Lesson 4.5 - Dilations in the Coordinate Plane

Learning Intent (Target): _Today I will_ be able to graph polygons in the coordinate plane using transformations.

Success Criteria: _I'll know I'll have it when_ I can accuratley graph dilations in the coordinate plane and determine the scale factor.

Accountable Team Task: _Therefore, I can_ practice using interactive flip charts for notes & invstigations using gizmos to graph transformations and determine scale factors.
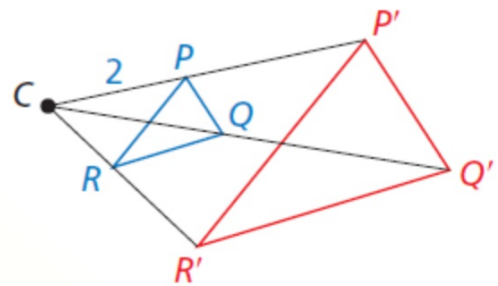
🌀 Core Concept

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.
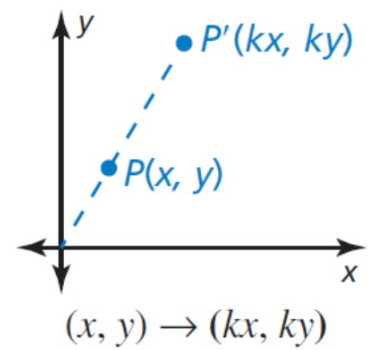
- If P is the center point C, then $P = P'$.

- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \dfrac{CP'}{CP}$.

- Angle measures are preserved.
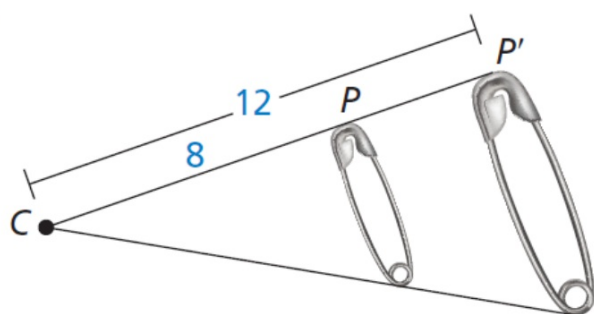
Core Concept

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.
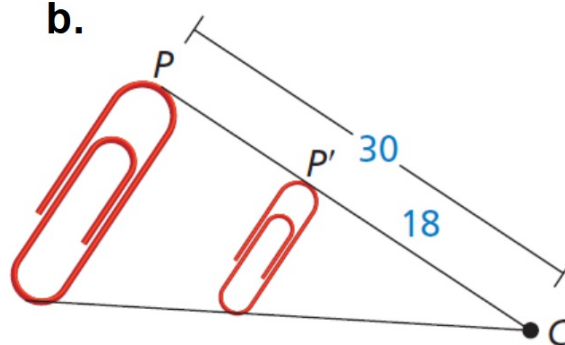

$(x, y) \rightarrow (kx, ky)$

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a.

b.

SOLUTION

a. Because $\dfrac{CP'}{CP} = \dfrac{12}{8}$, the scale factor is $k = \dfrac{3}{2}$. So, the dilation is an enlargement.

b. Because $\dfrac{CP'}{CP} = \dfrac{18}{30}$, the scale factor is $k = \dfrac{3}{5}$. So, the dilation is a reduction.

Graph △*ABC* with vertices *A*(2, 1), *B*(4, 1), and *C*(4, −1) and its image after a dilation with a scale factor of 2.

SOLUTION
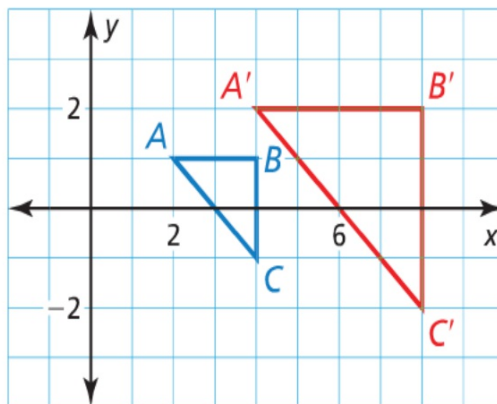
Use the coordinate rule for a dilation with $k = 2$ to find the coordinates of the vertices of the image. Then graph △*ABC* and its image.

