2.) all corners  $90^\circ$

### Special Properties of Rhombi

- 1.) diagonals NOT congruent
- 2.) diagonals  $\perp$
- 3.) all 4 sides  $\cong$

### Properties of Squares

diagonals bisect + are congruent +  $\perp$

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$$\text{Median} = \frac{1}{2}(\text{base} + \text{base})$$

### Properties of Isosceles Trapezoids

- 1.) lower two base  $\angle$ s  $\cong$   
    " " " "
- 2.) upper " " " "
- 3.) diagonals  $\cong$   
    Opposite  $\angle$ 's supplementary

### Properties of a Kite

- 1.) two pairs consec. sides  $\cong$
- 2.) opposite sides NOT  $\cong$
- 3.) diagonals  $\perp$   
    One pair  $\cong \angle$ s  
    one pair  $\angle$ s bisected

## Chapter Seven

The sum of the interior angles of a convex polygon is  $180(n-2)$

The sum of the exterior angles of a convex polygon is 360

Find each interior and each exterior angle of a regular polygon

Identify and describe convex, concave, regular, and irregular polygons

Know and apply the properties of parallelograms, rectangles, rhombi, squares, kites, trapezoids, and isosceles trapezoids

Using coordinate geometry, identify the quadrilateral

- 1.) Find the sum of the measures of the interior angles of a pentagon.

$$180(5-2) = 180(3) = 540^\circ$$

- 2.) Find the measure of an exterior angle of a regular 36-gon.

$$360 \text{ total} \div 360 \text{ sides} = 10$$

- 3.) Name the regular polygon that has an interior angle measuring  $140^\circ$ .

$$180(n-2) = 140n$$

$$40n = 360$$

$$n = 9$$

nonagon

- 4.) Find the value of  $x$ .

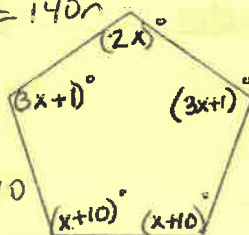
$540^\circ$  total (see #1)

$$\begin{matrix} 2x \\ 3x+1 \\ x+10 \\ x+10 \\ 3x+1 \end{matrix}$$

$$10x+22 = 540$$

$$10x = 518$$

$$x = 51.8$$



- 5.) Use parallelogram ABCD:

- a.) Name a pair of consecutive angles.  $\angle D, \angle C$

- b.) Name the diagonals.  $\overline{AC}, \overline{BD}$

- c.) Solve for  $x$  and  $y$ .

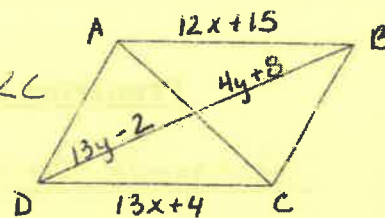
$$13y-2 = 4y+8$$

$$9y = 10$$

$$y = \frac{10}{9}$$

$$12x+15 = 13x+4$$

$$11 = x$$



- 6.) Use Trapezoid MATH:

- a.) Name a pair of base angles.  $\angle H, \angle T$

- b.) Find the length of the midsegment.

- c.) Solve for  $x$ , if  $NR = 10$ .

$$10 = 2x + \frac{1}{2}$$

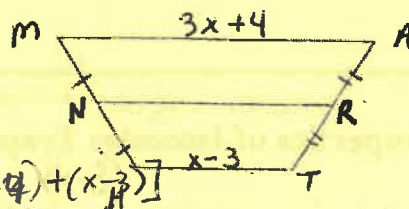
$$\frac{19}{2} = 2x$$

$$x = \frac{19}{4}$$

$$\frac{1}{2}[(3x+4) + (x-3)]$$

$$\frac{1}{2}(4x+1)$$

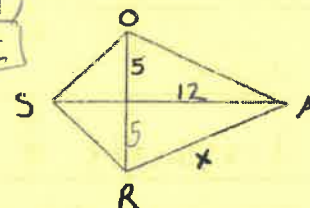
$$2x + \frac{1}{2}$$



- 7.) Find the value of  $x$ , using kite SOAR.

