Chapter 7 Quadrilaterals

Sunday, March 27, 2022 6:44 PM

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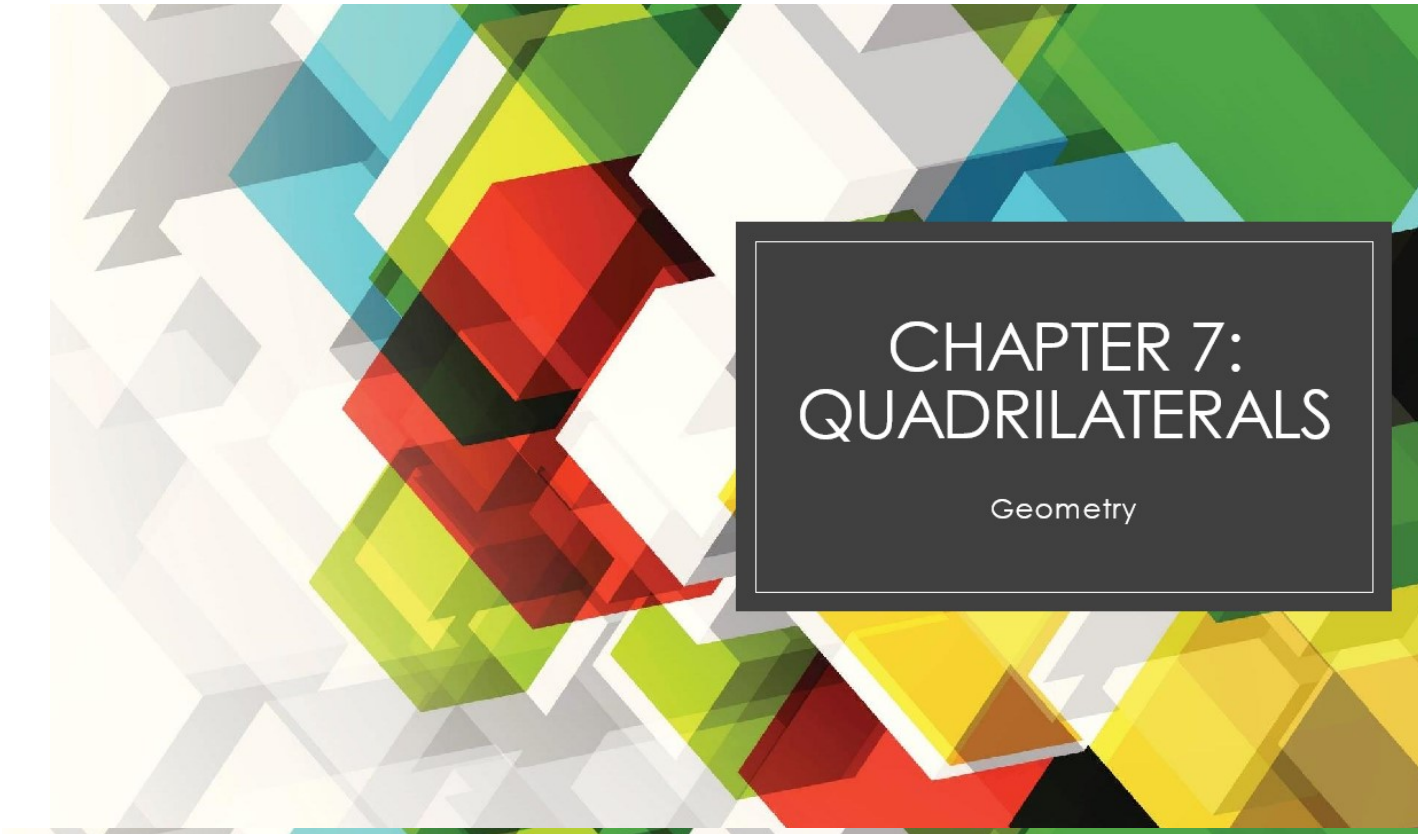
Quadrilat...



