Date: 10/13/20

Lesson 2.4-2.6: Reasoning and Proofs

Learning Intent (Target): *Today I will* be able to use algebraic and geometric properties to justify steps in problem solving.

Success Criteria: *I'll know I'll have it when* I'll be able to complete 2 column and flow chart proofs using properties.

Accountable Team Task: *Therefore, I can* practice writing proofs from interactive flip charts and apply it to problem solving.

Core Concept

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

Core Concept

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Distributive Property

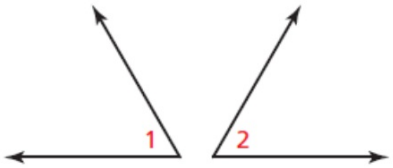
Let a, b, and c be real numbers.

Sum $a(b + c) = ab + ac$ **Difference** $a(b - c) = ab - ac$

Concept Summary

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

statements based on facts that you know or on conclusions from deductive reasoning

definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

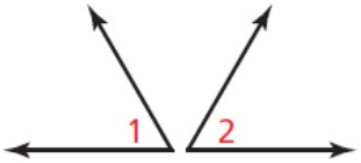
Concept Summary

Types of Proofs

Symmetric Property of Angle Congruence (Theorem 2.2)

Given $\angle 1 \cong \angle 2$

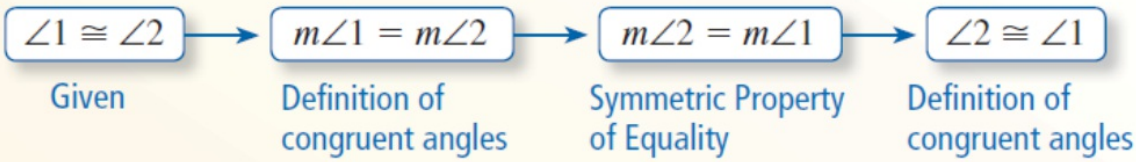
Prove $\angle 2 \cong \angle 1$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Flowchart Proof



Paragraph Proof

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

Name the property of equality that the statement illustrates.

7. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.

8. $34° = 34°$

9. $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 5$. So, $m\angle 1 = m\angle 5$.

Name the property of equality that the statement illustrates.

7. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.

Symmetric Property of Equality

8. $34° = 34°$

Reflexive Property of Equality

9. $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 5$. So, $m\angle 1 = m\angle 5$.

Transitive Property of Equality

Hide Answers

Name the property of equality that the statement illustrates.

10. If $JK = KL$ and $KL = 16$, then $JK = 16$.

11. $PQ = ST$, so $ST = PQ$.

12. $ZY = ZY$

10. If $JK = KL$ and $KL = 16$, then $JK = 16$. Transitive Property of Equality

11. $PQ = ST$, so $ST = PQ$. Symmetric Property of Equality

12. $ZY = ZY$ Reflexive Property of Equality

State the Property of Equality each statement illustrates.

a. If $AB = CD$, then $AB + FE = CD + FE$.

b. If $m\angle 2 = m\angle 4$ and $m\angle 4 = m\angle 6$, then $m\angle 2 = m\angle 6$.

c. If $XY = AB$, then $AB = XY$.

a. If $AB = CD$, then $AB + FE = CD + FE$.
Addition Property of Equality

b. If $m\angle 2 = m\angle 4$ and $m\angle 4 = m\angle 6$, then $m\angle 2 = m\angle 6$.
Transitive Property of Equality

c. If $XY = AB$, then $AB = XY$.
Symmetric Property of Equality

Solve the equation. Justify each step.

5. $4 = -10b + 6(2 - b)$

6. Solve the formula $A = \frac{1}{2}bh$ for b. Justify each step. Then find the base of a triangle whose area is 952 square feet and whose height is 56 feet.