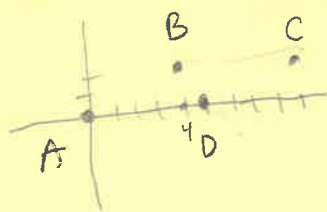
$$5^2 + 12^2 = x^2$$

$$x = 13$$



Yes

- 8.) If  $A(0,0)$ ,  $B(4,2)$ ,  $C(9,2)$ , and  $D(5,0)$ , is ABCD a parallelogram?



$$\overline{BC} \parallel \overline{AD}$$

$$m_{AB} = \frac{0-2}{0-4} = \frac{1}{2}$$

$$m_{CD} = \frac{0-2}{5-9} = \frac{-2}{-4} = \frac{1}{2}$$



## Chapter Eight

**Ratio** – is a comparison of two quantities using division.

**Proportion** – two ratios that are equal

Two figures are similar if

1.) Corresponding angles are =

2.) Corresponding sides are proportional

NO

Scale Factor:  $k = \frac{\text{new}}{\text{original}} = \frac{\text{image}}{\text{pre-image}}$

The scale factor depends on the order of comparison.

The scale factor is equal to the ratio of their corresponding perimeters.

### Ways to Prove Triangles Similar

1.) AA

2.) SAS

3.) SSS

### Theorems:

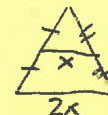
1.) If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



2.) If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.



3.) A midsegment of a triangle is **parallel** to one side of the triangle, and its length is **one half** the length of that side.



4.) If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.



5.) If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



6.) If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides.

- 7.) If two triangles are similar, the lengths of corresponding angle bisectors are proportional to the lengths of corresponding sides.
- 8.) If two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides.
- 9.) An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

### Types of Similarity Transformations

- 1.) reflection      3.) translation
- 2.) rotation

### **Real life application: Scale Drawings and models**

- 1.) Solve  $\frac{3x-1}{4} = \frac{2x+4}{5}$  for x.

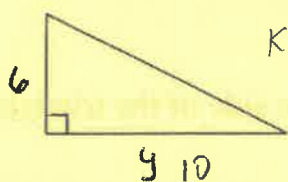
$$\begin{aligned} 5(3x-1) &= 4(2x+4) \\ 15x-5 &= 8x+16 \\ 7x &= 21 \\ \boxed{x=3} \end{aligned}$$

- 2.) The ratio of the measures of the three sides of a triangle is 9:7:5. Its perimeter is 191.1 inches. Find the measure of each side.

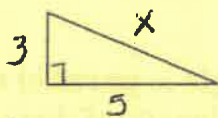
$$\frac{191.1}{21} = 9.1 \quad \text{sides: } 9(9.1), 7(9.1), 5(9.1)$$

$$\boxed{81.9, 63.7, 45.5} = 191.1 \checkmark$$

- 3.) Find the values for x and y in the following similar triangles.

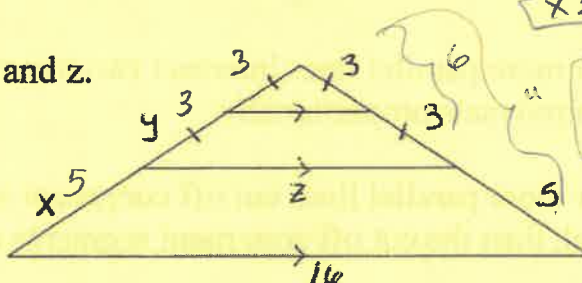


$$K = \frac{1}{2}$$



$$\begin{aligned} \boxed{y=10} \\ 3^2 + 5^2 &= x^2 \\ 36 &= x^2 \\ \boxed{x=6} \end{aligned}$$