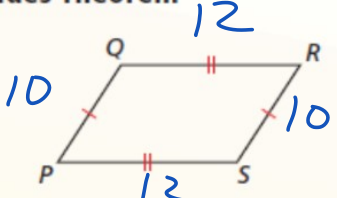
Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 368

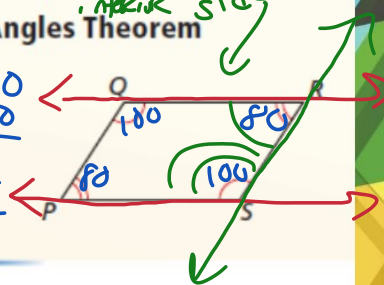


Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 373



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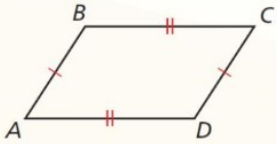
Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

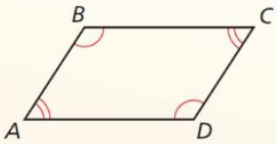


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383



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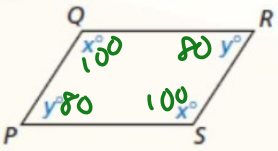
Theorems

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 373

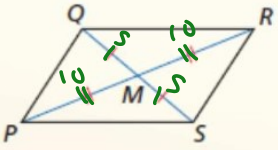


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 370



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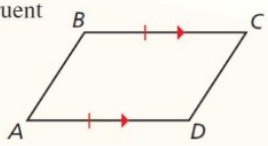
Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof Ex. 40, p. 383

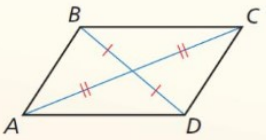


Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

Proof Ex. 41, p. 383



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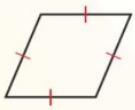
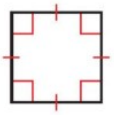
Core Concept

Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

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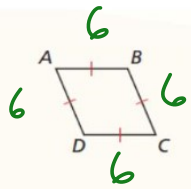
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

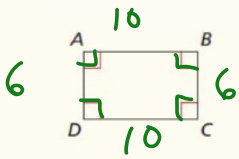


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396



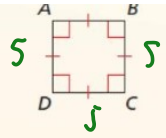
Corollary 7.4 Square Corollary

5

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



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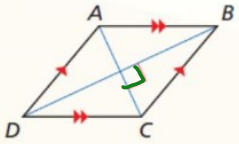
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

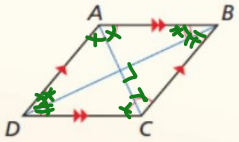


Theorem 7.12 Rhombus Opposite Angles Thoerem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395



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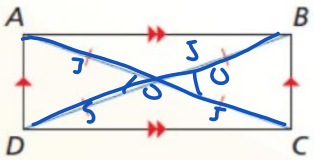
Theorem

Theorem 7.13 Rhombus Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



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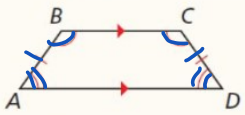
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405



Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

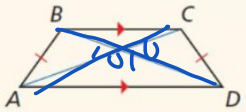
Proof Ex. 40, p. 405

Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



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Theorem

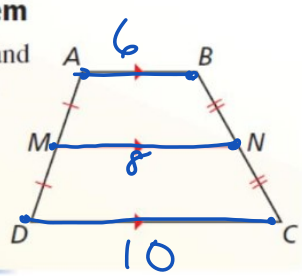
Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 406

6+10
½ (16)
8



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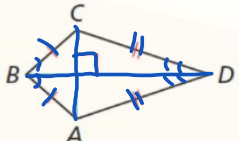
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

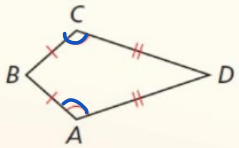


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406



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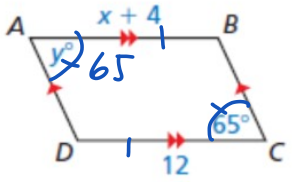
Find the values of x and y.

