

Chapter 7 Quadrilaterals

Sunday, March 27, 2022 6:44 PM

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Quadrilat...

CHAPTER 7: QUADRILATERALS

Geometry

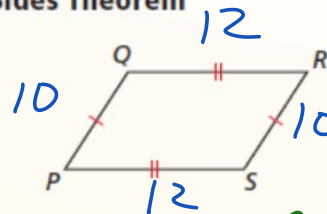
Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 368

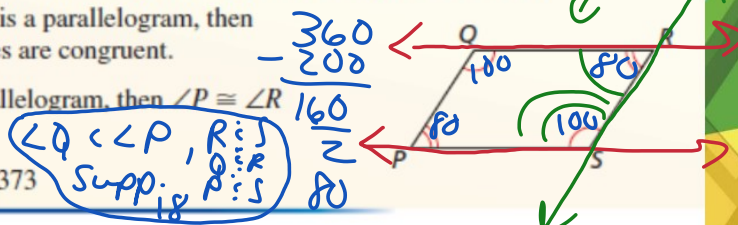


Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 373





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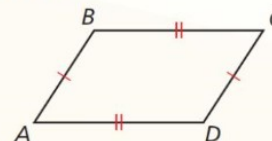
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Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

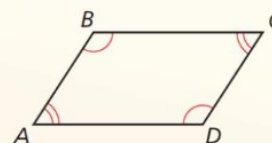


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383



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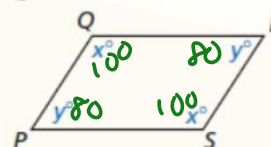
Theorems

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x^\circ + y^\circ = 180^\circ$.

Proof Ex. 38, p. 373

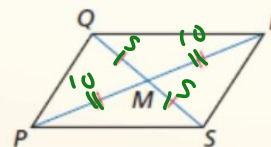


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 370



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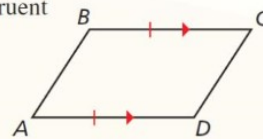
Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof Ex. 40, p. 383

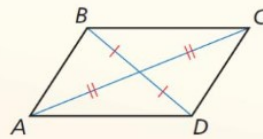


Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If \overline{BD} and \overline{AC} bisect each other, then $ABCD$ is a parallelogram.

Proof Ex. 41, p. 383



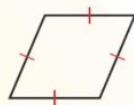
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Core Concept

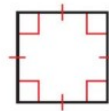
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.



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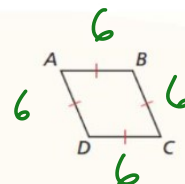
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

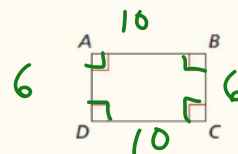


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396



Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it has four congruent sides and four right angles.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396