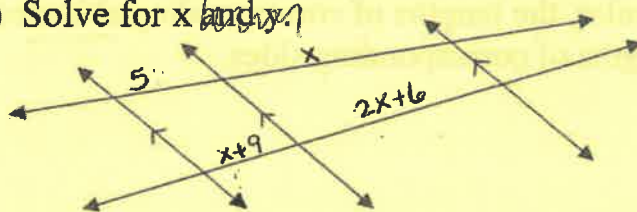
- 4.) Solve for x, y, and z.



$$\begin{aligned} \boxed{x=5} \\ \boxed{y=3} \\ \boxed{z=8.7} \end{aligned}$$

$$\begin{aligned} \frac{6}{2} &= \frac{11}{10} \\ 11z &= 6(16) \\ \boxed{z=8.7} \end{aligned}$$

- 5.) Solve for x and y.



$$\begin{aligned} \frac{5}{x+9} &= \frac{x}{2x+6} \\ 10x+30 &= x^2+9x \\ x^2-x-30 &= 0 \\ (x-6)(x+5) &= 0 \\ \boxed{x=6} \end{aligned}$$

## Chapter Nine

### I. Geometric Mean:

If the altitude is drawn to the hypotenuse of a right triangle, then...

- 1.) the two triangles formed are similar to the original triangle and to each other.
- 2.) the leg is the geometric mean between the segment it touches and the hypotenuse
- 3.) the altitude is the geometric mean between the two segments of the hypotenuse.

### II. Pythagorean Theorem and its converse

In a triangle where  $c$  is the longest side...

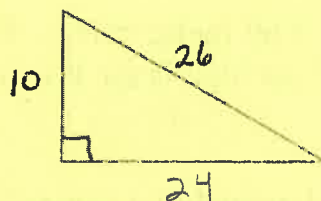
If  $c^2 = a^2 + b^2$  then it is a right triangle.

If  $c^2 > a^2 + b^2$  then it is an obtuse triangle.

If  $c^2 < a^2 + b^2$  then it is an acute triangle.

Pythagorean Triples: 3-4-5, 5-12-13, 7-24-25, 8-15-17

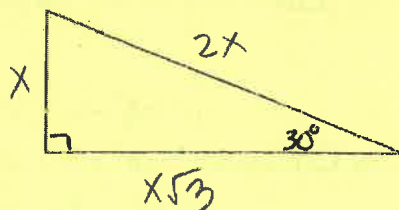
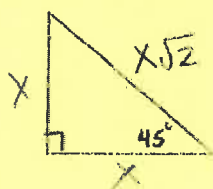
Find the missing side.



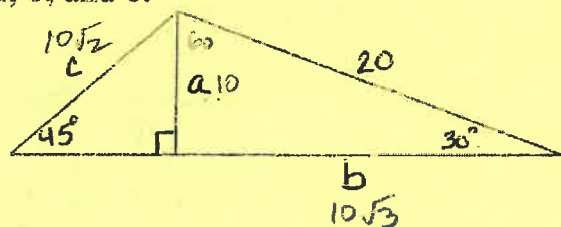
Classify the triangle with sides 4, 4, and 6. *isosceles*

Can a triangle have sides 4, 4, and 6? Why?   
 $4+4 > 6$    
 $4+6 > 4$    
 $6+4 > 4$    
*yes*

### III. Formulas for Special Right Triangles



Solve for  $a$ ,  $b$ , and  $c$ .



$$\begin{aligned} a &= 10 \\ b &= 10\sqrt{3} \\ c &= 10\sqrt{2} \end{aligned}$$

#### IV. Trigonometry: SOH CAH TOA

$$\sin 43^\circ = .682$$

$$\cos x = .6875$$

$$\cos^{-1}(.6875) = x$$

$$x = 46.567$$

$$\tan 45^\circ = \frac{3}{x}$$

$$1 = \frac{3}{x} \quad x = 3$$

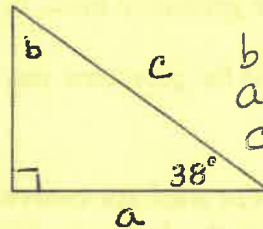
Solve the following triangle:

