Big Ideas Math: Geometry

Chapter Summary

Chapter 10: Circles

Core Vocabulary

A ***circle*** is the set of all points in a plane that are equidistant from a given point.

The point from which all points on a circle are equidistant is called the ***center*** of the circle.

A segment whose endpoints are the center and any point on a circle is a ***radius***.

A ***chord*** is a segment whose endpoints are on a circle.

A ***diameter*** is a chord that contains the center of the circle.

A ***secant*** is a line that intersects a circle in two points.

A ***tangent*** of a circle is a line in the plane of a circle that intersects the circle in exactly one point

The point at which a tangent line intersects a circle is called the ***point of tangency***.

Coplanar circles that intersect in one point are called ***tangent circles***.

Coplanar circles that have a common center are called ***concentric circles***.

A line or segment that is tangent to two coplanar circles is called a ***common tangent***.

A ***central angle*** of a circle is an angle whose vertex is the center of the circle.

An arc with a measure less than 180°is called a ***minor arc***.

An arc with a measure greater than 180°is called a ***major arc***.

A ***semicircle*** is an arc with endpoints that are the endpoints of a diameter.

The ***measure of a minor arc*** is the measure of its central angle.

The ***measure of a major arc*** is the measure of its central angle.

Arcs of a circle that have exactly one point in common are called ***adjacent arcs***.

Congruent circles are circles that can be mapped onto each other by a rigid motion or a composition of rigid motions.

Congruent arcs are arcs that have the same measure and are of the same circle or of congruent circles.

Arcs that have the same measure are ***similar arcs.***

An ***inscribed angle*** is an angle whose vertex is on a circle and whose sides contain chords of the circle.

An arc that lies between two lines, rays, or segments is called an ***intercepted arc***.

A polygon is an ***inscribed polygon*** when all its vertices lie on a circle.

If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to ***subtend*** the angle.

A circle that contains all the vertices of an inscribed polygon is a ***circumscribed circle***.

A ***circumscribed angle*** is an angle whose sides are tangent to a circle.

The segments formed from two chords that intersect in the interior of a circle are called ***segments of a chord***.

A ***tangent segment*** is a segment that is tangent to a circle at an endpoint.

A ***secant segment*** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle.

The part of a secant segment that is outside the circle is called an ***external segment***.

The ***standard equation of a circle*** is written as $(x - h)^2 + (y - k)^2 = r^2$, where r is the radius and (h, k) is the center.

Essential Questions

What are the definitions of the lines and segments that intersect a circle?

How are circular arcs measured?

What are two ways to determine when a chord is a diameter of a circle?

How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?

What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

What is the equation of a circle with center (h, k) and radius r in the coordinate plane?

Learning Goals

Identify special segments and lines.

Draw and identify common tangents.

Use properties of tangents.

Find arc measures.

Identify congruent arcs.

Prove circles are similar.

Use chords of circles to find lengths and arc measures.

Use inscribed angles.

Use inscribed polygons.

Find angle and arc measures.

Use circumscribed angles.

Use segments of chords, tangents, and secants.

Write and graph equations of circles.

Write coordinate proofs involving circles.

Solve real-life problems using graphs and circles.

Standards

Common Core:
HSG-CO.A.1,
HSG-CO.D.13,
HSG-C.A.1,
HSG-C.A.2,
HSG-C.A.3,
HSG-C.A.4,
HSG-MG.A.1,
HSG-MG.A.3,
HSG-GPE.A.1,
HSG-GPE.B.4

Theorems

10.1 Tangent Line to Circle Theorem
In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

10.2 External Tangent Congruence Theorem
Tangent segments from a common external point are congruent.

10.3 Congruent Circles Theorem
Two circles are congruent circles if and only if they have the same radius.

10.4 Congruent Central Angles Theorem
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.

10.5 Similar Circles Theorem
All circles are similar.

10.6 Congruent Corresponding Chords Theorem
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

10.7 Perpendicular Chord Bisector Theorem
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

10.8 Perpendicular Chord Bisector Converse
If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

10.9 Equidistant Chords Theorem
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

10.10 Measure of an Inscribed Angle Theorem
The measure of an inscribed angle is one-half the measure of its intercepted arc.