Proof Ex. 84, p. 396

Proof Ex. 85, p. 396

Proof Ex. 86, p. 396

Proof Ex. 87, p. 396

Proof Ex. 88, p. 396

Proof Ex. 89, p. 396

Proof Ex. 90, p. 396

Proof Ex. 91, p. 396

Proof Ex. 92, p. 396

Proof Ex. 93, p. 396

Proof Ex. 94, p. 396

Proof Ex. 95, p. 396

Proof Ex. 96, p. 396

Proof Ex. 97, p. 396

Proof Ex. 98, p. 396

Proof Ex. 99, p. 396

Proof Ex. 100, p. 396

Proof Ex. 101, p. 396

Proof Ex. 102, p. 396

Proof Ex. 103, p. 396

Proof Ex. 104, p. 396

Proof Ex. 105, p. 396

Proof Ex. 106, p. 396

Proof Ex. 107, p. 396

Proof Ex. 108, p. 396

Proof Ex. 109, p. 396

Proof Ex. 110, p. 396

Proof Ex. 111, p. 396

Proof Ex. 112, p. 396

Proof Ex. 113, p. 396

Proof Ex. 114, p. 396

Proof Ex. 115, p. 396

Proof Ex. 116, p. 396

Proof Ex. 117, p. 396

Proof Ex. 118, p. 396

Proof Ex. 119, p. 396

Proof Ex. 120, p. 396

Proof Ex. 121, p. 396

Proof Ex. 122, p. 396

Proof Ex. 123, p. 396

Proof Ex. 124, p. 396

Proof Ex. 125, p. 396

Proof Ex. 126, p. 396

Proof Ex. 127, p. 396

Proof Ex. 128, p. 396

Proof Ex. 129, p. 396

Proof Ex. 130, p. 396

Proof Ex. 131, p. 396

Proof Ex. 132, p. 396

Proof Ex. 133, p. 396

Proof Ex. 134, p. 396

Proof Ex. 135, p. 396

Proof Ex. 136, p. 396

Proof Ex. 137, p. 396

Proof Ex. 138, p. 396

Proof Ex. 139, p. 396

Proof Ex. 140, p. 396

Proof Ex. 141, p. 396

Proof Ex. 142, p. 396

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Proof Ex. 180, p. 396

Proof Ex. 181, p. 396

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Proof Ex. 183, p. 396

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Proof Ex. 198, p. 396

Proof Ex. 199, p. 396

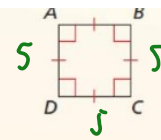
Proof Ex. 200, p. 396

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if

$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



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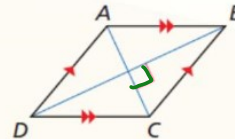
Theorems

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

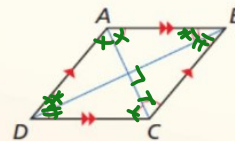


Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if \overline{AC} bisects $\angle BCD$ and $\angle BAD$, and \overline{BD} bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395



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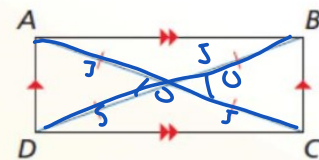
Theorem

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



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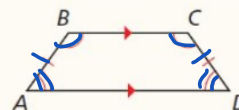
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

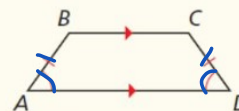


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof Ex. 40, p. 405

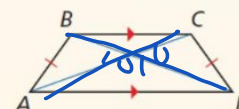


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



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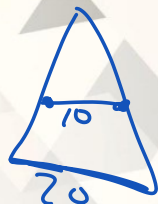
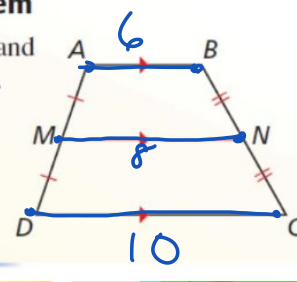
Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If \overline{MN} is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 406



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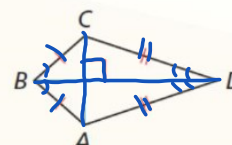
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

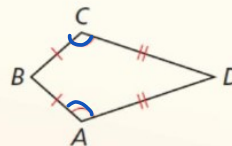


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.

Proof Ex. 47, p. 406



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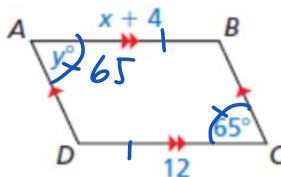
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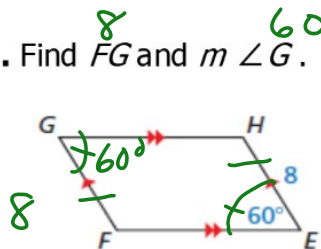
Find the values of x and y .

$$\begin{array}{r} x + y = 12 \\ -y \quad -y \\ \hline x = 8 \end{array}$$

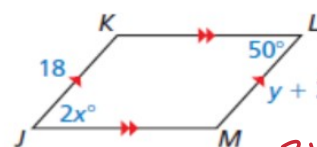
65°



1. Find FG and $m\angle G$.



2. Find the values of x and y .



$$\begin{array}{r} y + 3 = 18 \\ y - 3 = 3 \\ \hline y = 15 \end{array}$$

$$\begin{array}{r} 2x = 50 \\ \frac{2x}{2} = \frac{50}{2} \\ x = 25 \end{array}$$



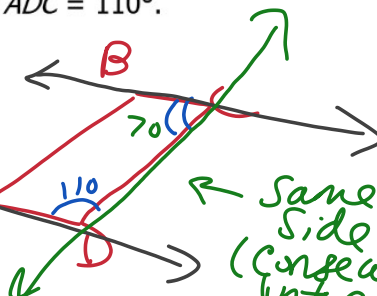
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As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110^\circ$.