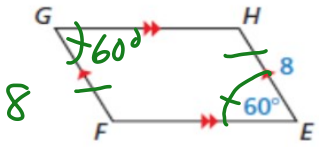
$x + y = 12$
$-y \quad -y$
$x = 8$

65°



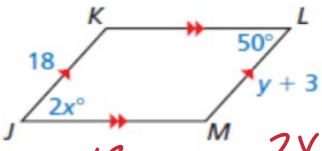
1. Find FG and $m\angle G$.

8 60°



2. Find the values of x and y.

25 15



$y + 3 = 18$
$-3 \quad -3$
$y = 15$

$\frac{2x}{2} = \frac{50}{2}$
$x = 25$

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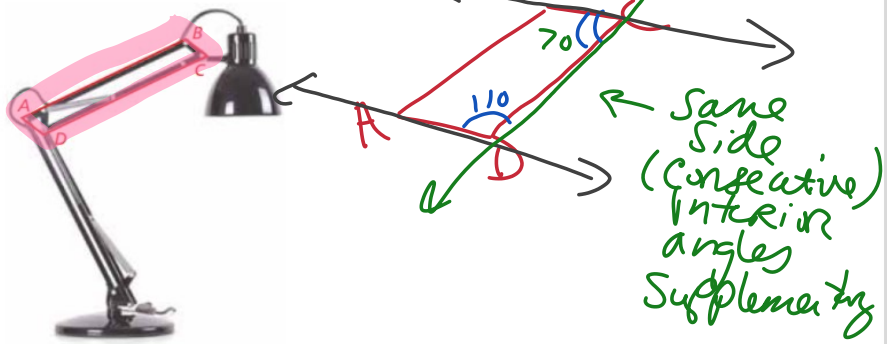
As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.

$180 - 110 = 70$



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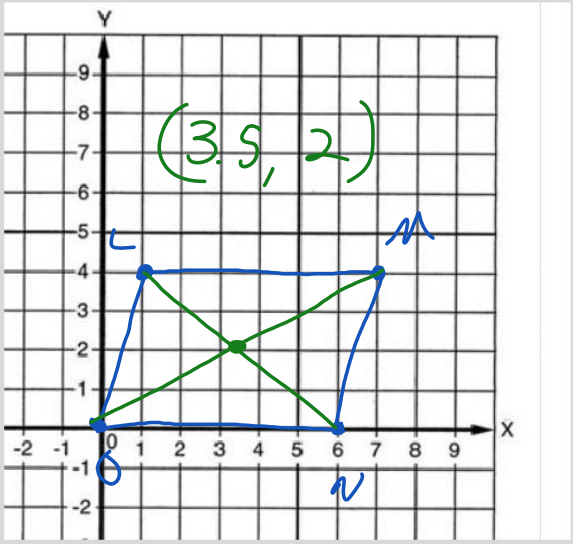
Find the coordinates of the intersection of the diagonals of ▫*LMNO* with vertices *L*(1, 4), *M*(7, 4), *N*(6, 0), and *O*(0, 0).



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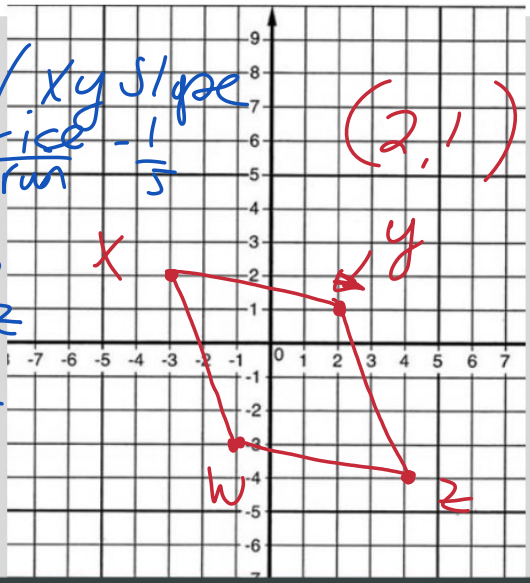
Three vertices of ▫*WXYZ* are *W*(−1, −3), *X*(−3, 2), and *Z*(4, −4). Find the coordinates of vertex *Y*.