$$(x, y) \rightarrow (2x, 2y)$$

$$A(2, 1) \rightarrow A'(4, 2)$$

$$B(4, 1) \rightarrow B'(8, 2)$$

$$C(4, -1) \rightarrow C'(8, -2)$$

Graph quadrilateral *KLMN* with vertices *K*(−3, 6), *L*(0, 6), *M*(3, 3), and *N*(−3, −3) and its image after a dilation with a scale factor of $\dfrac{1}{3}$.

SOLUTION

Use the coordinate rule for a dilation with $k = \frac{1}{3}$ to find the coordinates of the vertices of the image. Then graph quadrilateral *KLMN* and its image.
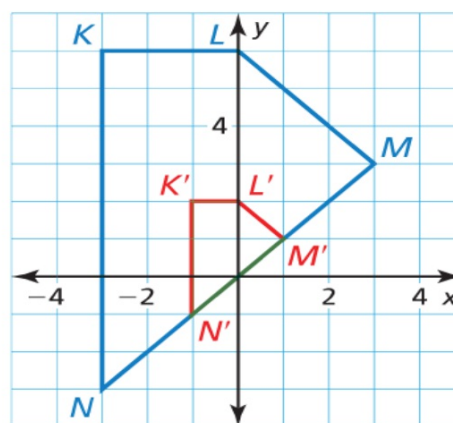
$$(x, y) \rightarrow \left(\tfrac{1}{3}x, \tfrac{1}{3}y\right)$$

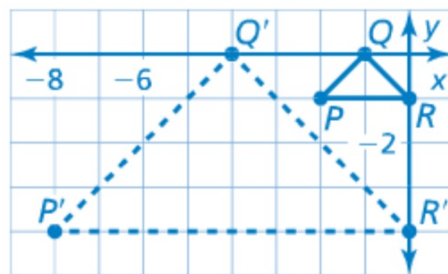$$K(-3, 6) \rightarrow K'(-1, 2)$$
$$L(0, 6) \rightarrow L'(0, 2)$$
$$M(3, 3) \rightarrow M'(1, 1)$$
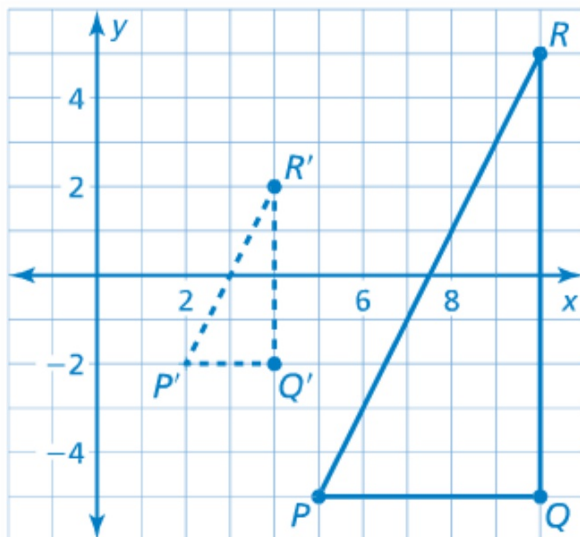$$N(-3, -3) \rightarrow N'(-1, -1)$$

Graph △*PQR* and its image after a dilation with scale factor *k*.

2. $P\,(-2,\,-1)$, $Q(-1,\,0)$, $R(0,\,-1)$; $k = 4$

3. $P\,(5,\,-5)$, $Q(10,\,-5)$, $R(10,\,5)$; $k = 0.4$

Graph $\triangle FGH$ with vertices $F(-4, -2)$, $G(-2, 4)$, and $H(-2, -2)$ and its image after a dilation with a scale factor of $-\dfrac{1}{2}$.
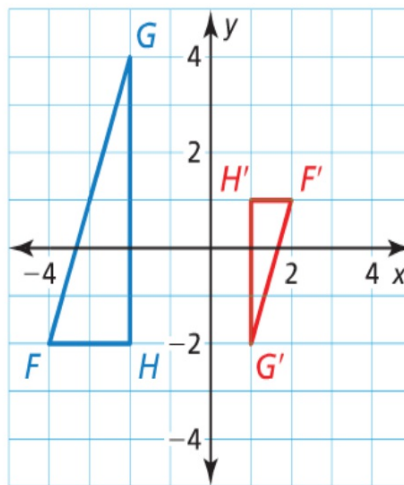
Use the coordinate rule for a dilation with $k = -\dfrac{1}{2}$ to find the coordinates of the vertices of the image. Then graph $\triangle FGH$ and its image.

