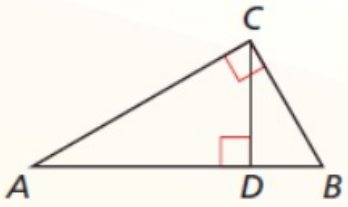
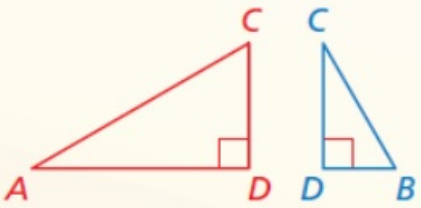
Theorem

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



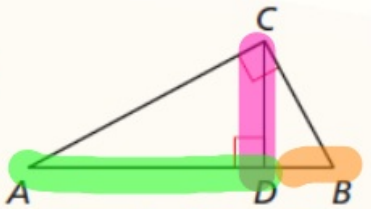
Proof Ex. 45, p. 484

Theorems

Theorem 9.7 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.



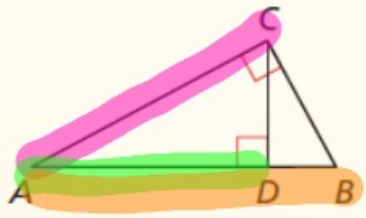
$CD^2 = AD \cdot BD$

Proof Ex. 41, p. 484

Theorem 9.8 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

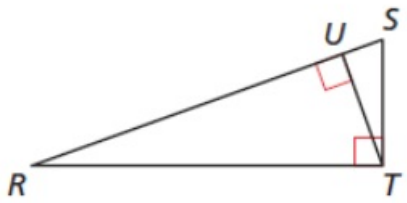


$CB^2 = DB \cdot AB$

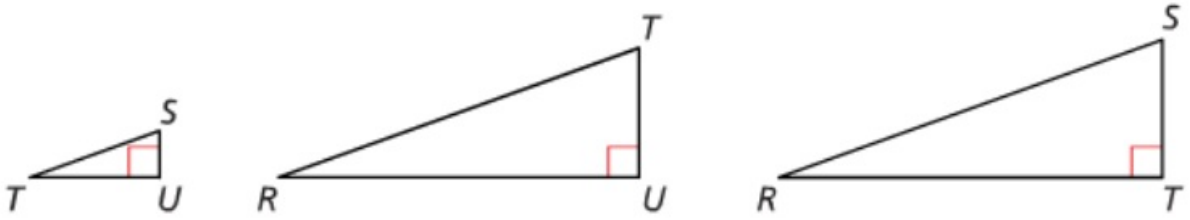
$AC^2 = AD \cdot AB$

Proof Ex. 42, p. 484

Identify the similar triangles in the diagram.

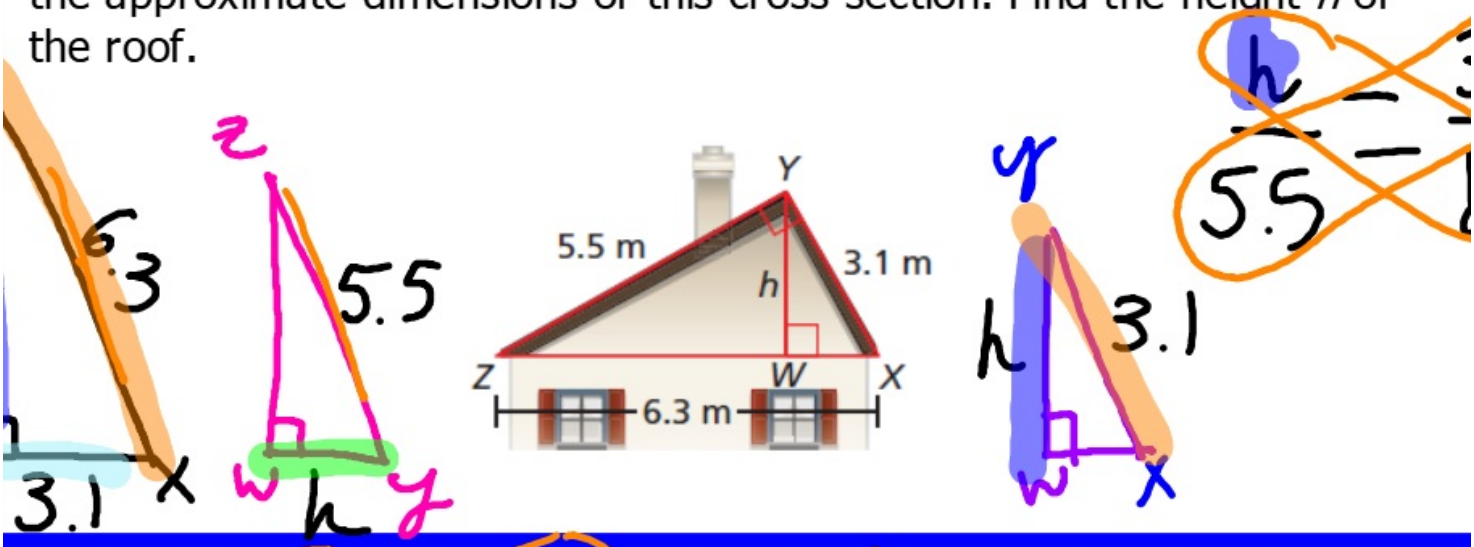


Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



▶ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the roof.



$$\frac{h}{5.5} = \frac{3.1}{6.3}$$

$$\frac{6.3h}{6.3} = \frac{17.05}{6.3}$$

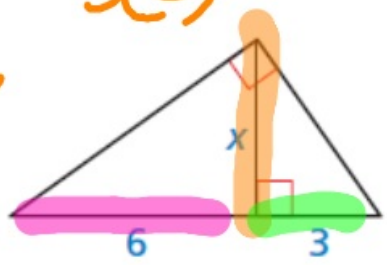
$$h = 2.7$$

Find the value of each variable.