Slope up
X W

Rise -5
Run 2

✓ XY Slope
rise -1
run 5

✓ Slope WZ
−1/5

1 S

From Z ↑ up 5
Left
← -2

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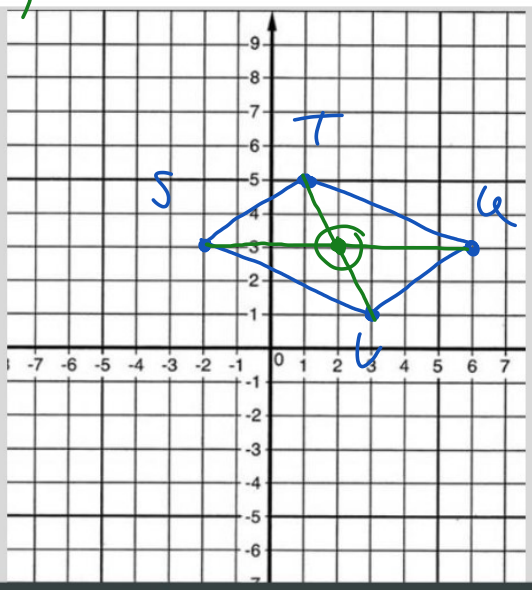
5. Find the coordinates of the intersection of the diagonals of ▫*STUV* with vertices *S*(−2, 3), *T*(1, 5), *U*(6, 3), and *V*(3, 1).

(2, 3)



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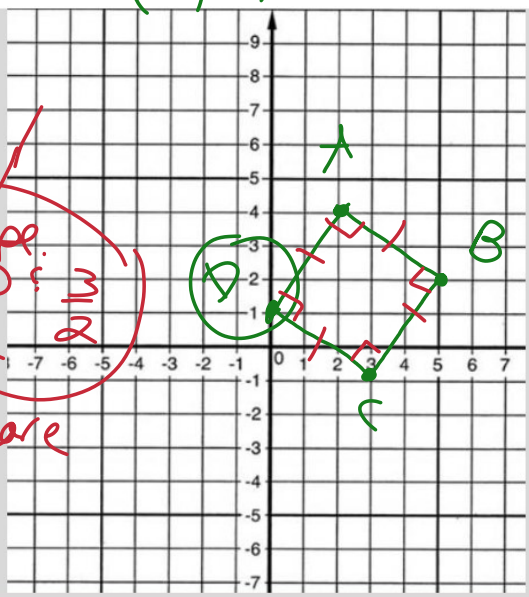
6. Three vertices of ▫*ABCD* are *A* (2, 4), *B* (5, 2), and *C* (3, −1). Find the coordinates of vertex *D*. (0, 1)



Slope of $\overline{AB}$

Rise / Run $= -\frac{2}{3}$

negative reciprocals perpendicular

Rectangle 90° Square

Slope AD $= \frac{3}{2}$

BC ✓

From C: up 2, left 3 $= -\frac{2}{3}$

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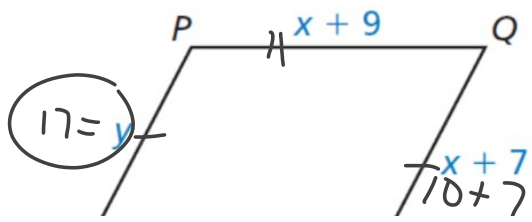
For what values of *x* and *y* is quadrilateral *PQRS* a parallelogram?