$$\sin 38^\circ = \frac{10}{c}$$

$$c = \frac{10}{\sin 38^\circ}$$

$$\tan 38^\circ = \frac{10}{a}$$

$$a = \frac{10}{\tan 38^\circ}$$



$$b = 52^\circ$$

$$a = 12.799$$

$$c = 16.243$$

check

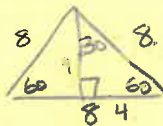
$$12.8^2 + 10^2 = 16.2^2$$

$$267.84 = 262.44$$

✓

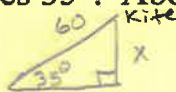
#### V. Word Problems (Angles of elevation and depression)

The length of each side of an equilateral triangle is 8 in. Find the length of the altitude.



$$4\sqrt{3}$$

Sam is flying a kite on a 60-meter string. The angle of elevation of the kite measures  $35^\circ$ . About how high is the kite off the ground.



$$\sin 35 = \frac{x}{60}$$

$$x = 34.415 \text{ meters}$$

Find the length of the diagonal of a square if a side is 10 centimeters long.



$$10\sqrt{2}$$

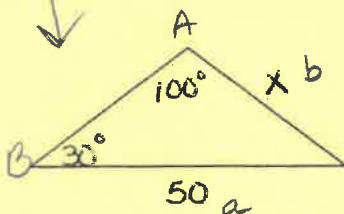
#### VI. Law of Sines and Law of Cosines

$$\text{Law of Sines: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Law of Cosines: } b^2 = a^2 + c^2 - 2ac \cos B$$

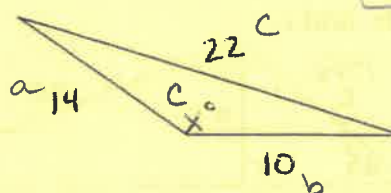
$$c^2 = a^2 + b^2 - 2ab \cos C$$



$$\frac{\sin 100}{50} = \frac{\sin 30}{x}$$

$$x = \frac{\sin 30}{\sin 100} \cdot 50$$

$$x = 25.386$$



$$x = 132.177^\circ$$

$$22^2 = 14^2 + 10^2 - 2(14)(10) \cos x$$

$$22^2 - 14^2 - 10^2 = -2(14)(10) \cos x$$

$$188 = -2(14)(10) \cos x$$

$$\frac{188}{-280} = \cos x$$

$$x = \cos^{-1}\left(-\frac{188}{280}\right)$$

$$x = 132.177^\circ$$



## Chapter Ten

- 1.) **Parts of a Circle:** center, radius, diameter, chord, tangent, secant, major arc, minor arc, semicircle, central angle, inscribed angle, chord-chord angle, tangent-chord angle, tangent-secant angle, secant-secant angle, interior point, exterior point, point on the circle, point of tangency
- 2.) **Common Tangents:** Common internal tangents and common external tangents
- 3.) **Measures of Arcs and Angles:**
  - a. Vertex of the angle inside the circle
  - b. Vertex of the angle outside the circle
  - c. Vertex on the circle
  - d. Vertex at the center of the circle
- 4.) **Segments of a Circle**
  - a. Two chords inside a circle
  - b. Tangent segments and Secant segments
- 5.) **Equations of a Circle**

$(x-h)^2 + (y-k)^2 = r^2$  where  $(h, k)$  is the center and  $r$  is the radius

$(x-3)^2 + (y+2)^2 = 25$

### **Postulate:**

**Arc Addition Postulate:** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

### **Theorems:**

- 1.) If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- 2.) In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.
- 3.) If two segments from the same exterior point are tangent to a circle, then they are congruent.
- 4.) In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- 5.) If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
- 6.) If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

- 7.) In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.
- 8.) If an angle is inscribed in a circle, then its measure is half of its intercepted arc.
- 9.) If two inscribed angles of a circle intercept the same arc, then the angles are congruent.
- 10.) If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.
- 11.) A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.
- 12.) If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.
- 13.) If two chords intersect in the interior of a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
- 14.) If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the intercepted arcs.
- 15.) If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.  
(piece \* piece = piece \* piece)
- 16.) If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.
- 17.) If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.  
(outside seg \* whole seg = outside seg \* whole seg)

\*\*\* Study the common internal tangent and common external tangent handout\*\*\*

## Chapter Ten

Define and identify the circumference, radius, diameter, minor arc, major arc, semicircle, intercepted arc, arc length, chord, secant, and tangent

Know and apply theorems to find the measures of angles and arcs.