$$180 - 110 = 70$$



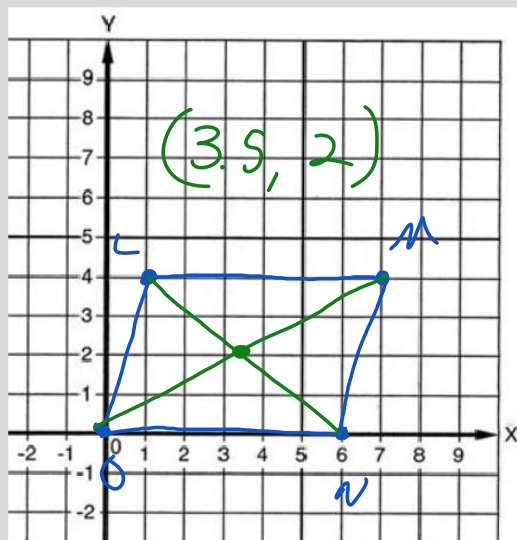
Same Side (consecutive) interior angles
Supplementary = 180



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Find the coordinates of the intersection of the diagonals of $\square LMNO$ with vertices $L(1, 4)$, $M(7, 4)$, $N(6, 0)$, and $O(0, 0)$.



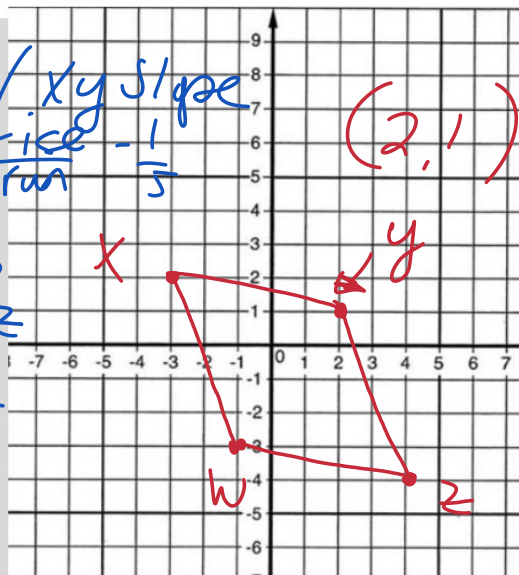
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Three vertices of $\square WXYZ$ are $W(-1, -3)$, $X(-3, 2)$, and $Z(4, -4)$. Find the coordinates of vertex Y .

Slope of \overline{XW}
Rise -5
Run 2

✓ \overline{XY} slope
Rise -1
Run 5
✓ Slope \overline{WZ}
 $-\frac{1}{5}$



15
From Z \uparrow up 5
Left 2
 $\leftarrow -2$

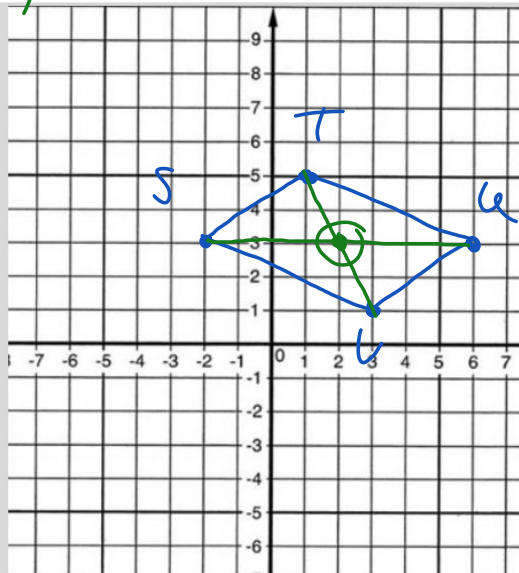


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5. Find the coordinates of the intersection of the diagonals of $\square STUV$ with vertices $S(-2, 3)$, $T(1, 5)$, $U(6, 3)$, and $V(3, 1)$.

(2,3)



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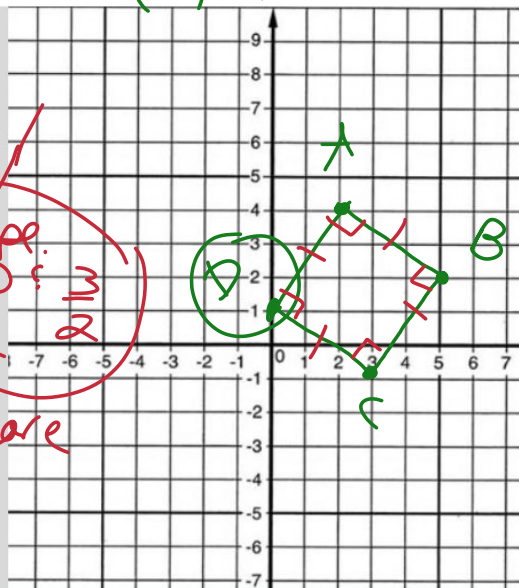
6. Three vertices of $\square ABCD$ are $A(2, 4)$, $B(5, 2)$, and $C(3, -1)$. Find the coordinates of vertex D .