$x + 9 = 2x - 1$

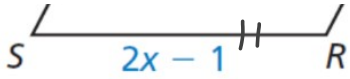
$+1 \quad +1$

$x + 10 = 2x$

$-x \quad -x$



$10 = x$



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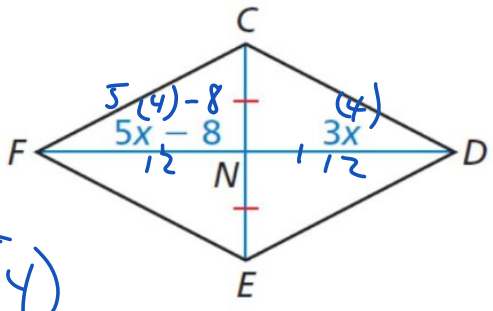
For what value of x is quadrilateral $CDEF$ a parallelogram?

$5x - 8 = 3x$
$+8 \quad +8$
$5x = 3x + 8$
$-3x \quad -3x$
$2x = 8$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$



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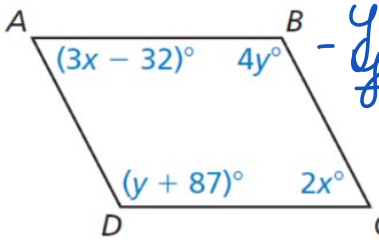
2. For what values of x and y is quadrilateral $ABCD$ a parallelogram? Explain your reasoning.

$3x - 32 = 2x$
$+32 \quad +32$
$3x = 2x + 32$
$-2x \quad -2x$
$x = 32$

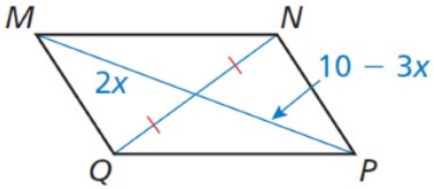


$y + 87 = 4y$
$-y \quad -y$
$87 = 3y$
$\frac{87}{3} = \frac{3y}{3}$
$29 = y$

Students, draw anywhere on this slide!

6. For what value of x is quadrilateral $MNPQ$ a parallelogram? Explain your reasoning.

$2x = 10 - 3x$
$+3x \quad +3x$
$\frac{5x}{5} = \frac{10}{5}$ $x = 2$



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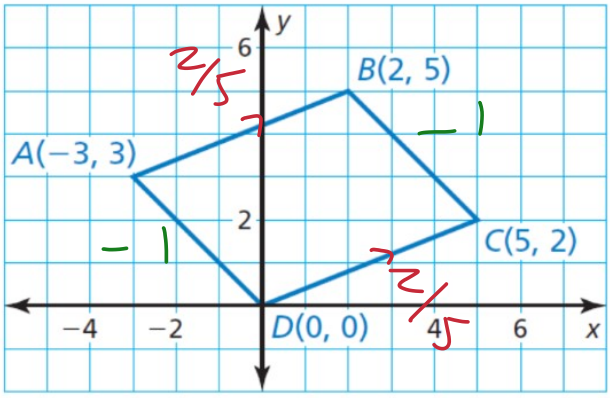
Show that quadrilateral $ABCD$ is a parallelogram.

Parallel lines Slopes Same



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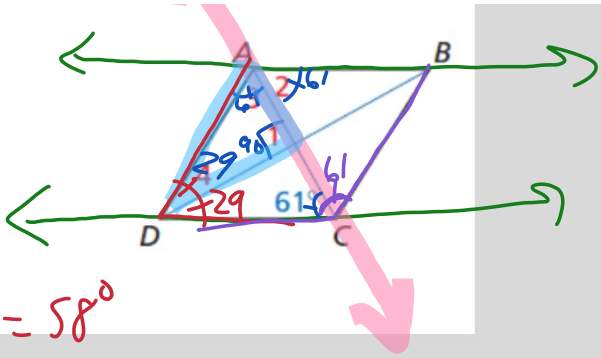
Find the measures of the numbered angles in rhombus $ABCD$.

$\angle 1 = 90$ 180

$\angle 2 = 61$
$\angle 3 = 61$
$\angle 4 = 29$

$61 + 90 - 151 = 29$



$\angle BCD = 61 + 61 = 122$

$\angle ADC = 29 + 29 = 58^\circ$

Students, draw anywhere on this slide!