Know and apply theorems to find the lengths of segments of a circle.

Solve real-world problems using measures of circumference, arc length, areas of circles, and areas of sectors.


Graph and write equations of circles.


State the radius and center of a circle if you are given the equation.

Know the terms inscribed in and circumscribed about.


### Formulas for Angles of a Circle

If the Vertex of an Angle is:

 INSIDE the circle =  $\frac{1}{2}$  Sum intercepted arcs  $\frac{1}{2}(\widehat{arc1} + \widehat{arc2})$

 OUTSIDE the circle =  $\frac{1}{2}$  difference intercepted arcs  $\frac{1}{2}(\widehat{Big\ arc} - \widehat{little\ arc})$

2x  ON the circle = inscribed angle =  $\frac{1}{2}$  intercepted arc

 AT THE CENTER = m central angle



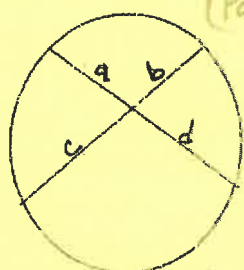
$$\angle 1 = \frac{1}{2}(x + y)$$



$$\angle 1 = \frac{1}{2}(x - y)$$

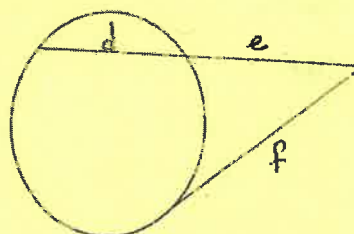


### Formulas for Segments of a Circle



$$cb = ad$$

$$\begin{aligned} (\text{Part})(\text{Part}) &= \\ (\text{Part})(\text{Part}) & \end{aligned}$$



$$e(d + e) = f^2$$

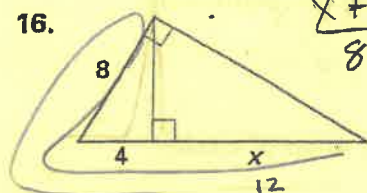
$$TO = TO$$

# Cumulative Review

For use with Chapters 1-10

Find the value of the given variable. (9.1, 9.2)

16.

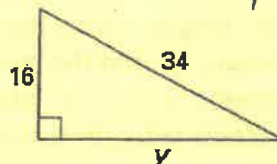


$$\frac{x+4}{8} = \frac{8}{4}$$

$$x+4=16$$

$$x=12$$

17.



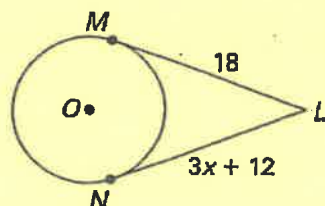
$$y^2 + 16^2 = 34^2$$

$$y = \sqrt{34^2 - 16^2}$$

$$y=30$$

$\overline{LM}$  and  $\overline{LN}$  are tangent to  $\odot O$ . Find the value of  $x$ . (10.1)

18.