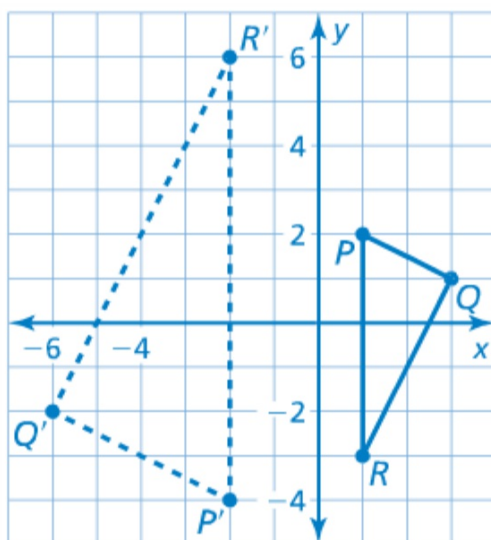$$(x, y) \rightarrow \left(-\tfrac{1}{2}x, -\tfrac{1}{2}y\right)$$

$$F(-4, -2) \rightarrow F'(2, 1)$$

$$G(-2, 4) \rightarrow G'(1, -2)$$

$$H(-2, -2) \rightarrow H'(1, 1)$$

4. Graph $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 1)$, and $R(1, -3)$ and its image after a dilation with a scale factor of -2.

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?
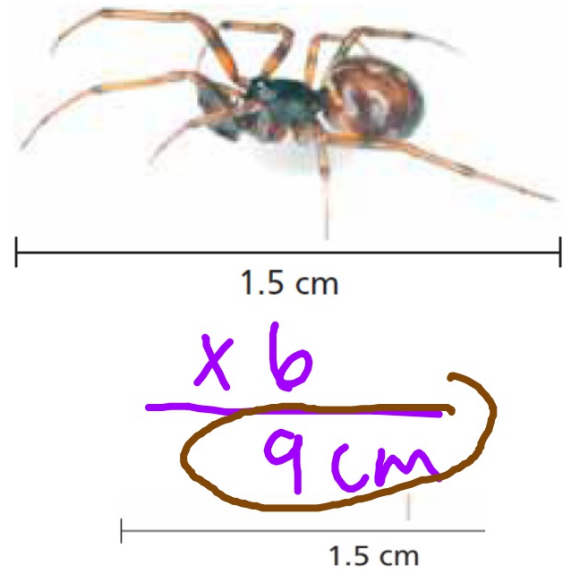
SOLUTION

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\dfrac{1.1 \text{ in.}}{4 \text{ in.}}$.

▶ So, in simplest form, the scale factor is $\dfrac{11}{40}$.

4 in.

1.1 in.

You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.

1.5 cm

$$\frac{\times 6}{9 \text{ cm}}$$

1.5 cm

SOLUTION

$$\frac{\text{image length}}{\text{actual length}} = k$$

$$\frac{x}{1.5} = 6$$

$$x = 9$$

▶ So, the image length through the magnifying glass is 9 centimeters.

6. An optometrist dilates the pupils of a patient's eyes to get a better look at the back of the eyes. A pupil dilates from 4.5 millimeters to 8 millimeters. What is the scale factor of this dilation?

enlargement $\frac{\text{new}}{\text{original}}$ $\frac{8}{4.5} \times \frac{\times 2}{\times 2} = \frac{16}{9}$ 1.7

7. The image of a spider seen through the magnifying glass in Example 6 is shown. Find the actual length of the spider.

2.1 cm

12.6 cm

reduce

$\frac{\text{new}}{\text{orig.}} = \frac{12.6}{6} = 2.1$