Big Ideas Math: Geometry

Chapter Summary

Chapter 2: Reasoning and Proofs

Core Vocabulary

A logical statement that has a hypothesis and a conclusion is a ***conditional statement.***

A conditional statement in the form "if *p*, then *q*" is in ***if-then form.***

The "if" part of a conditional statement written in if-then form is the ***hypothesis.***

The "then" part of a conditional statement written in if-then form is the ***conclusion.***

The opposite of a statement is a ***negation.***

The statement formed by exchanging the hypothesis and conclusion of a conditional statement is the ***converse.***

The statement formed by negating both the hypothesis and conclusion of a conditional statement is the ***inverse.***

The statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement is the ***contrapositive.***

Equivalent statements are two related conditional statements that are both true or both false.

Perpendicular lines are two lines that intersect to form a right angle.

A statement that contains the phrase "if and only if" is a ***biconditional statement.***

A ***truth table*** is a table that shows the truth values for a hypothesis, conclusion, and conditional statement.

An unproven statement that is based on observations is a ***conjecture.***

Inductive reasoning is a process that includes looking for patterns and making conjectures.

A specific case for which a conjecture is false is a ***counterexample.***

Deductive reasoning is a process that uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.

A statement that can be proven is a ***theorem.***

A ***truth value*** is a value that represents whether a statement is true (T) or false (F).

A ***line perpendicular to a plane*** is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

A logical argument that uses deductive reasoning to show that a statement is true is a ***proof.***

A ***two-column proof*** is a type of proof that has numbered statements and corresponding reasons that show an argument in a logical order.

A ***flowchart proof***, or ***flow proof*** is a type of proof that uses boxes and arrows to show the flow of a logical argument.

A style of proof that presents the statements and reasons as sentences in a paragraph, using words to explain the logical flow of an argument is a ***paragraph proof.***

Learning Goals

Write conditional statements.

Use definitions written as conditional statements.

Write biconditional statements.

Make truth tables.

Use inductive reasoning.

Use deductive reasoning.

Identify postulates using diagrams.

Sketch and interpret diagrams.

Use Algebraic Properties of Equality to justify the steps in solving an equation.

Use the Distributive Property to justify the steps in solving an equation.

Use properties of equality involving segment lengths and angle measures.

Write two-column proofs.

Name and prove properties of congruence.

Write flowchart proofs to prove geometric relationships.

Write paragraph proofs to prove geometric relationships.

Essential Questions

When is a conditional statement true or false?

How can you use reasoning to solve problems?

In a diagram, what can be assumed and what needs to be labeled?

How can algebraic properties help you solve an equation?

How can you prove a mathematical statement?

How can you use a flowchart to prove a mathematical statement?

Postulates

2.1 Two Point Postulate
Through any two points, there exists exactly one line.

2.2 Line-Point Postulate
A line contains at least two points.

2.3 Line Intersection Postulate
If two lines intersect, then their intersection is exactly one point.

2.4 Three Point Postulate
Through any three noncollinear points, there exists exactly one plane.

2.5 Plane-Point Postulate
A plane contains at least three noncollinear points.

2.6 Plane-Line Postulate
If two points lie in a plane, then the line containing them lies in the plane.

2.7 Plane Intersection Postulate
If two planes intersect, then their intersection is a line.

2.8 Linear Pair Postulate
If two angles form a linear pair, then they are supplementary.

Standards

Common Core:
HSG-CO.C.9,
HSG-CO.C.10,
HSG-CO.C.11,
HSG-SRT.B.4

Theorems

2.1 Properties of Segment Congruence
Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment *AB*, $\overline{AB} \cong \overline{AB}$.
Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

2.2 Properties of Angle Congruence
Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle *A*, $\angle A \cong \angle A$.
Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

2.3 Right Angles Congruence Theorem
All right angles are congruent.

2.4 Congruent Supplements Theorem
If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

2.5 Congruent Complements Theorem
If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

2.6 Vertical Angles Congruence Theorem
Vertical angles are congruent.

<u>Inductive Reasoning</u>

- A conjecture is an unproven statement that is based on observations.
- You use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.