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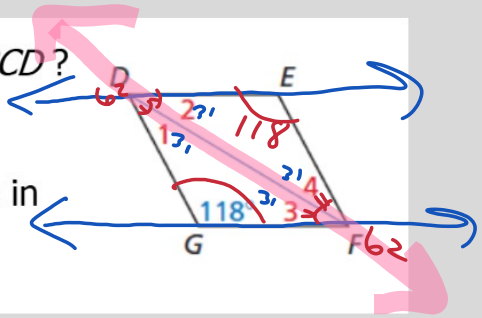
4. In Example 3, what is $m\angle ADC$ and $m\angle BCD$?

$360 - (118 + 118) = 124/2 = 62$

5. Find the measures of the numbered angles in rhombus *DEFG*.

each $= 31^\circ$



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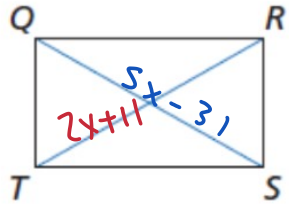
$5(14) - 31 = 39$ $2(14) + 11 = 39$

In rectangle *QRST*, $QS = 5x - 31$ and $RT = 2x + 11$.
Find the lengths of the diagonals of *QRST*.

$5x - 31 = 2x + 11$
$-2x \quad -2x$
$3x - 31 = 11$
$+31 \quad +31$



$\frac{3x}{3} = \frac{42}{3}$ $(x = 14)$

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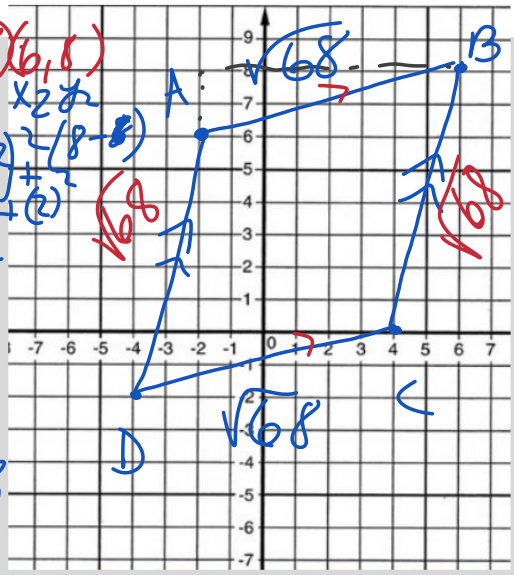
Decide whether ▫*ABCD* with vertices *A* (−2, 6), *B* (6, 8), *C* (4, 0), and *D* (−4, −2) is a *rectangle* , a *rhombus* , or a *square* . Give all names that apply.



Slopes
AB 1/4
DC 1/4
AD 4
BC 4
negative Reciprocals
NOT 90°

(−2, 6) (6, 8)
$(6 - -2)^2 + (8 - 6)^2$
$8^2 +$

(6, 8) (4, 0)
Parallelogram Rh

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$(4 - 6)^2 + (0 - 8)^2$
$(-2)^2 + (8)^2$
$4 + 64$
68

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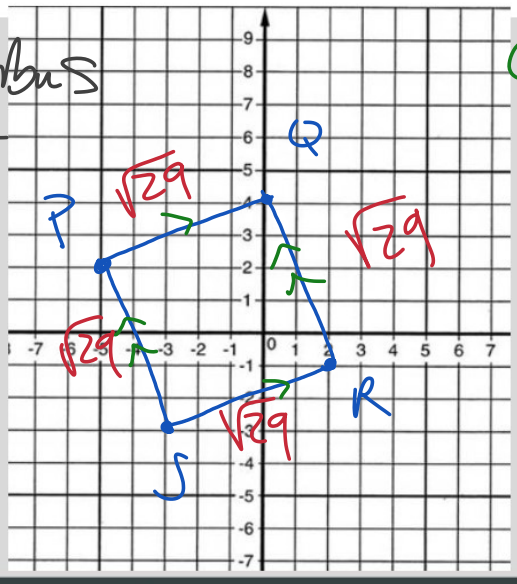
8. Decide whether ▫*PQRS* with vertices *P* (−5, 2), *Q* (0, 4), *R* (2, −1), and *S* (−3, −3) is a *rectangle* , a *rhombus* , or a *square* . Give all names that apply.



rise/run

Slopes
PQ 2/5
SR 2/5
PS − 5/2
QR − 5/2
neg. reciprocal
90° ✓
perp.

