Date: 10/23/20

Lesson 3.5 Equations of Parallel & Perpendicular Lines

Learning Intent (Target): *Today I will* be able to discover properties of parallel & perpendicular lines.

Success Criteria: *I'll know I'll have it when* I'll be able to use properties about parallel & perpendicular lines to write equations of parallel & perpendicular lines.

Accountable Team Task: *Therefore, I can* practice from interactive flip charts and matching card sort activities.

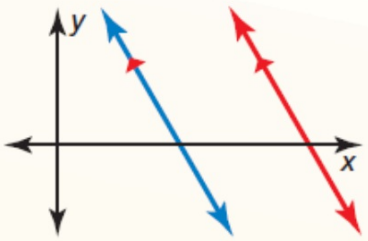
Theorems

Theorem 3.13 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Proof p. 439; Ex. 41, p. 444



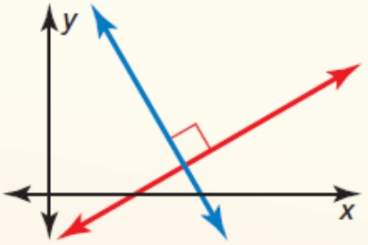
$m_1 = m_2$

Theorem 3.14 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

Proof p. 440; Ex. 42, p. 444



$m_1 \cdot m_2 = -1$

Write an equation of the line passing through the point (−1, 1) that is parallel to the line $y = 2x - 3$.

SOLUTION

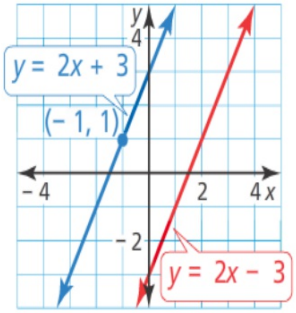
Step 1 Find the slope m of the parallel line. The line $y = 2x - 3$ has a slope of 2. By the Slopes of Parallel Lines Theorem, a line parallel to this line also has a slope of 2. So, $m = 2$.

Step 2 Find the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$y = mx + b$	Use slope-intercept form.
$1 = 2(-1) + b$	Substitute for m, x, and y.
$3 = b$	Solve for b.

▶ Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$. Use a graph to check that the line $y = 2x - 3$ is parallel to the line $y = 2x + 3$.

Check



Write an equation of the line passing through the point (2, 3) that is perpendicular to the line $2x + y = 2$.

SOLUTION

Step 1 Find the slope m of the perpendicular line. The line $2x + y = 2$, or $y = -2x + 2$, has a slope of -2. Use the Slopes of Perpendicular Lines Theorem.

$-2 \cdot m = -1$ The product of the slopes of $\perp$ lines is -1.

$m = \frac{1}{2}$ Divide each side by -2.

Step 2 Find the y-intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

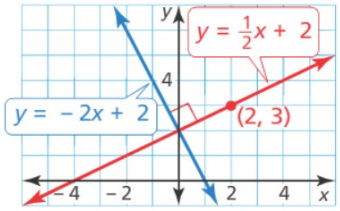
$y = mx + b$ Use slope-intercept form.

$3 = \frac{1}{2}(2) + b$ Substitute for m, x, and y.

$2 = b$ Solve for b.

▶ Because $m = \frac{1}{2}$ and $b = 2$, an equation of the line is $y = \frac{1}{2}x + 2$. Check that the lines are perpendicular by graphing their equations and using a protractor to measure one of the angles formed by their intersection.

Check



4. Write an equation of the line that passes through the point (1, 5) and is (a) parallel to the line $y = 3x - 5$ and (b) perpendicular to the line $y = 3x - 5$.

5. How do you know that the lines $x = 4$ and $y = 2$ are perpendicular?

4. Write an equation of the line that passes through the point (1, 5) and is (a) parallel to the line $y = 3x - 5$ and (b) perpendicular to the line $y = 3x - 5$.

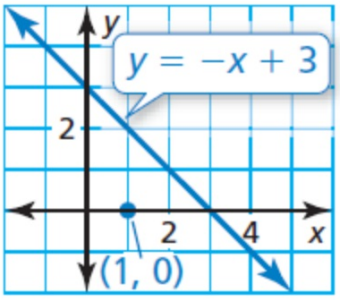
a. $y = 3x + 2$

b. $y = -\frac{1}{3}x + \frac{16}{3}$

5. How do you know that the lines $x = 4$ and $y = 2$ are perpendicular?

Line $x = 4$ is a vertical line, and line $y = 2$ is a horizontal line. So, they are perpendicular by the Slopes of Perpendicular Lines Theorem (Thm. 3.14).

Find the distance from the point (1, 0) to the line $y = -x + 3$.



Work Shown on Next Page

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SOLUTION

Step 1 Find the equation of the line perpendicular to the line $y = -x + 3$ that passes through the point $(1, 0)$.

First, find the slope m of the perpendicular line. The line $y = -x + 3$ has a slope of -1. Use the Slopes of Perpendicular Lines Theorem.

$-1 \cdot m = -1$ The product of the slopes of $\perp$ lines is -1.

$m = 1$ Divide each side by -1.

Then find the y-intercept b by using $m = 1$ and $(x, y) = (1, 0)$.

$y = mx + b$ Use slope-intercept form.

$0 = 1(1) + b$ Substitute for x, y, and m.

$-1 = b$ Solve for b.

Because $m = 1$ and $b = -1$, an equation of the line is $y = x - 1$.

Step 2 Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

$y = -x + 3$ Equation 1

$y = x - 1$ Equation 2

Substitute $-x + 3$ for y in Equation 2.

$y = x - 1$ Equation 2

$-x + 3 = x - 1$ Substitute $-x + 3$ for y.

$x = 2$ Solve for x.

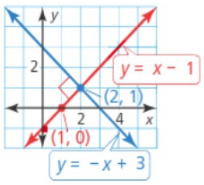
Substitute 2 for x in Equation 1 and solve for y.

$y = -x + 3$ Equation 1

$y = -2 + 3$ Substitute 2 for x.

$y = 1$ Simplify.

So, the perpendicular lines intersect at $(2, 1)$.



Step 3 Use the Distance Formula to find the distance from $(1, 0)$ to $(2, 1)$.

$$\text{distance} = \sqrt{(1 - 2)^2 + (0 - 1)^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \approx 1.4$$

▶ So, the distance from the point $(1, 0)$ to the line $y = -x + 3$ is about 1.4 units.

6. Find the distance from the point (6, 4) to the line $y = x + 4$.

7. Find the distance from the point $(-1, 6)$ to the line $y = -2x$.

6. Find the distance from the point $(6, 4)$ to the line $y = x + 4$.
about 4.2 units

7. Find the distance from the point $(-1, 6)$ to the line $y = -2x$.
about 1.8 units