<u>Counterexample</u>

- To show that a conjecture is true, you must show that it is true for all cases.
- You can show that a conjecture is false, however, by finding just one counterexample.
- A counterexample is a specific case for which the conjecture is false.

<u>Deductive Reasoning</u>

- Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.
- This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

<u>Laws of Logic</u>

Law of Detachment

- If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

- If hypothesis *p*, then conclusion *q*.
  If hypothesis *q*, then conclusion *r*. ↖ If these statements are true,
- If hypothesis *p*, then conclusion *r*. ← then this statement is true.

Core Concept

<u>Conditional Statement</u>
A conditional statement is a logical statement that has two parts, a *hypothesis p* and a *conclusion q*. When a conditional statement is written in if-then form, the "if" part contains the hypothesis and the "then" part contains the conclusion.

Words If *p*, then *q*.
Symbols $p \to q$ (read as "*p* implies *q*")

<u>Negation</u>
The negation of a statement is the *opposite* of the original statement. To write the negation of a statement *p*, you write the symbol for negation (~) before the letter. So, "not *p*" is written $\sim p$.

Words not *p*
Symbols $\sim p$

<u>Biconditional Statement</u>
When a conditional statement and its converse are both true, you can write them as a single *biconditional statement.* A biconditional statement is a statement that contains the phrase "if and only if."

Words *p* if and only if *q*
Symbols $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

<u>Related Conditionals</u>
Consider the conditional statement: If *p*, then *q*. ($p \to q$)

Converse

- To write the converse of a conditional statement, exchange the hypothesis and the conclusion.
- If *q*, then *p*. ($q \to p$)

Inverse

- To write the inverse of a conditional statement, negate both the hypothesis and the conclusion.
- If not *p*, then not *q*. ($\sim p \to \sim q$)

Contrapositive

- To write the contrapositive of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.
- If not *q*, then not *p*. ($\sim q \to \sim p$)

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called equivalent statements.

<u>Algebraic Properties of Equality</u>
Let *a*, *b*, and *c* be real numbers.

Addition Property of Equality

- If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

- If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality

- If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality

- If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

Substitution Property of Equality

- If $a = b$, then *a* can be substituted for *b* (or *b* for *a*) in any equation or expression.

<u>Distributive Property</u>
Let *a*, *b*, and *c* be real numbers.

Sum

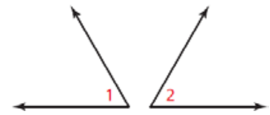
- $a(b + c) = ab + ac$

Difference

- $a(b - c) = ab - ac$

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$
Prove $\angle 2 \cong \angle 1$

statements based on facts that you know or on conclusions from deductive reasoning

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

Types of Proofs

Symmetric Property of Angle Congruence (Theorem 2.2)
Given $\angle 1 \cong \angle 2$
Prove $\angle 2 \cong \angle 1$

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

Paragraph Proof

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

Flowchart Proof

$\angle 1 \cong \angle 2$ → $m\angle 1 = m\angle 2$ → $m\angle 2 = m\angle 1$ → $\angle 2 \cong \angle 1$

Given → Definition of congruent angles → Symmetric Property of Equality → Definition of congruent angles

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Additional Review

- Making a Truth Table, *p. 70*
- Identifying Postulates, *p. 85*
- Sketching and Interpreting Diagrams, *p. 86*
- Reflexive, Symmetric, and Transitive Properties of Equality, *p. 94*
- Writing Flowchart Proofs, *p. 106*
- Writing Paragraph Proofs, *p. 108*

Games

- Equation Relay
- Equation Tic-Tac-Toe

These are available online in the *Game Closet* at www.bigideasmath.com.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 2: Tiger STEM Video is available online at www.bigideasmath.com.