Rhombus
Square

(0, 4) (2, −1)
Rectangle
Parallelogram

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$(2 - 0)^2 + (-1 - 4)^2$
$2^2 + (-5)^2$
$4 + 25$
$\sqrt{29}$

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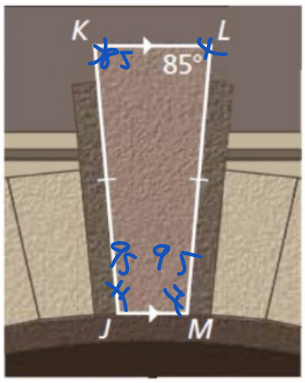
The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.



85 + 85 = 170

360 − 170 = 190

$\frac{190}{2} = 95$

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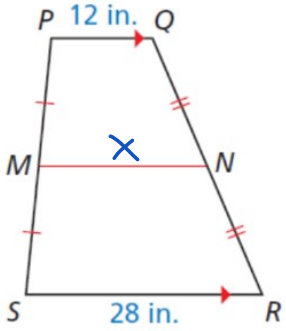
In the diagram, $\overline{MN}$ is the midsegment of trapezoid $PQRS$. Find MN.

$x = \frac{1}{2}(PQ + SR)$

$x = \frac{1}{2}(12 + 28)$

$x = \frac{1}{2}(40) = 20$



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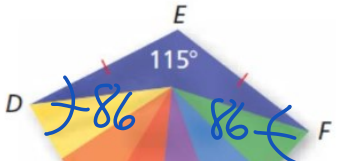
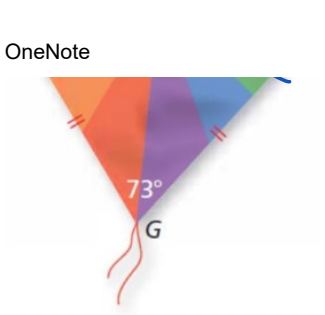
Find $m\angle D$ in the kite shown.

115 + 73 = 188

360 − 188 = 172



$\frac{1}{2} \cdot 12 \cdot 16 = 96$



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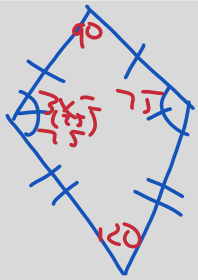
6. In a kite, the measures of the angles are $3x°$, $75°$, $90°$, and $120°$. Find the value of x. What are the measures of the angles that are congruent? 75°

$+ \quad + \quad + = 360$



$3x + 285 = 360$
$-285 \quad -285$
$\frac{3x}{3} = \frac{25}{3}$
$x = 25$

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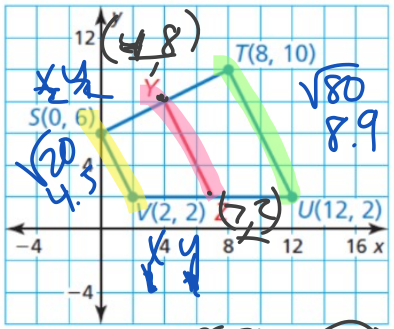
$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$a^2 + b^2 = c^2$

Find the length of midsegment $\overline{YZ}$ in trapezoid $STUV$.