$x^2 = 3(6)$

$y^2 = 2(7)$

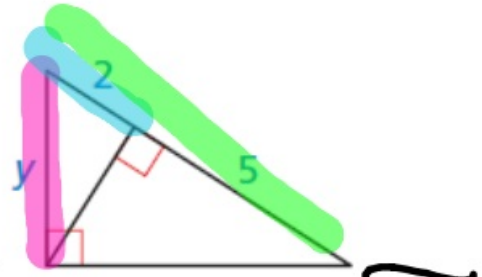
a.



$\therefore 18$

$\sqrt{18}$

b.



$y^2 = 14$

$y = \sqrt{14}$

SOLUTION

a. Apply the Geometric Mean (Altitude) Theorem.

$$x^2 = 6 \cdot 3$$

$$x^2 = 18$$

$$x = \sqrt{18}$$

$$x = \sqrt{9} \cdot \sqrt{2}$$

$$x = 3\sqrt{2}$$

▶ The value of x is $3\sqrt{2}$.

b. Apply the Geometric Mean (Leg) Theorem.

$$y^2 = 2 \cdot (5 + 2)$$

$$y^2 = 2 \cdot 7$$

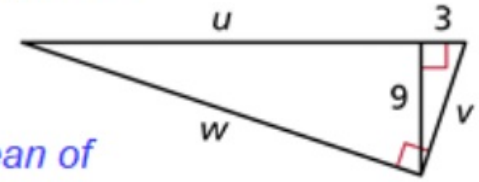
$$y^2 = 14$$

$$y = \sqrt{14}$$

▶ The value of y is $\sqrt{14}$.

Check It Out! Example 3

Find *u*, *v*, and *w*.



$9^2 = (3)(u)$ *9 is the geometric mean of u and 3.*

$u = 27$ *Divide both sides by 3.*

$w^2 = (27 + 3)(27)$ *w is the geometric mean of u + 3 and 27.*

$w = \sqrt{810} = 9\sqrt{10}$ *Find the positive square root.*

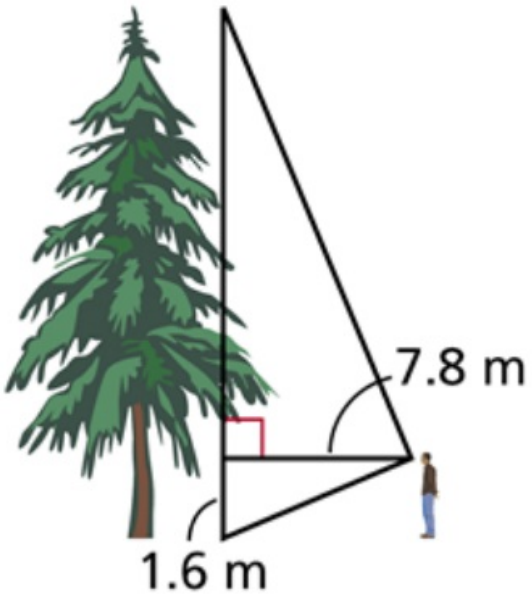
$v^2 = (27 + 3)(3)$ *v is the geometric mean of u + 3 and 3.*

$v = \sqrt{90} = 3\sqrt{10}$ *Find the positive square root.*

8-1 Similarity in Right Triangles

Example 4: Measurement Application

To estimate the height of a Douglas fir, Jan positions herself so that her lines of sight to the top and bottom of the tree form a 90° angle. Her eyes are about 1.6 m above the ground, and she is standing 7.8 m from the tree. What is the height of the tree to the nearest meter?



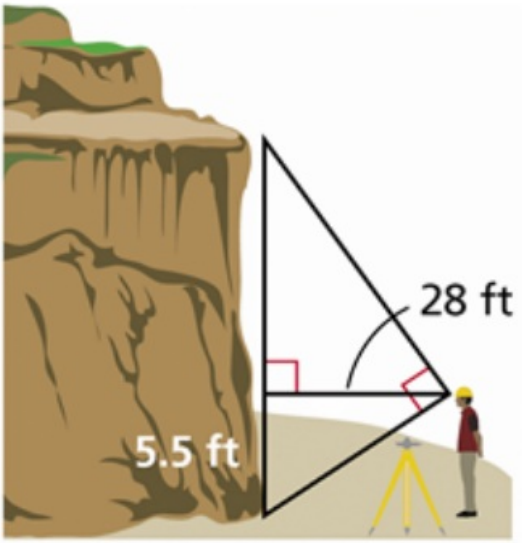
8-1 Similarity in Right Triangles

Example 4 Continued

8-1 Similarity in Right Triangles

Check It Out! Example 4

A surveyor positions himself so that his line of sight to the top of a cliff and his line of sight to the bottom form a right angle as shown. What is the height of the cliff to the nearest foot?



8-1 Similarity in Right Triangles

Check It Out! Example 4 Continued

To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall. You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the horizontal distance from you to the gym wall. Approximate the height of the gym wall.