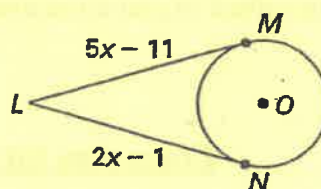


$$3x+12=18$$

$$3x=6$$

$$x=2$$

19.



$$5x-11=2x-1$$

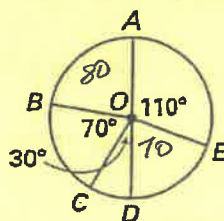
$$3x=10$$

$$x=\frac{10}{3}$$

$\overline{AD}$  is a diameter of  $\odot O$ . Find the indicated measure. (10.2)

20.  $m\widehat{ABC} = ?$   $150^\circ$

21.  $m\widehat{CDE} = ?$   $100^\circ$



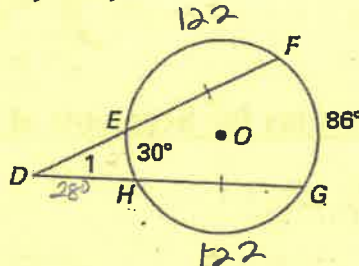
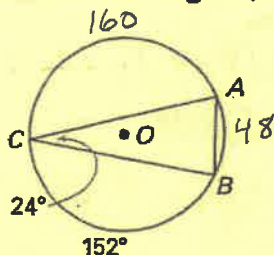
Find the measure of the arc or angle. (10.3, 10.4)

22.  $m\widehat{AB} = ?$   $48^\circ$

23.  $m\widehat{AC} = ?$   $160^\circ$

24.  $m\angle 1 = ?$   $28^\circ$

25.  $m\widehat{EF} = ?$   $122^\circ$



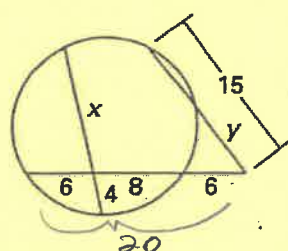
$$\angle 1 = \frac{1}{2}(86 - 30) = 28$$

$$360 - 86 - 30 = 244$$

$$\frac{244}{2} = 122$$

Find the value of  $x$  and  $y$ . (10.5)

26.



$$4x = 6(8)$$

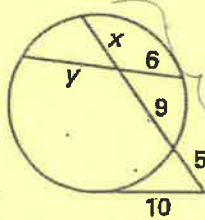
$$x=12$$

$$TO = TO$$

$$(20)(6) = 15y$$

$$y=8$$

27.



$$(9)(x) = 6(y)$$

$$54 = 6y$$

$$y=9$$

$$TO = TO$$

$$(x+4)5 = 10^2$$

$$5x+20=100$$

$$x=6$$

Write the standard equation of a circle with the given center and radius. (10.6)

28. center  $(2, 3)$  and radius  $= 4$

$$(x-2)^2 + (y-3)^2 = 16$$

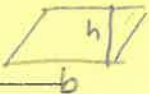
29. center  $(-1, 2)$  and radius  $= 3$

$$(x+1)^2 + (y-2)^2 = 9$$



## Chapter Eleven

Area formulas for polygons and circles:

Parallelogram:  $b h$  

Triangle:  $\frac{1}{2} b h$

Rectangle:  $L \times w$

Trapezoid:  $\frac{1}{2} (b_1 + b_2) h$

Square:  $s \times s$

Circle:  $\pi r^2$

Rhombus:  $\frac{1}{2} d_1 d_2$  or  $\frac{1}{2} a p$

Sector:  $\frac{\text{arc}}{360} \pi r^2$

Kite:  $\frac{1}{2} d_1 d_2$

Regular Polygon:  $\frac{1}{2} a p$

The area of a region is the sum of the areas of its nonoverlapping parts.

If two figures are congruent, then they have the same area.

If two polygons are similar, then their areas are proportional to the square of the scale factor between them.

### Questions:

- 1.) The area of a triangle is  $120\text{m}^2$ . If the length of the base is 24m, find the height of the triangle.

$$120 = \frac{1}{2} (24)(h)$$

$$120 = 12h$$

$$h = 10\text{m}$$

- 2.) Find the area of a regular polygon, if its apothem is 8ft long and the perimeter is 60 ft.

$$A = \frac{1}{2} (8)(60)$$

$$= 240\text{ft}^2$$

- 3.) The length of a rectangle is 3cm more than its width. If the area is  $40\text{cm}^2$ , find the length of the rectangle.

$$L = w + 3$$

$$A = 40$$

$$L = 8$$

$$w = 5$$

$$A = 40\checkmark$$

$$40 = (w + 3)(w)$$

$$40 = w^2 + 3w$$

$$w^2 + 3w - 40 = 0$$

$$(w + 8)(w - 5)$$

$$w = -8, w = 5$$

- 4.) One base of a trapezoid is 10in long and the altitude is 20in long. If the area is  $240\text{in}^2$ , find the length of the other base.