10.11 Inscribed Angles of a Circle Theorem
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

10.12 Inscribed Right Triangle Theorem
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

10.13 Inscribed Quadrilateral Theorem
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

10.14 Tangent and Intersected Chord Theorem
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.

10.15 Angles Inside the Circle Theorem
If two chords intersect inside a circle, then the measure of each angle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

10.16 Angles Outside the Circle Theorem
If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

10.17 Circumscribed Angle Theorem
The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.

10.18 Segments of Chords Theorem
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

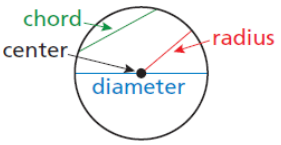
10.19 Segments of Secants Theorem
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

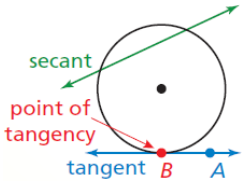
10.20 Segments of Secants and Tangents Theorem
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

Core Concept

Lines and Segments That Intersect Circles

- A segment whose endpoints are the center and any point on a circle is a radius.
- A chord is a segment whose endpoints are on a circle.
- A diameter is a chord that contains the center of the circle.
- A secant is a line that intersects a circle in two points.
- A tangent is a line in the plane of a circle that intersects the circle in exactly one point, the point of tangency. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.





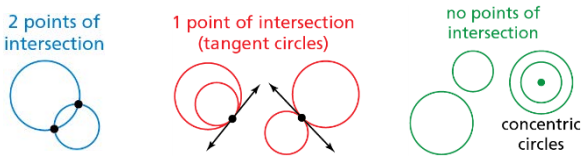
Postulate

10.1 Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Core Concept

Coplanar Circles and Common Tangents

In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called tangent circles. Coplanar circles that have a common center are called concentric circles.



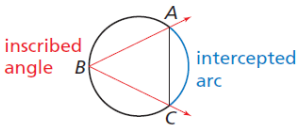
A line or segment that is tangent to two coplanar circles is called a common tangent. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

Measuring Arcs

- The measure of a minor arc is the measure of its central angle.
- The expression $\overset{\frown}{AB}$ is read as "the measure of arc AB."
- The measure of an entire circle is 360°.
- The measure of a major arc is the difference of 360° and the measure of the related minor arc.
- The measure of a semicircle is 180°.

Inscribed Angle and Intercepted Arc

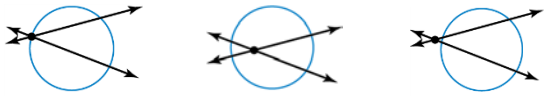
- An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.
- An arc that lies between two lines, rays, or segments is called an intercepted arc.
- If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to subtend the angle.



$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

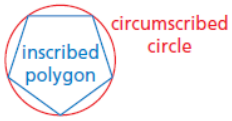
Intersecting Lines and Circles

If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



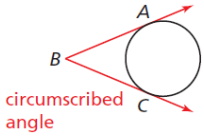
Inscribed Polygon

- A polygon is an inscribed polygon when all its vertices lie on a circle.
- The circle that contains the vertices is a circumscribed circle.



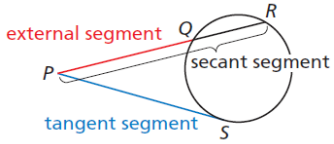
Circumscribed Angle

A circumscribed angle is an angle whose sides are tangent to a circle.



Tangent Segment and Secant Segment

- A tangent segment is a segment that is tangent to a circle at an endpoint.
- A secant segment is a segment that contains a chord of a circle and has exactly one endpoint outside the circle.
- The part of a secant segment that is outside the circle is called an external segment.



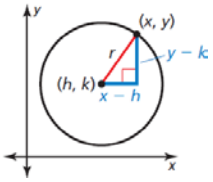
$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Standard Equation of a Circle

Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean Theorem (Theorem 9.1),

$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the standard equation of a circle with center (h, k) and radius r.



What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 10: Seismographs and Earthquake Epicenters STEM Video is available online at www.bigideasmath.com.

Additional Review

- Writing Coordinate Proofs Involving Circles, *p. 578*