$YZ = \frac{1}{2}(SV + TU)$



$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(0 - 2)^2 + (6 - 2)^2$

$(-2)^2 + (4)^2$

$4 + 16$

$\sqrt{20}$

$\frac{4.5 + 8.9}{2} = 6.7$

$\sqrt{80}$
8.9

$\sqrt{20}$
4.5

$(7, 2)(4,$
$x_1 \; y_1 \; x_2$
$\sqrt{(4-7)^2 + (8}$
$(-3)^2 + (6$
$9 + 36$
$\sqrt{45} =$

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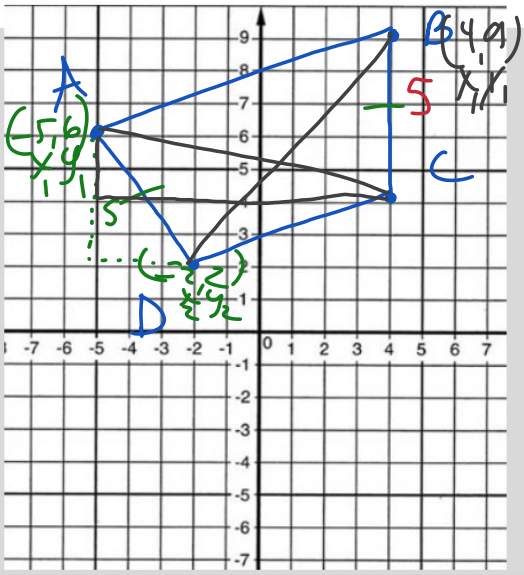
1. The points $A(-5, 6)$, $B(4, 9)$, $C(4, 4)$, and $D(-2, 2)$ form the vertices of a quadrilateral. Show that $ABCD$ is a trapezoid. Then decide whether it is isosceles.



yes

$a^2+b^2=c^2$

Is $AD = BC$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(-2+5)^2+(2-6)^2$

$3^2+(-4)^2$

$9+16$

$\sqrt{25}=5$

Does $AC = BD$ yes

2^2+9^2

$4+81$

$\sqrt{85}$

$(-2-4)^2+(2-9)^2$

-6^2+7^2

$36+49$

$\sqrt{85}$

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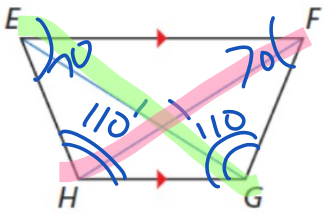
In Exercises 2 and 3, use trapezoid *EFGH*.

2. If $EG = FH$, is trapezoid *EFGH* isosceles? Explain.

yes

diagonals ≅

3. If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid *EFGH* isosceles? Explain.



$70+70+110+110 = 360$?

$360 = 360$ ✓

Base ∠'s ≅

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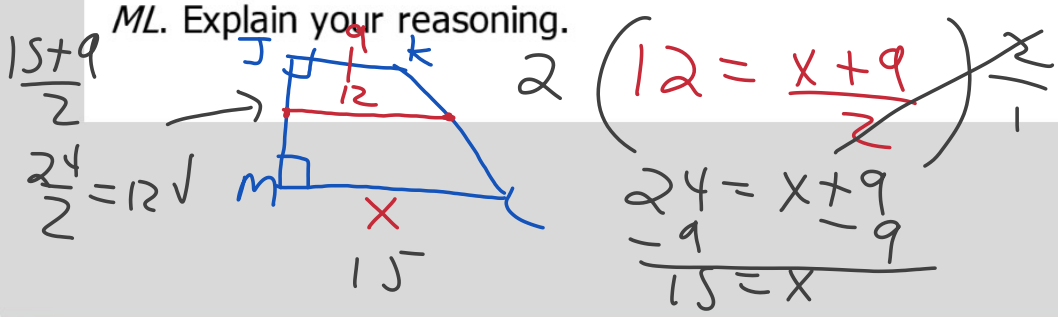
4. In trapezoid $JKLM$, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ centimeters. The length of midsegment $\overline{NP}$ of trapezoid $JKLM$ is 12 centimeters. Sketch trapezoid $JKLM$ and its midsegment. Find ML. Explain your reasoning.



$\frac{15+9}{2}$ → $\frac{24}{2} = 12\sqrt{}$

$2\left(12 = \frac{x+9}{2}\right)2$

$24 = x + 9$

$-9 \quad -9$

$15 = x$

Students, draw anywhere on this slide!

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