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$240 = \frac{1}{2}(10 + b_2)20$$

$$240 = 10(10 + b_2)$$

$$240 = 100 + 10b_2$$

$$140 = 10b_2$$

$$b_2 = 14$$

$$\frac{1}{2}(10 + 14)(20)$$

$$\frac{1}{2}(24)(20)$$

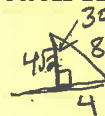
$$240 \checkmark$$

- 5.) Find the area of a regular hexagon with each side 8cm long.

$$P = 8 \times 6 = 48$$

$$A = \frac{1}{2}(4\sqrt{3})(48)$$

$$A = 50.912\text{cm}^2$$



$$\text{Central } \angle = \frac{360}{6} = 60$$

- 6.) Find the area of a rhombus with perimeter of 100cm and one diagonal 40cm long.

$$A = \frac{1}{2}d_1d_2$$

$$= \frac{1}{2}(40)(30)$$

$$A = 600\text{cm}^2$$

$$\div 4 = 25$$

$$\text{each side } 25$$

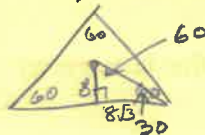


$$20^2 + x^2 = 25^2$$

$$x = 15$$

$$d_2 = 30$$

- 7.) Find the area of an equilateral triangle with apothem 8m long.



$$\text{side } 2(8\sqrt{3})$$

$$P = (3)(2)(8\sqrt{3})$$

$$A = \frac{1}{2}(8)(3)(2)(8\sqrt{3})$$

$$A = 332.554\text{m}^2$$

- 8.) Find the area of a square with an apothem 6cm long.



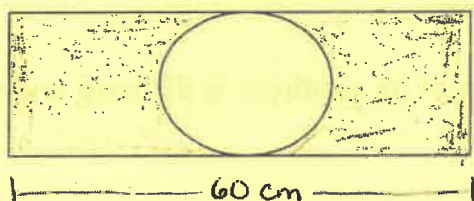
$$S = 2(6) = 12$$

$$P = 4(12) = 48$$

$$A = \frac{1}{2}(6)(48)$$

$$A = 144\text{cm}^2$$

- 9.) Find the area of the shaded region.

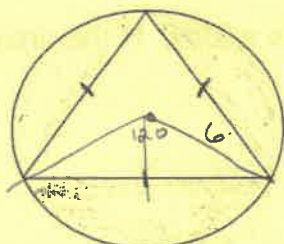


$$A_{\text{rectangle}} = 600\text{cm}^2$$

$$- A_{\text{circle}} = \pi 5^2$$

$$521.460\text{cm}^2$$

- 10.) Find the area of the shaded region.



$$\text{radius} = 6\text{cm}$$

$$S = 2(3\sqrt{3})$$

$$P = (3)(2)(3\sqrt{3})$$

$$a = 3$$

$$A = \frac{1}{2}(3)(3)(2)(3\sqrt{3})$$

$$A = 46.765\text{cm}^2$$

## Chapter Eleven

Know the difference between right and oblique solids

Determine the cross sections of solids

Use the Formulas for Lateral Area, Surface Area, and Volume to calculate missing values

### Prism

no base →  $LA = Ph = (2L + 2w)h$   
 $SA = 2(Lw + Lh + wh)$   
 $V = L \times w \times H$

### Cylinder

$$LA = 2\pi rh$$
$$SA = 2\pi rh + 2\pi r^2$$
$$V = \pi r^2 h$$

### Pyramid

$n$  sides →  $LA = n(\frac{1}{2}bh)$   
 $SA = n(\frac{1}{2}bh)$   
 $V = \frac{1}{3}Bh$   
↑  
 $\frac{1}{2}bh$  of  $\Delta$  or  
 $L \times w$  square

### Cone

$$LA = \pi r \sqrt{h^2 + r^2}$$
$$SA = \pi r^2 + \pi r l$$
$$V = \frac{1}{3}Bh$$

### Sphere

$$SA = 4\pi r^2$$
$$V = \frac{4}{3}\pi r^3$$

Two solids are similar if they have the same shape and the ratios of their corresponding linear measures are equal.