5. $4 = -10b + 6(2 - b)$

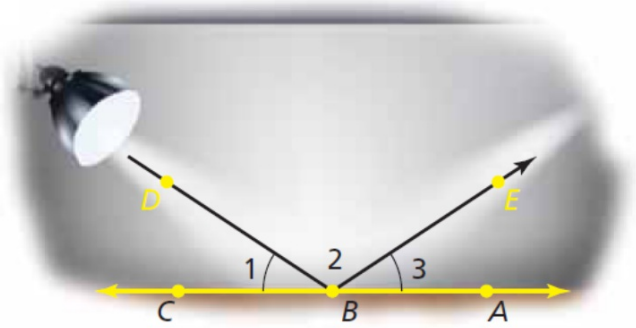
Equation	Explanation and Reason
$4 = -10b + 6(2 - b)$	Write the equation; Given
$4 = -10b + 12 - 6b$	Multiply; Distributive Property
$4 = -16b + 12$	Combine like terms; Simplify.
$-8 = -16b$	Subtract 12 from each side; Subtraction Property of Equality
$\frac{1}{2} = b$	Divide each side by -16; Division Property of Equality
$b = \frac{1}{2}$	Rewrite the equation; Symmetric Property of Equality

6. Solve the formula $A = \frac{1}{2}bh$ for b. Justify each step. Then find the base of a triangle whose area is 952 square feet and whose height is 56 feet.

Equation	Explanation and Reason
$A = \frac{1}{2}bh$	Write the equation; Given
$2A = bh$	Multiply each side by 2; Multiplication Property of Equality
$\frac{2A}{h} = b$	Divide each side by h; Division Property of Equality
$b = \frac{2A}{h}$	Rewrite the equation; Symmetric Property of Equality

$b = 34$ feet

You reflect the beam of a spotlight off a mirror lying flat on a stage, as shown. Determine whether $m\angle DBA = m\angle EBC$.



SOLUTION

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle DBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate (Post. 1.4)
$m\angle DBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle EBC$	Add measures of adjacent angles.	Angle Addition Postulate (Post. 1.4)
$m\angle DBA = m\angle EBC$	Both measures are equal to the sum $m\angle 1 + m\angle 2$.	Transitive Property of Equality

1. Six steps of a two-column proof are shown. Copy and complete the proof.

Given T is the midpoint of $\overline{SU}$.

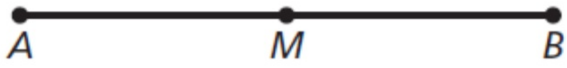
Prove $x = 5$

STATEMENTS	REASONS
1. T is the midpoint of $\overline{SU}$.	1. Given
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. Substitution Property
5. 4x = 20	5. Subtraction Property of Equality
6. $x = 5$	6. Division Property

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one-half AB.

Given M is the midpoint of $\overline{AB}$.

Prove $AB = 2AM$, $AM = \frac{1}{2}AB$

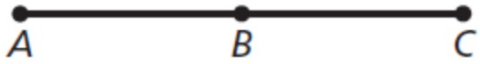


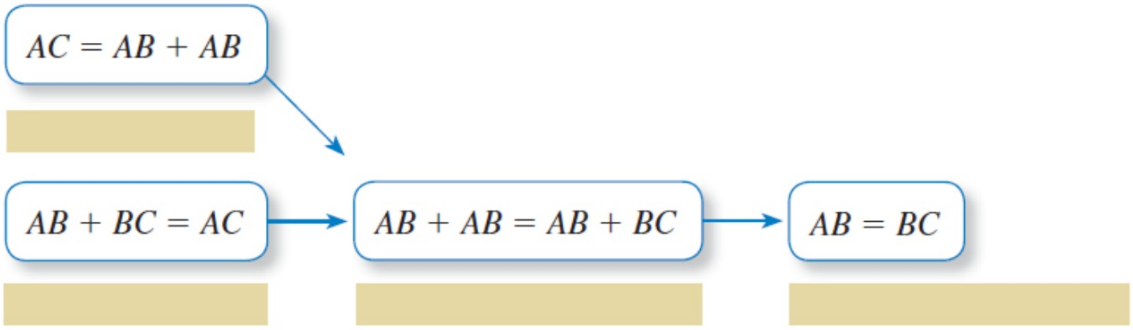
STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of midpoint
3. $AM = MB$	3. Definition of congruent segments
4. $AM + MB = AB$	4. Segment Addition Postulate
5. $AM + AM = AB$	5. Substitution Property of Equality
6. $2AM = AB$	6. Distributive Property
7. $AM = \frac{1}{2}AB$	7. Division Property of Equality

Work with a partner. Match each reason with the correct step in the flowchart.

Given $AC = AB + AB$

Prove $AB = BC$





A. Segment Addition Postulate (Post. 1.2) **B.** Given

C. Transitive Property of Equality **D.** Subtraction Property of Equality

Answers: B, A, C, D