(0,1)

Slope of \overline{AB}
Rise -2
Run 3

negative
reciprocals
perpendicular
Rectangle 90° Square

Slope:
 $\overline{AD} = 3$
 $\overline{BC} = 2$



From C $\uparrow 2$
left 3
 $-\frac{2}{3}$



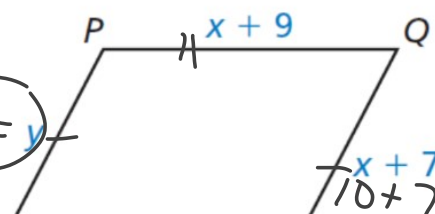
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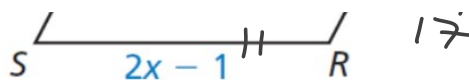
For what values of x and y is quadrilateral $PQRS$ a parallelogram?

$$\begin{array}{r} x+9 = 2x-1 \\ +1 \quad +1 \\ \hline x+10 = 2x \\ -x \quad -x \\ \hline 10 = x \end{array}$$

$$17 = y$$



$$10 = x$$

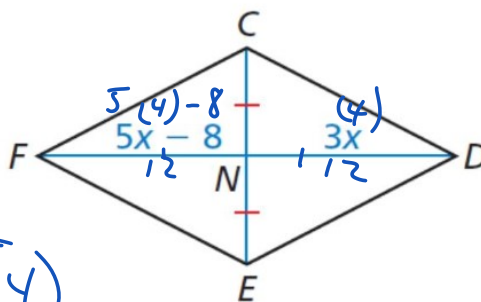


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For what value of x is quadrilateral $CDEF$ a parallelogram?

$$\begin{array}{r} 5x - 8 = 3x \\ +8 \quad +8 \\ \hline 5x = 3x + 8 \\ -3x \quad -3x \\ \hline 2x = 8 \\ \frac{2x}{2} = \frac{8}{2} \quad (x=4) \end{array}$$



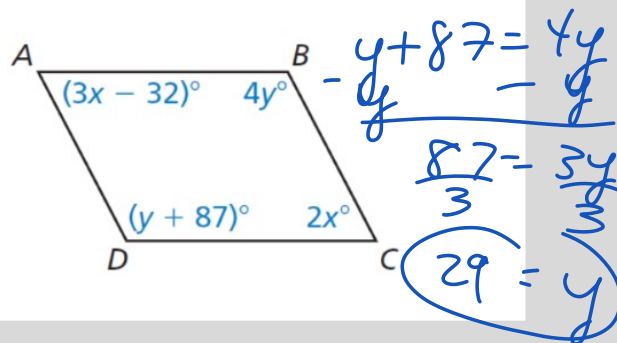
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2. For what values of x and y is quadrilateral $ABCD$ a parallelogram? Explain your reasoning.

$$\begin{array}{r} 3x - 32 = 2x \\ +32 \quad +32 \\ \hline 3x = 2x + 32 \\ -2x \quad -2x \\ \hline x = 32 \end{array}$$



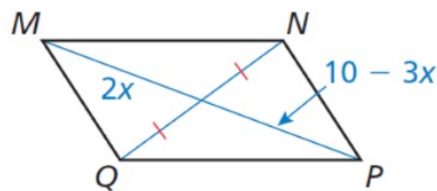


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6. For what value of x is quadrilateral $MNPQ$ a parallelogram?
 Explain your reasoning.

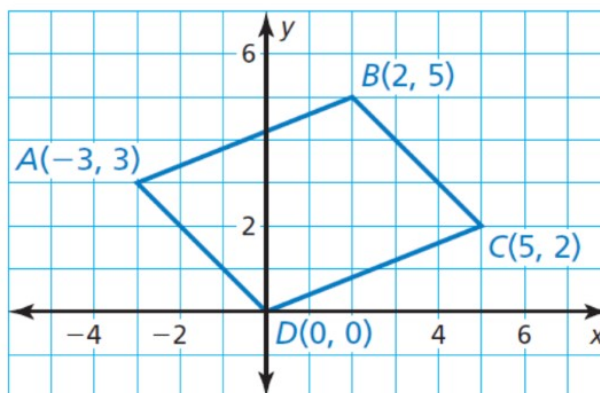
$$\begin{array}{r} 2x = 10 - 3x \\ + 3x \quad + 3x \\ \hline 5x = 10 \\ \frac{5x}{5} = \frac{10}{5} \quad \boxed{x=2} \end{array}$$



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Show that quadrilateral $ABCD$ is a parallelogram.



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Find the measures of the numbered angles in rhombus $ABCD$.