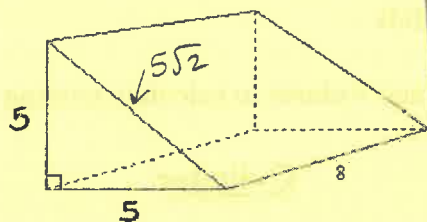
If two similar solids have a scale factor of  $a:b$ , then the surface areas have a ratio of  $a^2:b^2$ , and the volumes have a ratio of  $a^3:b^3$ .

Two solids are congruent if they have...

- 1.) Congruent corresponding angles
- 2.) Congruent corresponding edges
- 3.) Congruent corresponding faces
- 4.) Equal volumes

Find the lateral area, the surface area and the volume of the following solids.

1.)

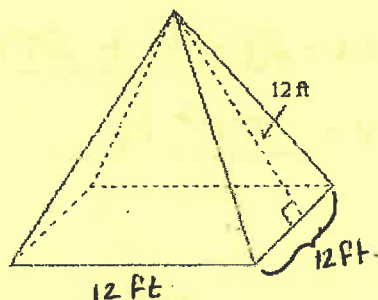


$$LA = (5 + 5 + 5\sqrt{2})8 = 136.569u^2$$

$$SA = 2(\frac{1}{2})(5)(5) + (8)(5) + (8)(5) + (8)(5\sqrt{2}) = 161.569u^3$$

$$V = Bh = (\frac{1}{2})5 \cdot 5 \cdot 8 = 100u^3$$

2.)

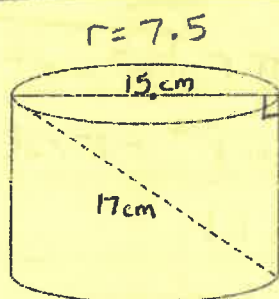


$$4(\frac{1}{2})(12)(12) = 288u^2$$

$$4(\frac{1}{2})(12)(12) + (12)(12) = 432ft^2$$

$$V = \frac{1}{3}Bh = \frac{1}{3}(12)(12)(12) = 576ft^3$$

3.)



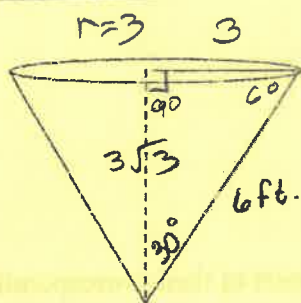
$$h = \sqrt{17^2 - 15^2} = 8$$

$$2\pi rh + \pi r^2 = 553.706 cm^2$$

$$SA = 2\pi rh + (\pi r^2)(2) = 730.402 cm^2$$

$$V = Bh = \pi(7.5)^2(8) = 1413.717 cm^3$$

4.)



$$LA = \pi r \sqrt{h^2 + r^2} = 3\pi \sqrt{(3\sqrt{3})^2 + 3^2} = 18\pi \approx 56.549 ft^2$$

$$SA = \pi r^2 + \pi r l = \pi(3)^2 + \pi(3)(6) = 27\pi \approx 84.823 ft^2$$

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi(3)^2(3\sqrt{3}) \approx 48.973 ft^3$$

5.) Find the surface area and volume of a sphere whose great circle has a circumference of  $26\pi$  cm.

$$26\pi = 2\pi r \\ r = 13 cm$$

$$SA = 4\pi r^2 = 4\pi(13)^2 = 2123.717 cm^2$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(13)^3 = 9202.772 cm^3$$